2.80

Solución

Ejercicios Propuestos

Capitulo Uno

1.- Las proposiciones no pueden ser preguntas, exclamaciones, ni expresiones poeticas

a) Si
b) No
c) Si
d) No
e) Si
f) Si
g) Si
h) Si
i) Si
j) Si
K) No
l) Si

2.-
a) Si
b) Si
c) Si
d) Si
e) No

3.-
a) No
b) No
c) No
d) No
e) Si

4.-
a) Si
b) Si
c) No
d) Si
e) Si

5. a) Sí
b) Sí
c) Sí
d) No
e) Sí

6.
I : No
II : Sí
III : Sí
IV : No
d) Correcto

7.
a) Si Elizabeth cumple sus obligaciones, entonces no es verdad que si aprueba el examen, no se va de vacaciones o trabaja.

b) Elizabeth aprueba el examen y no es verdad que decir que Elizabeth trabaja es igual a decir que no cumple con sus obligaciones ; o ; si Elizabeth se va de vacaciones o trabaja, entonces trabaja y no come.

c) Si Elizabeth se va de vacaciones, entonces, Elizabeth cumple sus obligaciones es igual a decir que es trabajadora y.

d) Elizabeth cumple sus obligaciones y aprueba el examen es igual a decir que se va de vacaciones o si trabaja, come.

8.—

a.-) Si como espinaca y la lógica es fácil, es igual a decir me divierto con esta lección.

b.-) Si la lógica es fácil y me divierto con esta lección, entonces como espinaca

c.-) Si no como espinaca, entonces, la lógica es difícil o no me divierto con esta lección

9.—

a	v	b
0	0	0

a) verdadero

10.— $\neg(\underbrace{\underbrace{a \vee b}_{0}}_{1})$ a: 0   b: 0

b) Falso

11.— p: estudio consentemente
q: apruebo el curso nivel cero

Traducción: $p \Rightarrow q$

contrarrecíproca $\neg q \Rightarrow \neg p$ si no apruebo el curso, no estudio consient.

$\therefore q \vee \neg p$ No estudio concientemente o apruebo el curso

b) Falso

12.- a) p: la decisión depende del juicio
q: la decisión depende de la intuición
r: la decisión depende del dinero

Traducción: $(p \vee q) \wedge \neg r$

b) p: iré al estadio
q: iré al cine
r: consigo dinero

Traducción: $r \Rightarrow (p \wedge q)$

c) p: el sol brilla
q: es el día del amor

Traducción: $q \Rightarrow p$

d) p: Juan le agrada el ejercicio
q: Juan no puede resolver el ejercicio

Traducción: $q \Rightarrow \neg p$

13.- a: la información es correcta
b: existe un incremento en los costos de producción
c: el analista tiene un error de apreciación.

Traducción①: $\neg a \Rightarrow (b \vee c)$

Traducción②: $\neg(b \vee c) \Rightarrow a$

14.-

a) p: Quito es capital de Argentina.

q: Buenos Aires es capital de Ecuador.

traducción: $p \vee q \equiv 0$

$0 \quad 0$

$\vee$

0

b) p: 5 es menor que 10

q: 8 no es un número primo.

traducción: $p \wedge q \equiv 1$

$1 \quad 1$

$\vee$

1

c) p: $9 - 16 = (3 \cdot 4)(3+4)$

q: $(-5)(-2) > 0$

Traducción: $p \vee q \equiv 1$

$1 \quad 1$

$\vee$

1

15.-

a) p: $2(3+5) = 16$

q: $5(6+1) = 35$

Traducción: $p \Rightarrow q \equiv 1$

$1 \quad 1$

$\vee$

1

b) $p: (4+5) = 20$

$q: (6+7) = 12$

Traducción : $p \Rightarrow q \equiv 1$

$0 \quad 0$

1

c) $p: (9+5) = 14$

$q: (6+5) = 11$

Traducción : $p \Rightarrow q \equiv 1$

$1 \quad 1$

1

d) $p: 9(4+2) = 54$

$q: 9(9+1) = 14$

Traducción : $p \Rightarrow q \equiv 0$

$1 \quad 0$

0

e) $p: 3(4+5) = 28$

$q: 7(6+5) = 37$

Traducción : $p \Rightarrow q \equiv 1$

$0 \quad 0$

1

16.- p: Carlos llega impuntual

q: Carlos se levanta tarde

Traducción : $q \Rightarrow p$

recíproca : $p \Rightarrow q$

Si Carlos llega impuntual, entonces se levanta tarde

b) Correcto

17.– m: tu eres inteligente
n: tu actuas con prudencia.
p: tu eres un ignorante en la materia.

traducción: $(m \wedge \neg n) \Rightarrow p$

$\therefore \neg(m \wedge \neg n) \vee p$

$\therefore \neg m \vee n \vee p$

$\therefore \neg m \vee (n \vee p)$

$\therefore m \Rightarrow (n \vee p)$ a) Correcto.

18.– p: tengo hambre
q: no tengo tiempo para comer
r: no me siento bien
s: no puedo estudiar

traducción: $(p \wedge q) \Rightarrow (r \wedge s)$

contra recíproca: $\neg(r \wedge s) \Rightarrow \neg(p \wedge q)$

$\therefore (\neg r \vee \neg s) \Rightarrow (\neg p \vee \neg q)$

Si me siento bien o puedo estudiar, entonces no tengo hambre o tengo tiempo para estudiar

e) correcto

19.– p: El pais está bien economicamente
q: yo tengo empleo

traducción: $\neg p \vee q$

$\therefore p \Rightarrow q$ condición necesaria

b) correcto

20.-

p: es función diferenciable.
q: es función continúa

Traducción : $p \Rightarrow q$

a) $p \Rightarrow q$.
b) $q \Rightarrow p$
c). $q \Rightarrow p$
d) $p \Rightarrow q$.
e) NO

21.-

I) p: Compro un traje gris
q: Uso el traje gris.
r: me pagan el traje gris.

traducción : $r \Rightarrow (p \wedge q)$.
recíproca : $(p \wedge q) \Rightarrow r$ a). Correcto
inversa : $\neg r \Rightarrow \neg(p \wedge q)$.
$\neg r \Rightarrow (\neg p \vee \neg q)$ d) Correcto

Contrarecíproca : $\neg(p \wedge q) \Rightarrow \neg r$.
$(\neg p \vee \neg q) \Rightarrow \neg r$. b) Correcto

22.-

a: Hoy es lunes.
b: Obtengo un buen resultado.

Traducción : $a \Rightarrow b$.

b) Falso.

23.-

m: la municipalidad realiza obras.

n: los ciudadanos colaboren en el aseo de las calles

p: Guayaquil mejora su imagen.

Traducción: $(m \vee n) \Rightarrow p$.

b) Falso

24.-

a: Utilizo mis habilidades Matemáticas.

b: resuelvo bien los ejercicios

c: hago un buen examen.

Traducción: $(b \wedge c) \Rightarrow a$

b) Falso.

25.-

m: Mis padres me compran un carro

n: me porto bien.

p: apruebo este curso.

Traducción: $m \Rightarrow (n \wedge p)$.

b) Correcto.

26.- $\neg(p \wedge \neg q \wedge \neg r) \equiv 0$

$(\underbrace{p}_{1} \wedge \underbrace{\neg q}_{1} \wedge \underbrace{\neg r}_{1}) \equiv 1$

p: 1

q: 0

r: 0

$\underbrace{\underbrace{p}_{1} \Rightarrow \underbrace{(\underbrace{q}_{0} \wedge \underbrace{r}_{0})}_{0}}_{0} \equiv 0$

b) Correcto.

27. m: viajo al exterior.

n: apruebo el curso de nivel cero

p: obtengo una beca

traducción: $m \Rightarrow (n \wedge p)$

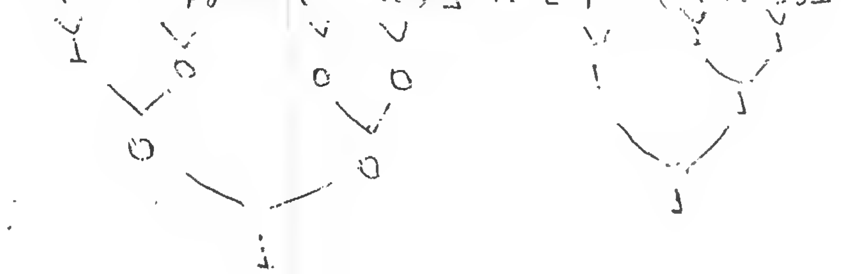
$\therefore \neg(n \wedge p) \Rightarrow \neg m$

$\therefore (n \wedge p) \vee \neg m$

d) Correcto

28.

$$[(p \Rightarrow \neg q) \Rightarrow (r \wedge \neg s)] \wedge [p \wedge (\neg r \wedge s)] \equiv 1$$



p : 1
r : 0
s : 1
q : 1

a) Falso
b) Verdadero
c) Falso
d) Verdadero
e) Falso

29.

a: te gustan las matematicas.

b: te gusta este deber.

a) No gustas de las matematicas ó gustas del deber

b) Gustas de las matematicas y no gustas del deber.

c) Si gustas de las matematicas, gustas del deber.

d). Si no gustas del deber, no gustas de las matemáticas

e) Si gustas o no de las matematicas, gustas del deber.

30.-

p: Necesito un doctor.
q: Necesito un abogado
r: Tengo un accidente
s: estoy enfermo.

Traducción: $(s \Rightarrow p) \wedge (r \Rightarrow q)$.

b) Correcto.

31.-

p: la guerra se detiene.
q: sigo estudiando
r: sigo trabajando

Traducción: $p \Rightarrow (q \vee r)$.

Negación: $\neg [p \vee \neg (q \vee r)]$
: $\neg (\neg p \vee (q \vee r))$
: $\neg (\neg p \vee q \vee r)$
: $p \wedge \neg q \wedge \neg r$.

e) Correcto.

32.-

p: Pedro realizó un paseo en grupo
q: Pedro preparó el mejor informe de la clase
r: Yo encontré a Pedro visitando el CC. San Marino.

Traducción: $r \Rightarrow \neg (p \wedge q)$

$r \Rightarrow (\neg p \vee \neg q)$

b) Correcto.

33.- p : Hoy es domingo.
q : Tengo que estudiar teorías de aprendizaje.
r : Aprobaré el curso.

Traducción : $(p \wedge q) \vee \neg r$.
$\neg r \vee (p \wedge q)$.
$r \Rightarrow (p \wedge q)$.

d) Correcto.

34.- a : Luis llega a tiempo.
b : Luis se levanta temprano.
c : Luis desayuna

Traducción : $(a \wedge c) \Rightarrow b$.
$\neg(a \wedge c) \vee b$.
$\neg a \vee \neg c \vee b$.

d) Correcto

35.-

m : Se realiza una gran fiesta.
n : Salgo bien en esta lección
p : Mis amigos están de acuerdo.

Traducción : $m \Rightarrow (n \wedge p)$
$\therefore \neg m \vee (n \wedge p)$.
$\therefore (n \wedge p) \vee \neg m$.

d) Correcto

36.- p: Estudio historia

q: Estudio geografía

r: estudio matemáticas

Traducción: $(p \vee q) \Rightarrow r$

$\therefore\ \neg(p \vee q) \vee r$

$\therefore\ (\neg p \wedge \neg q) \vee r$

$\therefore\ (\neg p \vee r) \wedge (\neg q \vee r)$

$\therefore\ (p \Rightarrow r) \wedge (q \Rightarrow r)$

a) Correcto.

37.- $\overset{0}{\overbrace{[\overset{1}{\overbrace{(\underset{1}{a} \wedge \underset{1}{\neg b})}} \Rightarrow \underset{0}{\overset{0}{d}}]}} \vee \overset{0}{\overbrace{\neg(\underset{0}{d} \vee \underset{1}{e})}} \equiv 0$

a) $\underset{0}{b} \vee \underset{1}{g} \equiv 1$

b) $\underset{0}{\neg e} \vee \underset{1}{\neg d} \equiv 1$

c) $\underset{0}{d} \vee \underset{1}{g} \equiv 1$

d) $\underset{1}{a} \Rightarrow \underset{0}{d} \equiv 0$

d) Correcto.

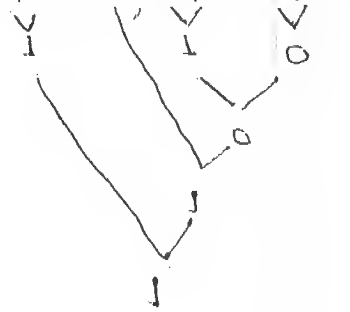
38.-

$\underset{1}{p} \Rightarrow \underset{0}{q} \equiv 0$

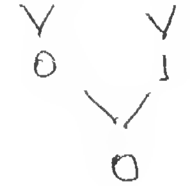
p: 1

q: 0

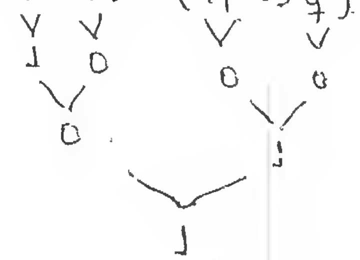
a) $p \vee \neg(\neg q \wedge \neg p)$



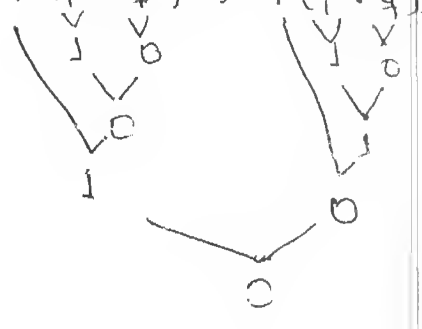
b) $\neg p \wedge \neg q$



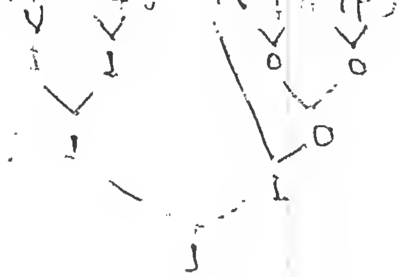
c) $(p \wedge q) \vee (\neg p \Rightarrow q)$



d) $\neg(p \wedge q) \Rightarrow \neg(p \vee q)$



e) $(p \wedge \neg q) \vee \neg(q \wedge \neg p)$



39.- a) p : es un numero divisible para 2

q : es un numero primo.

Traducción : $p \Rightarrow \neg q$

b) p : estudias.

q : aprendes

r : te arrepientes

traducción : $(p \Rightarrow q) \wedge (\neg p \Rightarrow r)$

c). p : x satisface la ecuación

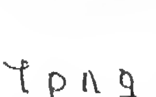
q : el Δ es rectangulo.

r : la longitud de la hipotenusa es 4.

s : No hay manera de calcular el area

Traducción : $(p \Rightarrow (q \wedge r)) \wedge (\neg p \Rightarrow s)$

40.- $\neg p \wedge q \equiv 1$



p : 0

q : 1

a) $p \Rightarrow (\neg q \wedge r) \equiv 1$

b) $q \vee (\neg p \Leftrightarrow r) \equiv 1$

1 1

1

e) $p \vee (q \vee r) \equiv 1$

0 1

1

c) $q \Rightarrow (p \wedge q) \equiv 0$

1 0 1

0

0

d) $\neg p \veebar q \equiv 0$

1 1

0

41.- $p \Rightarrow q \equiv 0$ $p:1$

1 0 $q:0$

a) $p \vee \neg(\neg q \wedge \neg p) \equiv 1$

1 1 0

0

1

1

b) $\neg q \wedge \neg p \equiv 0$

1 0

0

c) $(p \wedge q) \vee (\neg p \Rightarrow q) \equiv 1$

1 0 0 0

0 1

1

d) $\neg(p \wedge q) \Rightarrow \neg(p \vee q) \equiv 0$

1 0 1 0

0 1

1 0

0

e) $(p \wedge \neg q) \vee \neg(q \wedge \neg p) \equiv 1$

1 1 0 0

1 0

1

1

42.- Si $f(p,q,r)$ es tautología, cualquier valor de una de las proposiciones deberá ser 1

b) correcto

43.- Si $f(p,q,r)$ es contradicción, cualquier valor de una de las proposiciones deberá ser 0

b) correcto

44.-

$$\underbrace{f(1,0,1,1)}_{0} \Rightarrow \underbrace{f(0,1,0,0)}_{0}$$

$$1$$

b) correcto

45.-

$$\neg p \Rightarrow (q \vee \neg r)$$

0, 0, 0; 0; 1

No se puede asegurar si es contradicción

b) correcto

46.-

$$[\overbrace{(\neg p \vee q)}^{1} \wedge \overbrace{(\neg r \Rightarrow q)}^{1}] \Rightarrow \overbrace{(p \Rightarrow r)}^{0}$$

0 1 → 1; 1 1 → 1; 1 0 → 0

p: 1
q: 1
r: 0

No es tautología

b) correcto

47.— a) $(\neg q \Rightarrow \neg p) \Rightarrow (\neg p \vee q)$

p: 1
q: 0 Tautología.

b) $(p \vee q) \Rightarrow (\neg p \Rightarrow q)$

p: 0
q: 0 Tautología

c) $[(p \Rightarrow q) \wedge p] \Rightarrow q$

p: 1
q: 0 Tautología

d) $(p \Rightarrow q) \Rightarrow (q \Rightarrow p)$

q: 1 Falacia
p: 0

e) $[(p \wedge q) \wedge r] \Rightarrow [(p \vee r) \wedge (q \vee r)]$ Tautología

48.— a) $\neg(\neg p \wedge \neg q)$

$\therefore p \vee q$ Falacia

b) $\neg(\neg p \wedge q)$

$\therefore p \vee \neg q$ Falacia

c) $p \vee (p \wedge q)$ Falacia

d) $[p \wedge (p \Rightarrow q)] \Rightarrow q$ Tautología

49.- $[p \wedge (p \Rightarrow q)] \Rightarrow M$

a) correcto $p \wedge q$

50.- a) $(p \vee q) \Rightarrow (\neg p \Rightarrow q)$ $p: 0$ $q: 0$ tautología

b) $[(p \Rightarrow r) \wedge (q \Rightarrow r)] \Rightarrow [(p \vee q) \Rightarrow r]$ $p: 1$ $q: 0$ $r: 0$ Tautología

c) $[(p \vee q) \wedge \neg p] \Rightarrow q$ tautología $q: 0$ $p: 0$

d) $[(\neg q \Rightarrow \neg p)] \Rightarrow \neg q$

Falacia

e) $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$ $p: 1$ $q: 0$ $r: 0$ Tautología

1.— a) Falso

b) Falso

c) Falso

d) Verdadero

e) Falso

2.— a) $[(p \Rightarrow q) \wedge (r \Rightarrow s)] \Rightarrow [(p \wedge r) \Rightarrow (q \wedge s)]$ Tautología

b) $[p \wedge (p \Rightarrow q)] \Rightarrow q$ Tautología

c) $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \wedge r)$ No es tautología.

3.— p: estoy satisfecho

q: me dieron el aumento de sueldo

Traducción: $\neg q \Rightarrow \neg p$

$\therefore p \Rightarrow q$

$\therefore \neg p \vee q$

a) $q \Rightarrow p$ Falso

b) $\neg q \Rightarrow \neg p$

c) $p \Rightarrow q$

d) $q \vee \neg p$

e) $\neg p \vee q$

54.— a) $\neg(p \vee q) \Rightarrow (\neg p \vee \neg q)$ Tautología

b) $(\neg p \wedge \neg q) \Rightarrow \neg(p \vee q)$ Tautología

p: 0
q: 0

c) $\neg(\neg p \wedge \neg q) \vee \neg q$ Tautología

d) $(\neg p \wedge [\neg p \vee q]) \vee p$ Tautología

e) $[p \wedge (p \vee q)] \Rightarrow q$ Falacia

55.— $(p \Rightarrow q) \wedge \neg(q \Rightarrow r)$

(p	⇒	q)	∧	¬	(q	⇒	r)
0	1	0	0	0	0	1	0
0	1	0	0	0	0	1	1
0	1	1	1	1	1	0	0
0	1	1	0	0	1	1	1
1	0	0	0	0	0	1	0
1	0	0	0	0	0	1	1
1	1	1	1	1	1	0	0
1	1	1	0	0	1	1	1

Contingencia

1 — $[(p \wedge q) \Rightarrow r] \Rightarrow [p \Rightarrow (q \Rightarrow r)]$

$p: 1$
$q: 1$
$r: 0$

Tautologia

2 — a) $(r \wedge s) \vee \neg s$

r	∧	s	∨	¬s
0	0	0	1	1
0	0	1	0	0
1	0	0	1	1
1	1	1	1	0

c) $\neg p \Rightarrow (p \wedge q)$

¬p	⇒	p	∧	q
1	0	0	0	0
1	0	0	0	1
0	1	1	0	0
0	1	1	1	1

b) $(p \Rightarrow q) \Rightarrow (q \Rightarrow \neg p)$

p	⇒	q	⇒	q	⇒	¬p
0	1	0	1	0	1	1
0	1	1	1	1	1	1
1	0	0	1	0	1	0
1	1	1	0	1	0	0

d) $(p \wedge q) \wedge (p \Rightarrow \neg q)$

p	∧	q	∧	p	⇒	¬q
0	0	0	0	0	1	1
0	0	1	0	0	1	0
1	0	0	0	1	1	1
1	1	1	0	1	0	0

e) $(p \vee q) \Rightarrow [p \vee (\neg p \wedge q)]$

p	∨	q	⇒	p	∨	¬p	∧	q
0	0	0	1	0	0	1	0	0
0	1	1	1	0	1	1	1	1
1	1	0	1	1	1	0	0	0
1	1	1	1	1	1	0	0	1

58. — i) $\neg a \wedge (a \Leftrightarrow b)$

¬a	∧	a	⇔	b
1	1	0	1	0
1	0	0	0	1
0	0	1	0	0
0	0	1	1	1

Contingencia

II) $(p \wedge q) \Rightarrow (p \Rightarrow \neg q)$

p	$\wedge$	q	$\Rightarrow$	p	$\Rightarrow$	$\neg q$
0	0	0	1	0	1	1
0	0	1	1	0	1	0
1	0	0	1	1	1	1
1	1	1	0	1	0	0

Contingencia

III) $(a \wedge b) \wedge \neg c$

a	$\wedge$	b	$\wedge$	$\neg c$
0	0	0	0	1
0	0	0	0	0
0	0	1	0	1
0	0	1	0	0
1	0	0	0	1
1	0	0	0	0
1	1	1	1	1
1	1	1	0	0

Contingencia

IV. $[(p \Rightarrow q) \wedge \neg b] \Rightarrow \neg b$

p	$\Rightarrow$	q	$\wedge$	$\neg b$	$\Rightarrow$	$\neg b$
0	1	0	0	0	1	0
0	1	0	1	1	1	1
0	1	1	0	0	1	0
0	1	1	1	1	1	1
1	0	0	0	0	1	0
1	0	0	0	1	1	1
1	1	1	0	0	1	0
1	1	1	1	1	1	1

Tautología

59.— p: me comporto bien en mi hogar

q: soy un buen hijo

traducción: $p \Rightarrow q$

negación: $\neg(p \Rightarrow q)$

$\therefore \neg(\neg p \vee q)$

$\therefore p \wedge \neg q$ a) verdadero

60.– a) correcto

61.– a) correcto

62.– a) verdadero

63.–

a) $a \wedge (b \vee c) \equiv (a \wedge b) \vee (a \wedge c)$

a	$\wedge$	$(b$	$\vee$	$c)$
0	0	0	0	0
0	0	0	1	1
0	0	1	1	0
0	0	1	1	1
1	0	0	0	0
1	1	0	1	1
1	1	1	1	0
1	1	1	1	1

$(a$	$\wedge$	$b)$	$\vee$	$(a$	$\wedge$	$c)$
0	0	0	0	0	0	0
0	0	0	0	0	0	1
0	0	1	0	0	0	0
0	0	1	0	0	0	1
1	0	0	0	1	0	0
1	0	0	1	1	1	1
1	1	1	1	1	0	0
1	1	1	1	1	1	1

son iguales

b) $\neg(a \wedge b) \equiv \neg a \vee \neg b$

$\neg$	$(a$	$\wedge$	$b)$
1	0	0	0
1	0	0	1
1	1	0	0
0	1	1	1

$\neg a$	$\vee$	$\neg b$
1	1	1
1	1	0
0	1	1
0	0	0

son iguales

1.– p: se es inteligente
q: se es estudioso
r: se es aplicado

traducción: $(p \vee q) \Rightarrow r$

b) $(p \Rightarrow r) \wedge (q \Rightarrow r)$
$\therefore (\neg p \vee r) \wedge (\neg q \vee r)$
$\therefore r \vee (\neg p \wedge \neg q)$
$\therefore r \vee \neg(p \vee q)$
$\therefore (p \vee q) \Rightarrow r$

65.— a) correcto

66.— p: te gustan las matemáticas
q: eres hábil para la geometría

traducción: $\overbrace{(\underbrace{\underset{1}{p} \Rightarrow \underset{1}{q}}_{1})}^{1} \Rightarrow \overbrace{\neg p}^{0}$

b) Falso

67.— p: Trabajo arduamente
q: gano un buen sueldo

traducción: $[\overbrace{(\underbrace{\underset{1}{p} \Rightarrow \underset{0}{q}}_{0})}^{1} \wedge \overbrace{\neg q}^{1}] \Rightarrow \overbrace{\neg p}^{0}$

P: 1
q: 0

a) correcto

68.— p: lo intento con ahínco
q: tengo talento
r: me convierto en músico
s: seré feliz

a) $\{\overbrace{[\underbrace{(p \wedge q)}_{0} \Rightarrow \underset{0}{r}]}^{1} \wedge \overbrace{[\underbrace{\underset{0}{r} \Rightarrow \underset{0}{s}}_{1}]}^{1}\} \Rightarrow \overset{0}{s}$ FALSO

b) $\{\overbrace{[\underbrace{\underbrace{(\underset{0}{p} \wedge q)}_{0} \Rightarrow \underset{1}{r}}_{1}]}^{1} \wedge \overbrace{[\underbrace{\underset{1}{r} \Rightarrow \underset{1}{s}}_{1}]}^{1}\} \Rightarrow \overbrace{(\underset{1}{r} \Rightarrow \underset{0}{p})}^{0}$ FALSO

$\{[(p \wedge q) \Rightarrow r] \wedge [r \Rightarrow s]\} \Rightarrow \neg r$

r: 1 Falso
s: 1

$\{[(p \wedge q) \Rightarrow r] \wedge [r \Rightarrow s]\} \Rightarrow \neg q$

Falso
q: 1

$\{[(p \wedge q) \Rightarrow r] \wedge [r \Rightarrow s]\} \Rightarrow (\neg s \Rightarrow (\neg p \vee \neg q))$

CORRECTO

s: 0
p: 1
q: 1

p: apruebo todas las materias
q: me voy de vacaciones por un mes
r: compraré muchos recuerdos

$\{[p \Rightarrow q] \wedge [q \wedge r]\} \Rightarrow \neg q$ Falso

q: 1

$\{[p \Rightarrow q] \wedge [q \wedge r]\} \Rightarrow (p \wedge r)$

Falso

$\{(p \Rightarrow q) \wedge (q \wedge r)\} \Rightarrow (\neg p \wedge \neg q)$ Falso

$\{(p \Rightarrow q) \wedge (q \wedge r)\} \Rightarrow q$

correcto

70.— p: el gobierno realiza las gestiones apropiadas
q: el concurso de miss universo se realiza en nuestro país
r: el turismo se reactiva en el país.

a) $[(\neg p \Rightarrow \neg q) \wedge r \wedge q] \Rightarrow \neg q$ q:1 r:1 Falso

b) $[(\neg p \Rightarrow \neg q) \wedge r \wedge q] \Rightarrow \neg r$ Falso

c) $[(\neg p \Rightarrow \neg q) \wedge r \wedge q] \Rightarrow (r \wedge \neg p)$ p:1 Falso

d) $[(\neg p \Rightarrow \neg q) \wedge r \wedge q] \Rightarrow \neg p$ p:1 Falso

e) $[(\neg p \Rightarrow \neg q) \wedge r \wedge q] \Rightarrow (p \Rightarrow r)$ Correcto

71.— a) $[p \wedge (p \Rightarrow q)] \Rightarrow \neg q$ q:1 Falso

b) $[p \wedge (p \Rightarrow q)] \Rightarrow (\neg p \wedge q)$ Falso

c) $[p \wedge (p \Rightarrow q)] \Rightarrow (\neg p \wedge \neg q)$ Falso

d) $[p \wedge (p \Rightarrow q)] \Rightarrow (p \wedge q)$ Correcto

12.— p: estudio

q: apruebo nivel cero

r: viajo a Galapago

a) $\{[p \Rightarrow q] \wedge [q \wedge r]\} \Rightarrow \neg q$ $q: 1$ Falso

b) $\{[p \Rightarrow q] \wedge [q \wedge r]\} \Rightarrow (\neg p \wedge \neg q]$ Falso

c) $\{[p \Rightarrow q] \wedge [q \wedge r]\} \Rightarrow (p \wedge r)$ Falso

d) $\{[p \Rightarrow q] \wedge [q \wedge r]\} \Rightarrow q$ Correcto

–a) $[(\neg p \Rightarrow q) \wedge (p \wedge \neg r) \wedge (\neg p \Rightarrow r)] \Rightarrow (p \wedge q)$ $p: 1$ $r: 0$ Falso

b) $[(\neg p \Rightarrow q) \wedge (p \wedge \neg r) \wedge (\neg p \Rightarrow r)] \Rightarrow (p \Rightarrow q)$ $p: 1$ $q: 0$ $r: 0$ Falso

c) $[(\neg p \Rightarrow q) \wedge (p \wedge \neg r) \wedge (\neg p \Rightarrow r)] \Rightarrow (\neg p \Rightarrow q)$ $p: 0$ $r: 0$

Correcto

74.— p: se concluye con éxito la construcción del nuevo parque en el barrio del centenario

q: se coopera para el embellecimiento de la urbe.

r: se incrementa la captación de turistas

a) $\{(p \Rightarrow r) \wedge (q \wedge r)\} \Rightarrow \neg q$

$\underbrace{(p \Rightarrow \overset{}{r})}_{1} \wedge \underbrace{(q \wedge r)}_{1} \qquad \neg q = 0 \qquad q:1$ Falso

b) $\{\overset{1}{(p \Rightarrow r)} \wedge \overset{1}{(q \wedge r)}\} \Rightarrow q$

$\underbrace{(\underset{0}{q} \wedge \underset{1}{r})}_{0} \qquad \underset{0}{q}$ correcto

75.— $p: \alpha = \beta$ $\qquad u: \beta \neq 90$

$q: \beta = 45$

$r: \alpha = 90$

$s: \theta = 90$

$t: \beta = 90$

a) $[\overset{1}{(p \Rightarrow q)} \wedge \overset{1}{(q \Rightarrow r)} \wedge \overset{1}{(t \vee u)} \wedge \neg s] \Rightarrow p$ Falso

$\underbrace{(\underset{0}{p} \Rightarrow \underset{0}{q})}_{1} \wedge \underbrace{(\underset{0}{q} \Rightarrow \underset{0}{r})}_{1} \wedge \underbrace{(\underset{0}{t} \vee \underset{1}{u})}_{1} \wedge \underset{1}{\neg s} \Rightarrow \underset{0}{p}$

b) $[(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge (t \vee u) \wedge \neg s] \Rightarrow \neg p$

$\underbrace{(\underset{1}{p} \Rightarrow \underset{1}{q})}_{1} \wedge \underbrace{(q \Rightarrow r)}_{1} \wedge \underbrace{(t \vee u)}_{1} \wedge \underset{1}{\neg s} \Rightarrow \underset{0}{\neg p} \qquad p:1$ Falso

c) $[\underbrace{(p \Rightarrow q)}_{1} \wedge \underbrace{(q \Rightarrow r)}_{1} \wedge \underbrace{(t \vee u)}_{1} \wedge \underset{1}{\neg s}] \Rightarrow \underset{0}{s}$ Falso

d) $[(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge (t \vee u) \wedge \neg s] \Rightarrow q$ Falso

$\underbrace{(\underset{0}{p} \Rightarrow \underset{0}{q})}_{1} \wedge \underbrace{(\underset{0}{q} \Rightarrow \underset{1}{r})}_{1} \wedge \underbrace{(\underset{0}{t} \vee \underset{1}{u})}_{1} \wedge \underset{1}{\neg s} \Rightarrow \underset{0}{q}$

e) $[(p\Rightarrow q)\wedge(q\Rightarrow r)\wedge(t\veebar u)\wedge\neg s]\Rightarrow \cancel{q}$

correcto

— p: el reloj está andando

q: juan llegó antes de las 10

r: juan vió partir el coche de Andrés

s: Andrés dice la verdad

t: Andrés estaba en el edificio en el momento del crimen.

$[(p\Rightarrow(q\wedge r))\wedge(\neg s\Rightarrow\neg r)\wedge(s\vee t)\wedge p]\Rightarrow\neg q$ Falso

q: 1

p: 1

r: 1

s: 1

$[(p\Rightarrow(q\wedge r))\wedge(\neg s\Rightarrow\neg r)\wedge(s\vee t)\wedge p]\Rightarrow t$ Falso

$[(p\Rightarrow(q\wedge r))\wedge(\neg s\Rightarrow\neg r)\wedge(s\vee t)\wedge p]\Rightarrow(\neg r\vee t)$ Correcto

— p: estudio

q: soy un genio

r: apruebo nivel @

s: puedo Tomar nivel 100

$\{[(p\vee q)\Rightarrow r]\wedge(r\Rightarrow s)\}\Rightarrow(\neg s\Rightarrow\neg q)$

Válido

78.— p: el congreso asigna fondos

q: el proyecto será ejecutado

r: hay consenso ~~entre~~ los diputados

$$\{[(p \Rightarrow \underset{1}{q})] \wedge [\underset{0}{p} \Rightarrow \underset{0}{r}] \wedge \underset{1}{\neg r}\} \Rightarrow \underset{0}{\neg q}$$

1 1

q: 1

r: 0

p: 0

Falso

79.— p: esta ley será aprobada en esta sesión del congreso

q: la ley es apoyada por la mayoría legislativa

r: el presidente se opone a ella

s: la ley será pospuesta en deliberaciones del congreso

$$\{\overset{1}{(\underset{0}{p} \Rightarrow \underset{0}{q})} \wedge \overset{1}{(\underset{0}{q} \vee \underset{0}{r})} \wedge \overset{1}{(\underset{0}{r} \Rightarrow \underset{0}{s})}\} \Rightarrow \overset{0}{(\underset{0}{p} \vee \underset{0}{s})}$$

1 1 1

Falso

80.— p: la dolarización es difícil

q: la dolarización no les gusta a muchas personas

r: las medidas económicas son viables

$$\{\overset{1}{(p \wedge \underset{0}{q})} \wedge (\underset{0}{r} \Rightarrow \neg p)\} \Rightarrow \overset{0}{(\underset{1}{\neg r} \Rightarrow \underset{0}{q})}$$

0

Válido

(81) $p \Rightarrow q \equiv 1 \quad \therefore \quad p \neq 1 \wedge q \neq 0$

esta condición al mismo tiempo

$\neg p \equiv 1 \qquad q \equiv 0$

* Tenemos: $0 \Rightarrow 0 \equiv 1$

~~a)~~ Verdadero b) Falso

(82) a) $(p \Leftrightarrow q) \equiv (p \vee q) \Rightarrow (p \wedge q)$

* $(p \vee q) \Rightarrow (p \wedge q)$ · Tautología Util

$\neg(p \vee q) \vee (p \wedge q)$ · De Morgan

$(\neg p \wedge \neg q) \vee (p \wedge q)$ - Distributiva

$[(\neg p \wedge \neg q) \vee p] \wedge [(\neg p \wedge \neg q) \vee q]$

$[\underbrace{(\neg p \vee p)}_{**} \wedge (\neg q \vee p)] \wedge [(\neg p \vee q) \wedge \underbrace{(\neg q \vee q)}_{**}]$

En ** Ley del Tercero o Medio Excluido

$[1 \wedge (\neg q \vee p)] \wedge [(\neg p \vee q) \wedge 1]$ - Identidad

$(\neg q \vee p) \wedge (\neg p \vee q)$ - Tautología Util

$(q \Rightarrow p) \wedge (p \Rightarrow q)$ - Conmutativa

$(p \Rightarrow q) \wedge (q \Rightarrow p)$ - Equivalencia

$\therefore p \Leftrightarrow q$ - L. Q. Q. D.

b) $\neg(p \vee q) \vee (\neg p \wedge q) \equiv \neg p$

$\neg(p \vee q) \vee (\neg p \wedge q)$ - De Morgan

$(\neg p \wedge \neg q) \vee (\neg p \wedge q)$ · Distributiva

$\neg p \wedge (\neg q \vee q)$ - Tercero o Medio excluido

$\neg p \wedge 1$ - Identidad

$\neg p$ - L. Q. Q. D.

(83) Utilizar el método de reducción al absurdo para demostrar las siguientes proposiciones:

a) $[(\underset{1}{p} \Rightarrow \underset{0}{q}) \wedge p] \Rightarrow q$ $p \equiv 1$, $q \equiv 0$

$$\underbrace{\underbrace{\underbrace{(p \Rightarrow q)}_{1} \wedge \underbrace{p}_{1}}_{1}}_{1} \Rightarrow \underbrace{q}_{0}$$

$[(1 \Rightarrow 0) \wedge 1] \Rightarrow 0$

$(0 \wedge 1) \Rightarrow 0$ $0 \Rightarrow 0$ $\equiv 1$

b) $[(\underset{0}{p} \vee \underset{0}{q}) \wedge \neg q] \Rightarrow p$ $\neg q \equiv 1$ $q \equiv 0$, $p \equiv 0$

$$\underbrace{\underbrace{\underbrace{(p \vee q)}_{1} \wedge \underbrace{\neg q}_{1}}_{1}}_{1} \Rightarrow \underbrace{p}_{0}$$

$[(0 \vee 0) \wedge 1] \Rightarrow 0$

$(0 \wedge 1) \Rightarrow 0$ ▲ ▲ Contradicción $\equiv$ Falacia

$0 \Rightarrow 0$ $\equiv 1$

c) $\underset{0}{p} \Rightarrow (\underset{0}{p} \vee \underset{0}{q})$ $p \equiv 0$, $q \equiv 0$

$$\underbrace{p}_{1} \Rightarrow \underbrace{(p \vee q)}_{0}$$

$0 \Rightarrow (0 \vee 0)$

$0 \Rightarrow 0$ $\equiv 1$

* En los literales a, b y c todas han resultado ser tautología.

** Este método indica si una forma proposicional es tautológica o no, por medio de determinados valores para las proposiciones presentes.

*** En el caso de no ser una tautología, se deberá resolver por medio de las tablas de verdad para determinar si la forma proposicional es ▲falacia▲ o contingencia

CONJUNTOS

84.-

e) $x / x \neq x$ → no existen elementos que cumplan esta condición, por lo tanto la solución es vacío

85.-

a) Falso

b) Verdadero

c) Falso

d) Verdadero

e) Verdadero

86.-

a) $5 = \{5\}$ → subconjunto FALSO

b) $\{\} \in \phi$ FALSO

c) $1 \in \{\{1,4\}, \{2,4\}\} \Rightarrow 1 \in \{1,4\}$ FALSO

d) $\{4, 8, 2^3, 3\} = \{(-2)^2, 8, 3\}$ VERDADERO

e) $\underbrace{\{2, 4\}}_{\text{elementos}} = \{\underbrace{\{2\}, \{4\}}_{\text{subconjuntos}}\}$ FALSO

87.- $A = \{a, \{b\}, c, \{d, e\}\}$,

$B = \{b, c\}$

a) Verdadero

b) Falso ; $b \notin A$

c) Falso

d) Verdadero

e) Falso ; $b \in B$

88.-

a) $A = \{a, i, o, u\}$

b) $B = \{a, e, i, o\}$

c) $C = \{a, e, o\}$

d) $D = \{a, e, i, o, u\}$

I) Verdadero

II) Falso

III Falso

89.- a) Finito ; tiene inicio y fin

b) infinito ; tiene inicio, pero no fin

c) finito

d) unitario ; es el $\mathbb{Z}$

CUANTIFICADORES

10.- $Re = \{1, 2, 3, 4, 5\}$

a) FALSO; no hay elementos

b) FALSO; 2, 3, 4, 5 no cumplen la condición

c) FALSO; 1 no cumple la condición

d) VERDADERO

e) FALSO; 2, 4, 5 no cumplen la condición.

11.- a) Todo vegetariano come zanahorias

b) Algunos vegetarianos comen zanahorias

c) No todo vegetariano come zanahorias

d) $\forall x\,[p(x) \vee \neg q(x)]$

$\forall x\,[\neg q(x) \vee p(x)]$

$\forall x\,[q(x) \Rightarrow p(x)]$ ∴ todo vegetariano come zanahorias

e) $\forall x\,[\neg p(x) \vee \neg q(x)]$

∴ $\forall x\,[p(x) \Rightarrow \neg q(x)]$ ∴ todos los que comen zanahorias no son vegetarianos

92. a) $A = \{1, 2, 3, 4\}$

$P(A) = \{\{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}$
$\{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, A, \phi\}$

$N\,P(A) = 2^4 = 2 \times 2 \times 2 \times 2 = 16$

b) $B = \{\square, \circ, \Delta\}$

$P(B) = \{\{\square\}, \{\circ\}, \{\Delta\}, \{\square, \circ\}, \{\square, \Delta\}, \{\circ, \Delta\}, B, \phi\}$

$N\,P(B) = 2^3 = 2 \times 2 \times 2 = 8$

c) $C = \{\phi, \{\phi\}\}$

$P(C) = \{\{\phi\}, \{\{\phi\}\}, C, \phi\}$

$N\,P(C) = 2^{\cancel{4}} = 2 \times 2 = 4$

93. $A = \{a, \{b\}\}$

a) $\{\emptyset\} \subseteq A$; falso.
b) $a \in A$; falso
c) $\{b\} \in A$; falso.
d) $N(P(P(A))) = 16$; falso
e) $\{\{b\}\} \in P(A)$; Verdadero.

94.-

a) $A \subseteq B$; Todo elemento de A está en B

VERDADERO

b) $A \subseteq B \therefore A = B \therefore \underbrace{\underbrace{(A \subseteq B)}_{1} \wedge \neg \underbrace{(B \subseteq A)}_{0}}_{0}$ FALSO

c) $\underbrace{\underbrace{(A \subset B)}_{1} \Rightarrow \underbrace{[(A \subseteq B)}_{0} \wedge \underbrace{(B \subseteq A)}_{0}}_{0}$ FALSO

d) $\underbrace{\underbrace{(x \in \phi)}_{1} \Rightarrow \underbrace{(x \notin A)}_{1}}_{1}$; VERDADERO

e) $\underbrace{\underbrace{(x \in \phi)}_{1} \Rightarrow \underbrace{(x \in A)}_{1}}_{1}$; VERDADERO

f) $(A \subseteq B) \Leftrightarrow \underbrace{[\forall x [\underbrace{(x \in A)}_{1} \Rightarrow \underbrace{(x \in B)}_{1}]}_{1} \wedge \underbrace{\exists x [\underbrace{(x \in B)}_{1} \wedge \underbrace{(x \in A)}_{0}]]}_{0}$ FALSO

g) $\underbrace{\underbrace{(A = B)}_{1} \Rightarrow \underbrace{[(A \subset B) \wedge (B \subset A)}_{0}}_{0}$

FALSO

95- a) A subconjunto propio de B, es equivalente a: si x elemento de A entonces x elemento de B

b) Si A subconjunto propio de B, entonces A subconjunto propio de B y no es verdad que B sea subconjunto propio de

c) Si A subconjunto de B, entonces, A subconjunto propio de B y B subconjunto propio de A.

d) Si x elemento de vacío, entonces x no elemento de A

e) Si x elemento de vacío, entonces x elemento de A

f) A subconjunto propio de B es equivalente a para todo elemento de A, entonces x elemento de B y para algunos x elemento de B, entonces x elemto de A.

g) Si $A=B$, entonces: A subconjunto de B $\wedge$ B subconjunto

9.6.- $B = \{*, \alpha\}$

a) $N(P(B)) = 2^2$
$\Rightarrow 4$
$N(P(P(B))) = 2^4$ FALSO
$\Rightarrow 16$

b) $* \in P(B)$ Falso, $\{*\} \in P(B)$

c) Verdadero

d) $\{\{*, \alpha\}\} \in P(B)$

e) Falso

97.- I) $p(x)$: x es número

$q(x)$: x es impar

$\forall x\,[\,p(x) \Rightarrow q(x)]$ FALSO

II)

$\exists x\,(x \in \mathbb{Z} \,/\, 3x^2 - 5 = 0)$

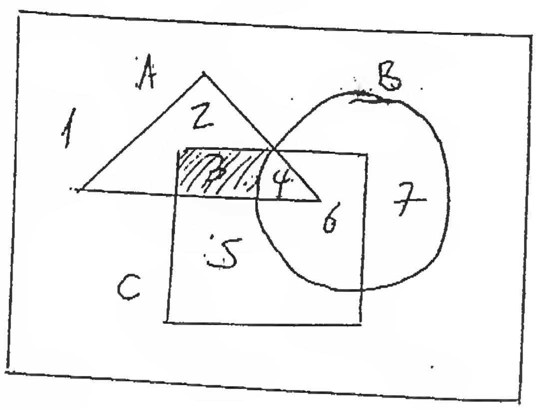
III) $\exists x\,(x \in \mathbb{Z} \,/\, x+1 < 0) \Rightarrow \forall x\,(x \in \mathbb{N} \Rightarrow x \in \mathbb{Z})$

98.- $A = \{\phi, \{\phi\}\}$

$P(A) = \{\{\phi\}, \{\{\phi\}\}, \{A\}\}$

$A \cap P(A) = \{\phi\}$ VERDADERO

99.-



$A = \{2, 3, 4\}$

$C = \{3, 4, 5, 6\}$

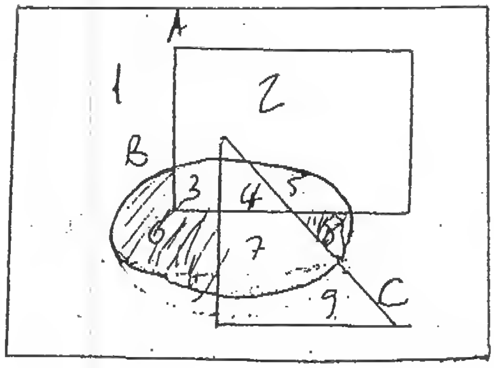
$B = \{4, 6, 7\}$

$A - B = \{2, 3\}$

d) $(A-B) \cap C = \{3\}$

CORRECTO

100.–



$A = \{2, 3, 4, 5\}$

$B = \{3, 4, 5, 6, 7, 8\}$

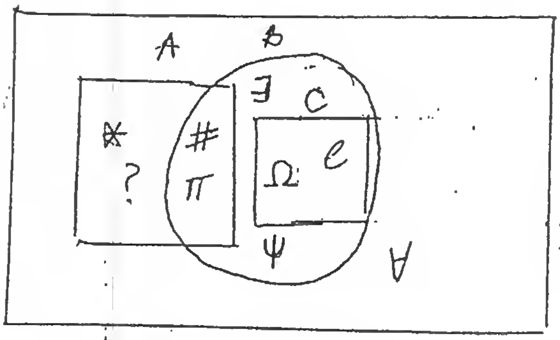
$C = \{4, 7, 9\}$

$A \cup C = \{2, 3, 4, 5, 7, 9\}$

$B - (A \cup C) = \{6, 8\}$

VERDADERO

101.–



$A = \{*, ?, \#, \pi\}$

$B = \{\#, \pi, \exists, \psi, \Omega, e\}$

$C = \{\Omega, e\}$

102.– $Re = \{*, !, \#, \$, \%, \&, ?\}$

$A = \{*, !, \#, \$\}$

$B = \{!, \%, \&, ?\}$

$C = \{\%, \&, ?\}$

$[(A-B)^c \cup C]^c$

$(A-B) = \{*, \#, \$\}$

$(A-B)^c = \{!, \%, \&, ?\}$

$(A-B)^c \cup C = \{!, \%, \&, ?\}$

$[(A-B)^c \cup C]^c = \{*, \#, \$\}$ e) correcto

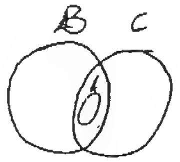
03.- $S = \{\{3\}, \{1,4\}\}$

$P(S) = \{\{\{3\}\}; \{\{1,4\}\}; \{S\}\}$

b) correcto

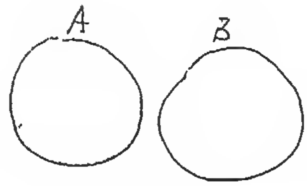
4.-

a)



VERDADERO

b)



NO HAY ELEMENTOS EN COMUN

VERDADERO

c)

$N(A) = 2$

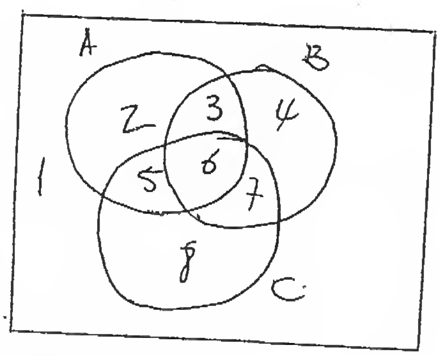
$N(P(A)) = 2^2$

$= \underline{\underline{4}}$

FALSO

d)

$A-(B \cup C)^c = (A-B^c) \cup (A-C^c)$



$A = \{2, 3, 5, 6\}$

$B = \{3, 4, 6, 7\}$

$C = \{5, 6, 7, 8\}$

$(B \cup C) = \{3, 4, 5, 6, 7, 8\}$

$B^c = \{1, 2, 5, 8\}$

$C^c = \{1, 2, 3, 4\}$

$(B \cup C)^c = \{1, 2\}$

$(A-B^c) = \{3, 6\}$

$A-C^c = \{5, 6\}$

$A-(B \cup C)^c = \{3, 5, 6\}$

$\cup$

$\{3, 5, 6\}$ CORRECTO

e) $N(A) = 4$

$N(B) = 3$

$N(A \cap B) = 2$

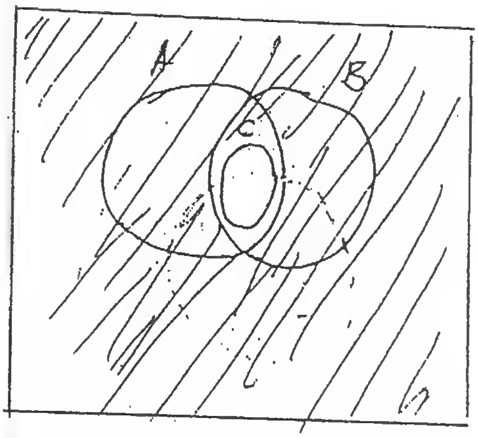
$N(A \cup B) = 4+3-2$

$\Rightarrow \underline{\underline{5}}$

$N(P(A \cup B)) = 2^5$ FALSO

$\Rightarrow \underline{\underline{32}}$

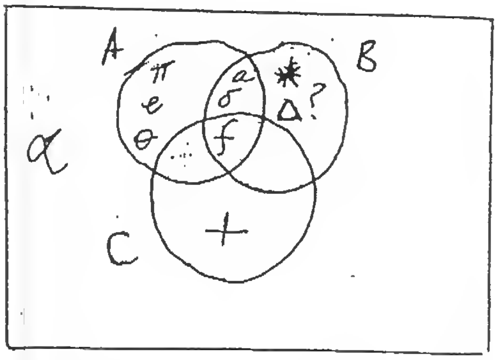
f)



FALSO

$C^c \cap (A \cup B) \neq \emptyset$

105.-



a) $A = \{\pi, e, \theta, a, \delta, f\}$

$B = \{a, \delta, f, *, \Delta, ?\}$

$C = \{f, +\}$

b) $N(C) \doteq N(B) - N(A)$

$2 = 6-6$

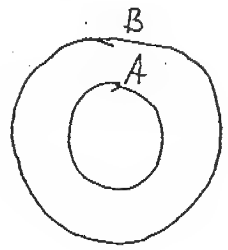
$2 \neq 0$ FALSO

$A \cup B = \{\theta, \pi, e, *, ?, \delta\}$

FALSO; faltan: Δ, a, f

6.—

$$A \cup B = B$$



a) $A \cap B = A$; VERDADERO

b) $A - B = A$; VERDADERO $\therefore A - B = \phi$

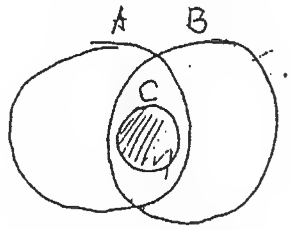
c) $B - A = A$; FALSO

d) $B^c = A$; FALSO

e) $B \subseteq A$; FALSO $\therefore A \subseteq B$

7.—

$$C \subset (A \cap B)$$



a) $(A \cup C) \subset (B \cap C)$; FALSO

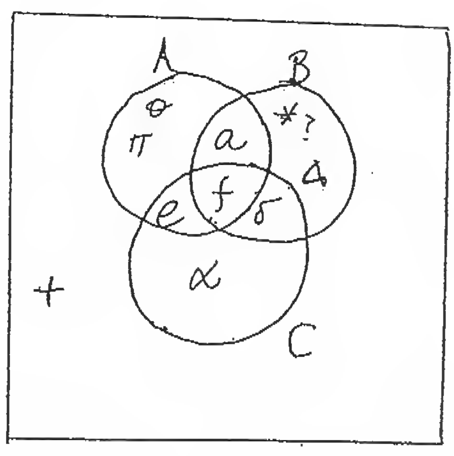
b) $A - B = \phi$; FALSO

c) $(A - B) \subseteq C$; FALSO

d) $(A - B) \cup (B - A) = C$; FALSO

e) $(C - A) = \phi$; VERDADERO

8.—



a) $A \cup B = \{\theta, \pi, e, *, ?, \delta, \Delta, f, a\}$; VERDADERO

b) $(B \cap C) - (A \cap B \cap C) = \emptyset$; FALSO ; RESPUESTA δ.

109.— $A = \{\phi, 1, \{1\}, \{2\}, \{\phi\}, \{1,2\}\}$

a) $\underbrace{(1 \cup A)}_{0} \veebar \underbrace{(\{1\} \in A)}_{1} \equiv 1$

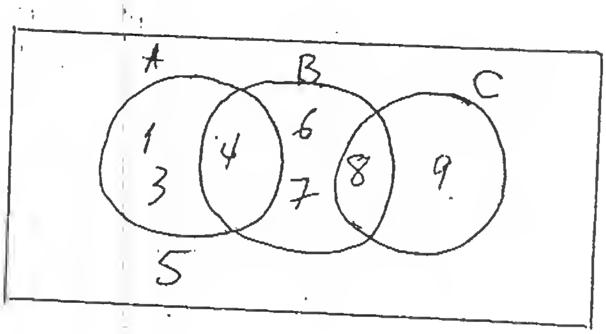
b) $\underbrace{(\phi \subseteq A)}_{1} \wedge \underbrace{(\phi \in A)}_{1} \equiv 1$

c) $\underbrace{(1 \in A)}_{1} \Rightarrow \underbrace{(2 \in A)}_{0} \equiv 0$

d) $\underbrace{\underbrace{(\{1,2\}}_{1} \in A \vee \underbrace{(\phi \subseteq A)}_{1})}_{1} \equiv 1$

e) $\underbrace{(x \in A)}_{1} \Rightarrow \underbrace{(2 \in A)}_{0} \equiv 0$

110.—

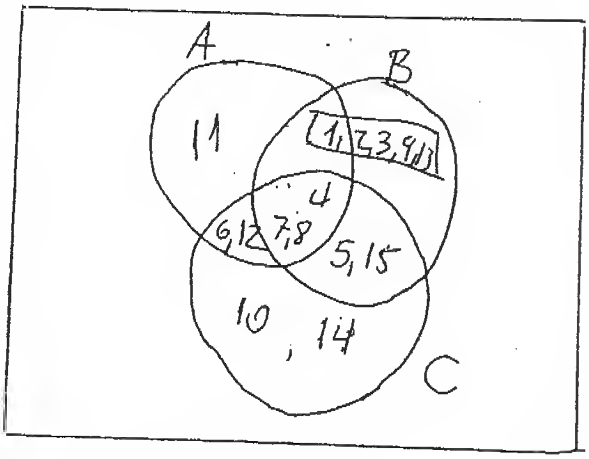


$A = \{1, 3, 4\}$

$B = \{4, 6, 7, 8\}$

$C = \{8, 9\}$

11.-

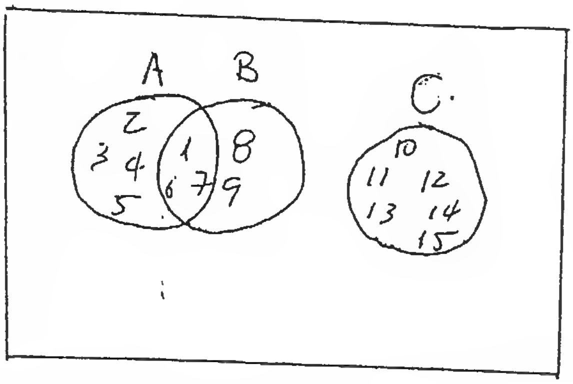


$A = \{3, 4, 6, 7, 8, 11, 12\}$

$B = \{1, 2, 3, 4, 5, 7, 8, 9, 13, 15\}$

$C = \{4, 5, 6, 7, 8, 10, 12, 14, 15\}$

12.-



$A = \{1, 2, 3, 4, 5, 6, 7\}$

$B = \{1, 6, 7, 8, 9\}$

$C = \{10, 11, 12, 13, 14, 15\}$

$A = \{a, i, o, u\}$

$B = \{a, i\}$

$C = \{o, u\}$

a) FALSO; $n(C) = 2$

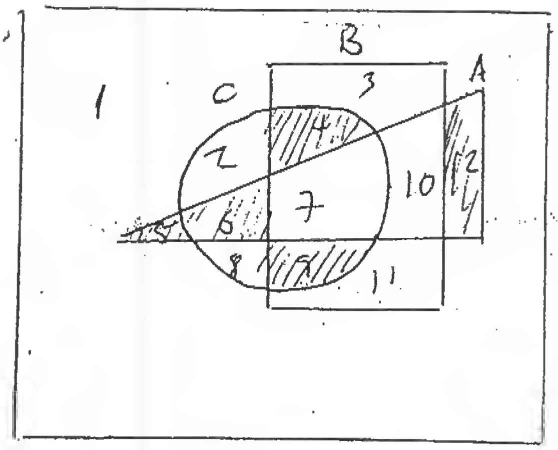
b) FALSO; $C - A = \phi$

c) FALSO; $N(P(B)) = 2^2 \Rightarrow 4$

d) FALSO;

e) $A - B = \{o, u\} = C$; FALSO

114.-



$A = \{5, 6, 7, 10, 12\}$

$B = \{3, 4, 7, 10, 9, 12\}$

$C = \{2, 4, 6, 7, 8, 9\}$

a) $(A-B) = \{5, 6\}$

$(C \cap B) = \{4, 7, 9\}$

$\cap = \phi$ FALSO

b) $A \cap B \cap C = \{7\}$

$(A \cap B \cap C)^c = \{1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12\}$ FALSO

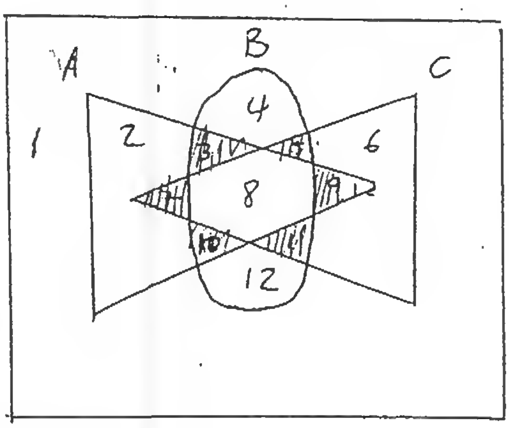
c) $(C-A) = \{2, 4, 8, 9\}$

$(C-A) \cap B = \{4, 9\}$

$(A-B) = \{5, 6, 12\}$

$\cup = \{4, 5, 6, 9, 12\}$ VERDADERO

115.-



$A = \{2, 3, 7, 8, 9, 10\}$

$B = \{3, 4, 5, 8, 10, 11, 12\}$

$C = \{5, 6, 7, 8, 9, 11\}$

b) $(A \cap B) = \{3, 8, 10\}$

$(A \cap B) - C = \{3, 10\}$

$(A-B) = \{2, 7, 9\}$

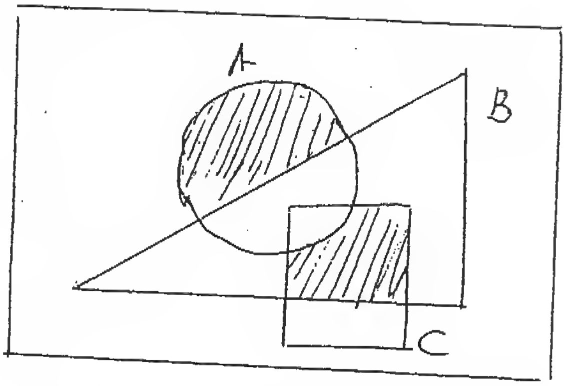
$(A-B) \cap C = \{7, 9\}$

$(B-A) = \{4, 5, 11, 12\}$

$(B-A) \cap C = \{5, 11\}$

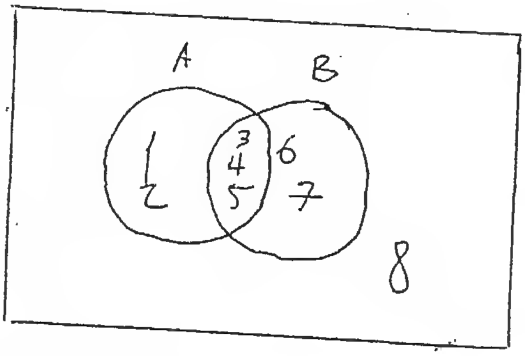
$U = \{3, 5, 7, 9, 10, 11\}$

16.-



$(A-B) \cup [(B \cap C) - A]$

2.-



$A = \{1,2,3,4,5\}$

$B = \{3,4,5,6,7\}$

a) $A-B = \{1,2\}$; FALSO

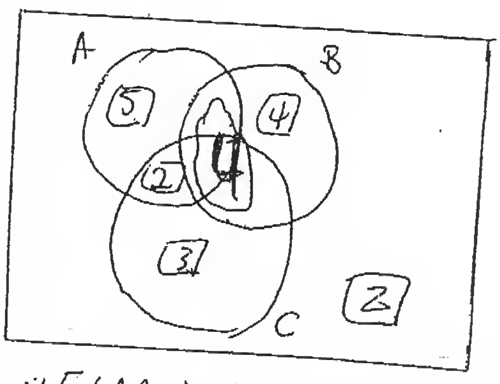
b) $B = \{3,4,5,6,7\}$; FALSO

c) $(A-B) \cap (B \cup A)$

$\{1,2\}$ $\{1,2,3,4,5,6,7\}$

$\cap = \{1,2\}$ VERDADERO

-

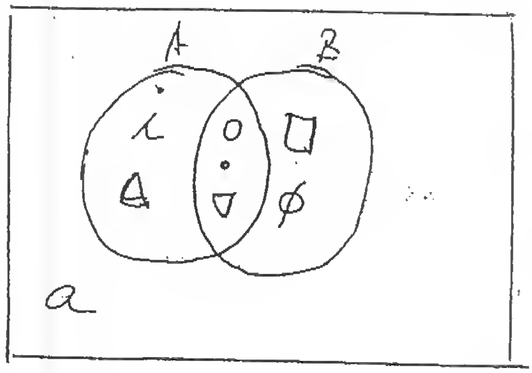


$N[(A \cap B) \cup (A \cap C) \cup (B \cap C)]$

4 2

$4+2=6$

119.—



$A = \{ i, \Delta, O, \circ, \nabla \}$

$B = \{ O, \circ, \nabla, \square, \phi \}$

a) $A - B = \{ i, \Delta \}$; FALSO

b) $B = \{ O, \circ, \nabla, \square, \phi \}$

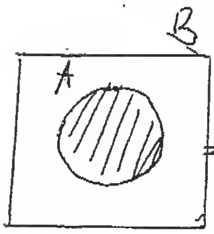
c) $(A-B) \cap (A \cup B)$

$\{ i, \Delta \}$ $\{ i, \Delta, O, \circ, \nabla, \square, \phi \}$

$\cap$

$\{ i, \Delta \}$ VERDADERO

20.- $A \subseteq B \therefore (A \cap B) = B$



$\Rightarrow A \cap B = A$.

FALSO

21.- $P(A) \Rightarrow$ Son todos los subconjuntos que existen en A.

a) correcto

22.- $[\underbrace{(A \cap (B \cup A))}_{A}] \cap A^c = Re$

$\underbrace{A \cap A^c}_{\phi}$

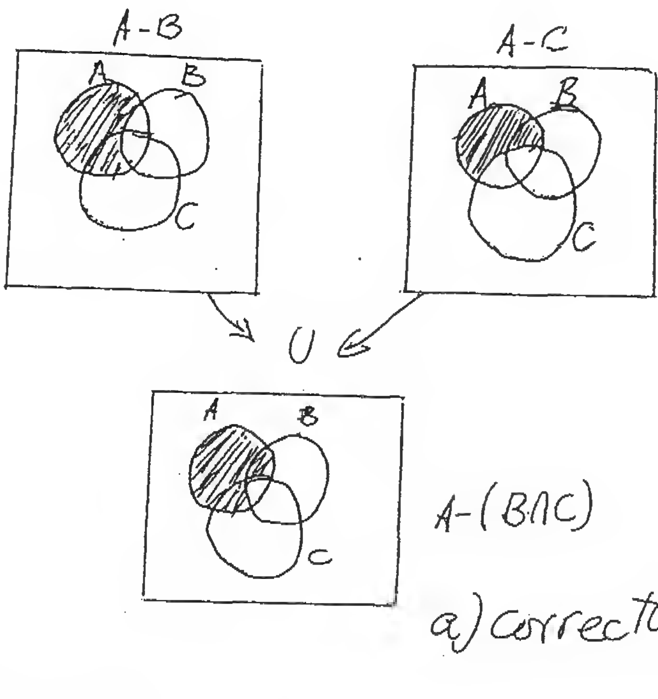
b) correcto

3.- $A = \phi \wedge B = \phi$

$A \cup B = \phi$

a) correcto

5.-

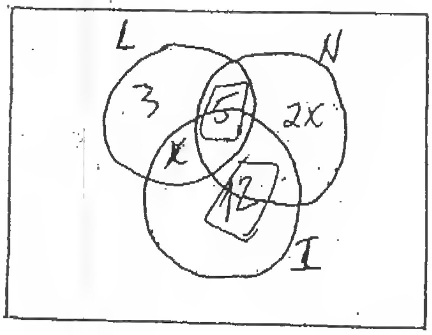


$A - (B \cap C)$

a) correcto

126.-

26: Total
23: $N \cup I$ ×
5: $L \cap N$
12: I no L
Solo N: 2 solo I ∧ L no N

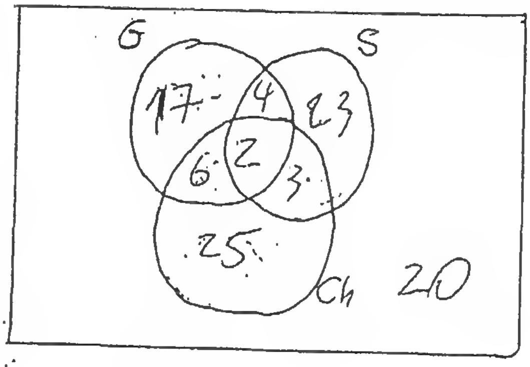


$$\underline{3}+\underline{x}+5+\underline{2x}+\underline{12}=26$$
$$3x+20=26$$
$$3x=6$$
$$x=2$$

locales: $3+5+x$
$: \underline{10}$

127.-

2% $G\cap S\cap C$ × (a)
6% $G\cap S$ ×
5% $S\cap C$ ×
29% G ×
32% S ×
36% C ×
8% $G\cap C$ ×



$$29+26+25+x=100$$
$$80+x=100$$
$$x=20$$

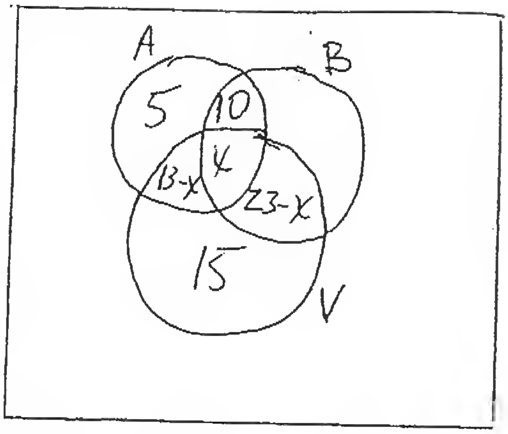
b) 20%

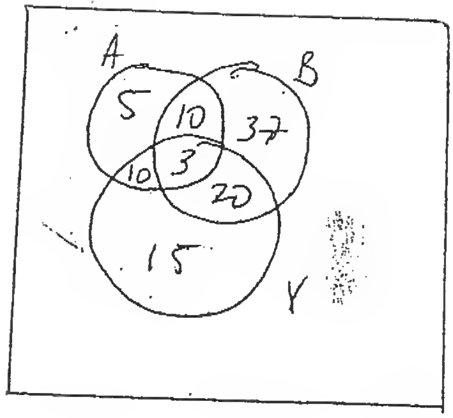
c) 2%

d) 40 personas

8.-

: total





$5+13-x+15=30$

$33-x=30$

$-x=30-33$

$x=3$

a) 3

b) $5+37+15 \Rightarrow 57$

c) 100

d) 97

9.-

300 : total

110 > 20 años

120 : mujeres

50 : mujeres > 20 años

	Mujeres	Hombres
>20	50	60
<20	70	120

a) 180

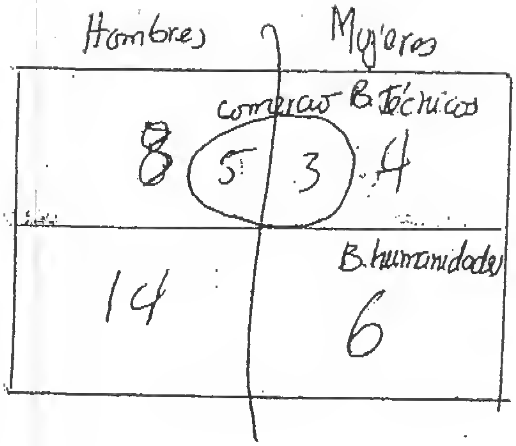
b) 60

c) 70

d) 120

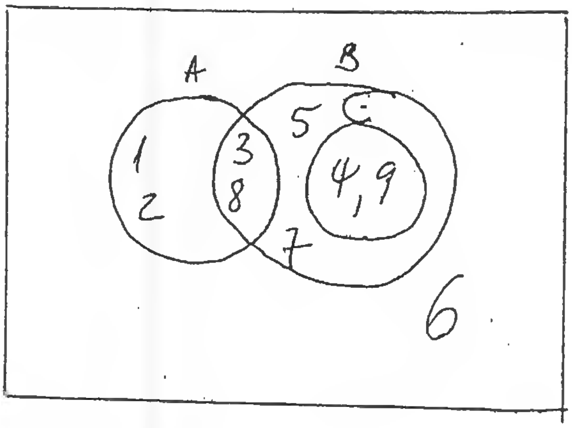
e) 190

130.–
40: total



a) 4
b) 14

131.–



$A = \{1, 2, 3, 8\}$
$B = \{3, 4, 5, 7, 8, 9\}$
$C = \{4, 9\}$

132.–
a) $A^c \cap B^c = (A \cup B)^c$

$x \notin A \wedge x \notin B$

$(A \cup B)^c = \neg(x \in A \vee x \in B)$
$\Rightarrow x \notin A \wedge x \notin B$

b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cap (B \cup C) = x \in A \wedge (x \in B \vee x \in C)$
$= (x \in A \wedge x \in B) \vee (x \in A \wedge x \in C)$

$(A \wedge B) \cup (A \wedge C) = (x \in B \wedge x \in A) \vee (x \in A \wedge x \in C$

c) $(A \cap B)^c = A^c \cup B^c$

$(A \cap B)^c = \neg(x \in A \wedge x \in B)$

$= x \notin A \vee x \notin B$

$A^c \cup B^c = x \notin A \vee x \notin B$

d) $A \cup \phi = A$

$x \in A \vee x \in \phi$

$x \in A \vee \phi$

$x \in A$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cup (B \cap C) = x \in A \vee (x \in B \wedge x \in C)$

$\Rightarrow (x \in A \vee x \in B) \wedge (x \in A \vee x \in C)$

$(A \cup B) \cap (A \cup C) \Rightarrow (x \in A \vee x \in B) \wedge (x \in A \vee x \in C)$

$A - B = A \cap B^c$

$A - B = x \in A \wedge x \notin B$

$A \cap B^c = x \in A \wedge x \notin B$

$A - B = (A^c \cup B)^c$

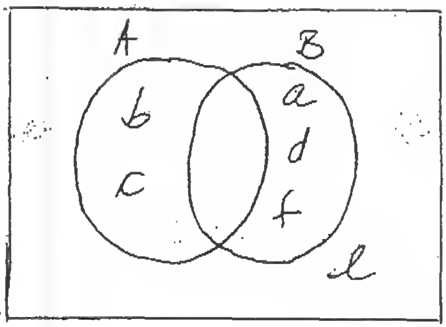
$A - B = x \in A \wedge x \notin B$

$A^c \cup B \Rightarrow x \notin A \vee x \in B$

$(A^c \cup B)^c \Rightarrow \neg(x \notin A \vee x \in B)$

$\Rightarrow x \in A \wedge x \notin B$

134.- $Re = \{a, b, c, d, e, f\}$



a) $N(B-A)=3$; incorrecto

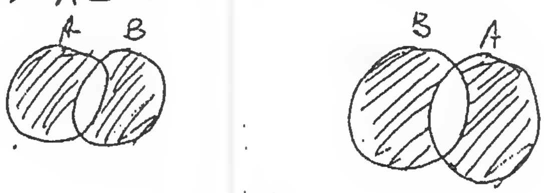
b) $N(A \cap B)=\phi$; incorrecto

c) $A=\{b, c\}$; incorrecto

d) $N(B)=3$; incorrecto

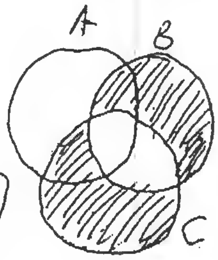
e) $N(A^c \cup B) = \{a, d, f, e\} = \underline{4}$

135.- a) $A \Delta B$ $= B \Delta A$

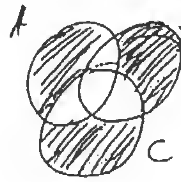


b)

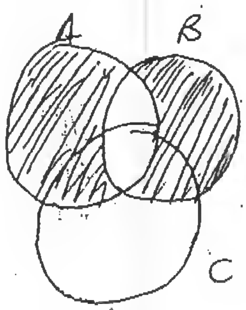
$\boxed{B \Delta C}$



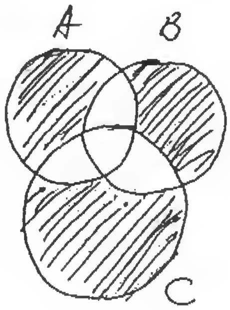
$\Rightarrow$ $A \Delta (B \Delta C)$



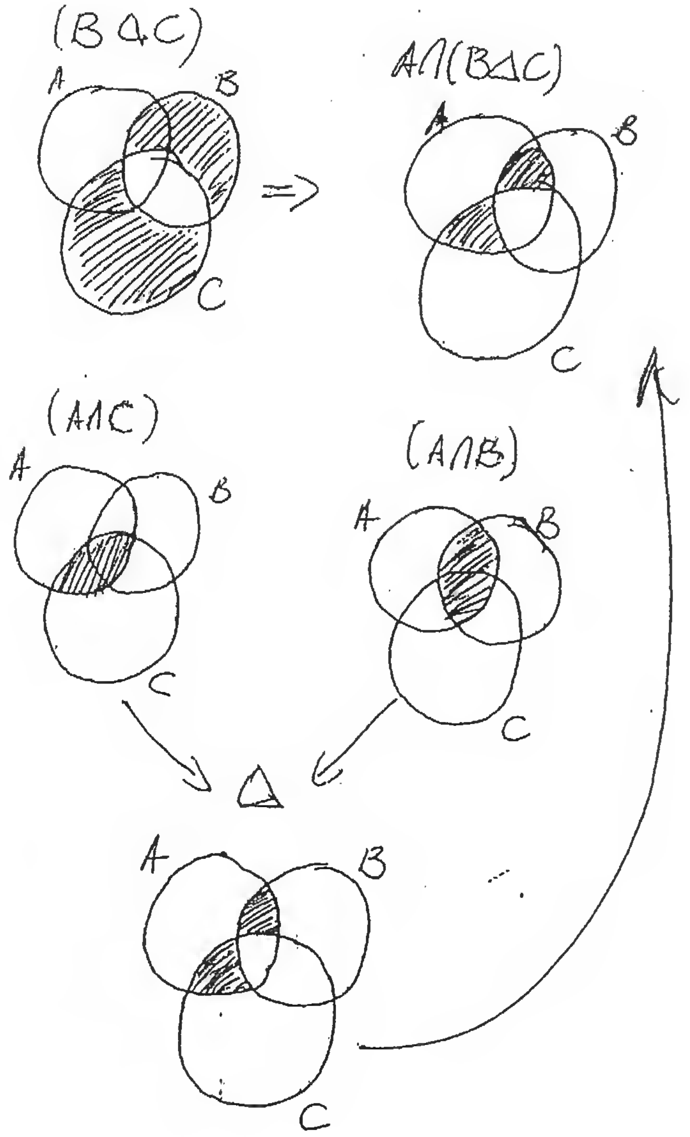
$(A \Delta B)$



$\Rightarrow$ $(A \Delta B) \Delta C$



36.— $A\cap(B\Delta C) = (A\cap B)\Delta(A\cap C)$



7.—

Total 335

C : 215

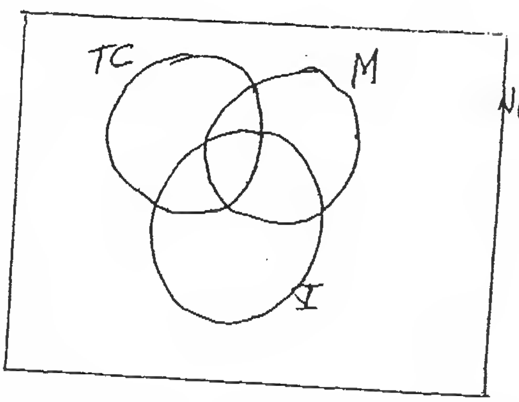
: 190

5 : x lo menos M

: TC ∩ I

: I ∩ M

: TC ∩ M



$N(A\cup B\cup C) = N(A\cup B) + N(A\cup C) + N(B\cup C) - N(A\cap B) - N(A\cap C) - N(B\cap C) + x$

$x = 335 - 215 - 190 - 255 + 70 + 110 + 145$

$x = 335 + 70 + 110 + 145 - (215 + 190 + 255)$

$x = 0$

138.-

total: 100

B: 50

F: 40

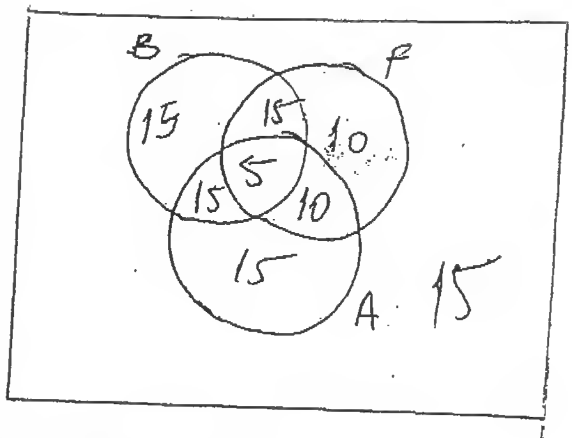
A: 45

$B \cap F$: 20

$B \cap A$: 20

$F \cap A$: 15

$F \cap A \cap B$: 5



a) Verdadero

b) Verdadero

c) Verdadero

d) verdadero

e) Falso

139.- $p(x)$: x es impar $\Rightarrow A_{p(x)}: \{1,3,5,7,9\ldots\}$

$q(x)$: x es par $\Rightarrow A_{q(x)}: \{2,4,6,8\ldots\}$

a) $A(p(x) \Rightarrow q(x)) \subseteq A_{q(x)}$

$A^c_{p(x)} \cup A_{q(x)}$

$A_{q(x)} \cup A_{q(x)}$

$A_{q(x)} \subseteq A_{q(x)}$

Verdadero

b) $R_e = A_{p(x)} \cup A_{q(x)}$

Verdadero

c) $A_{p(x)} = A^c_{q(x)}$

$A_{p(x)} = A_{p(x)}$

Verdadero

d) $A_{q(x)} - A_{p(x)} = \phi$

$A_{q(x)} \neq \phi$

Falso

140.– $p(x)$: x es divisor de 12 $\Rightarrow Ap(x): \{1, 2, 3, 4\}$

$q(x)$: x es primo $\Rightarrow Aq(x): \{2, 3, 5\}$

a) $Ap(x) \cup Aq(x) = Re$

Verdadero

b) $\exists x \in Re \, [p(x) \wedge q(x)] = \{2, 3\}$

Verdadero

c) $\exists x \in Re \, [\underbrace{\neg p(x)} \wedge q(x)]$

$5 \wedge \{2, 3, 5\} \Rightarrow \{5\}$

Verdadero

141.– $Re = \{-3, -2, -1, 1, 2, 3\}$

$p(x): x(x+2) = 0 \Rightarrow Ap(x): \{-2\}$

$q(x): x^2 > 0 \Rightarrow Aq(x) \{-3, -2, -1, 1, 2, 3\}$

a) $A[p(x) \wedge q(x)] = \{-2\}$; incorrecto

b) $A[p(x) \cup q(x)] = \{-3, -2, -1, 1, 2, 3\} = Re$

c) $A[p(x) \Rightarrow q(x)] = [\neg p(x) \cup q(x)]$

$\downarrow$

$\{-3, -1, 1, 2, 3\} \cup \{-3, -2, -1, 1, 2, 3\}$

$\therefore Re$; correcto

d) $A^c[q(x)] = \phi$

e) $A[q(x) \Rightarrow p(x)] = A[\underbrace{\neg q(x)} \cup p(x)]$

$\phi \cup \{-2\}$

$\therefore \{-2\}$

142 — $Re = \{0,1,2,3,4,5,6\}$

$p(x)$: x es número par $\Rightarrow Ap(x): \{2,4,6\}$

$q(x)$: x es mayor que 7 $\Rightarrow Aq(x): \phi$

$r(x)$: x es menor que 10 $\Rightarrow Ar(x): \{0,1,2,3,4,5,6\}$

$s(x)$: x es un número impar $\Rightarrow As(x): \{1,3,5\}$

a) $Ap(x) \cup Aq(x): \{2,4,6\}$

b) $A(s) \cap Ar(x): \{1,3,5\}$

c) $Ap(x) \cup As(x): \{1,2,3,4,5,6\}$

d) $A(p(x) \Rightarrow q(x))$

$A(\underbrace{\neg p(x)} \cup q(x))$

$\Downarrow$

$\{0,1,3,5\} \cup \phi$

$\therefore \{0,1,3,5\}$

e) $A[(p(x) \Rightarrow s(x)) \Rightarrow (q(x) \Rightarrow r(x))]$

$A[(\neg p(x) \cup s(x)) \Rightarrow (\neg q(x) \cup r(x))]$

$A[(\underbrace{p(x)} \cap \neg s(x)) \cup (\underbrace{\neg q(x)} \cup r(x))]$

$[\{2,4,6\} \cap \{0,2,4,6\}] \cup [Re \cup Re]$

$\{0,2,4,6\} \cup Re$

$\therefore Re$

f) $\underbrace{A^c r(x)}_{\phi} \cap \underbrace{A s(x)}_{\square}$

$\underbrace{\phi \cap \square}_{\phi}$

g.-) $\underbrace{(Re - Ap(x))}_{\{0,1,3,5\}} \cap \underbrace{(Aq(x) \cup As(x))}_{\{1,3,5\}}$

$\{1,3,5\}$

143.- $Re = \{1,2,3,4,5\}$

$p(x): x^2 - x + 41$ es primo

a) $Ap(x): \{1,2,3,4,5\}$

$(1)^2 - 1 + 41 \Rightarrow 41$ ✓

$(2)^2 - 2 + 41 \Rightarrow 43$ ✓

$(3)^2 - 3 + 41 \Rightarrow 47$ ✓

$(4)^2 - 4 + 41 \Rightarrow 53$ ✓

$(5)^2 - 5 + 41 \Rightarrow 61$ ✓

b.1) $\exists x \in Re \; \neg p(x)$; Falso

b.2) $\forall x \in Re \; p(x)$; verdadero

144.- $Re = \{1,2,3,4,5,6,7,8,9\}$

$p(x)$: x es divisor de 284 — $Ap(x): \{1,2,4\}$

$q(x): x + 3 < 9$ — $Aq(x): \{1,2,3,4,5\}$

$r(x): x + 2 = 8$ — $Ar(x): \{6\}$

$m(x)$: x es primo — $Am(x): \{1,2,3,5,7\}$

a) $A\,p(x): \{1,2,4\}$

b) $A\,q(x): \{1,2,3,4,5\}$

c) $A\,r(x): \{6\}$

d) $A\,m(x): \{1,2,3,5,7\}$

e) $A[p(x) \vee r(x)]$

$\{1,2,4,6\}$

f) $A[q(x) \wedge m(x)]: \{1,2,3,5\}$

g) $A[m(x) \Rightarrow \neg r(x)]$

$\therefore A[\underbrace{\neg m(x)} \vee \underbrace{\neg r(x)}]$

$: \{4,6,8,9\} \cup \{1,2,3,4,5,7,8,9\} = \{1,2,3,4,5,6,7,8,9\}$

h) $A[\underbrace{\neg r(x)} \wedge q(x)]$

$q(x) \rightarrow \{1,2,3,4,5\}$

$\{1,2,3,4,5,7,8,9\} \longrightarrow \{1,2,3,4,5\}$

145.- Para Todo x

Negación: existe un x.

Para todo número natural n; $n+2>8$

Negación Existe algún número natural tal que:

$n+2 \leq 8$

148.- $Re = \{1, 2, 3, 4, 5\}$

a) $x-3=1 \Rightarrow x=4 \therefore \exists x\,(x-3=1)$; correcto

b) $x+3<5 \Rightarrow x=1 \therefore \exists x\,(x+3<5)$; FALSO

c) $x>1 \Rightarrow x=2,3,4,5 \therefore \exists x\,(x>1)$; FALSO

d) $x+3<4 \Rightarrow x=\phi \therefore$ FALSO

e) $x^2-4x+3=0$; es una ecuación cuadrática, tendrá 2 raíces y no es el referencial

149.- $N(A)=2$

$N(B)=1$

$N(P\times A)=2\times 1$

$\Rightarrow 2$

$N(P(A\times B))=2^2$

$\Rightarrow 4$ //

b) Falso

150.- $A\times\underbrace{(B\cap C)}_{\phi} \neq \underbrace{(A\times B)}_{No\ \phi} \cup \underbrace{(A\times C)}_{No\ \phi}$

b) Falso

151.- $A=\{1,\{1\},\phi\}$

$N(A)=3$ b) Falso

152.- (a,b) ; (c,d)

$(a,b)=(c,d) \therefore a=c \wedge b=d$

a) correcto

146. $\forall x\,(x+\cancel{8}2 = 5 \;\wedge\; x-1 \leq 3)$

Negación : $\neg \forall x\; (x+2 \neq 5 \wedge x-1 \leq 3)$

$\therefore\; \exists x\; (x+2 \neq 5 \vee x-1 > 3)$

b) Correcto

147. $\forall x\,[(a(x) \wedge (\neg a(x) \Rightarrow \neg b(x)))]$

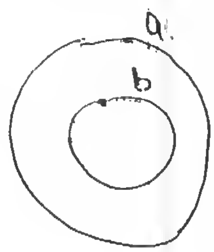
Negación : $\neg\forall x\,[(a(x) \wedge (b(x) \Rightarrow a(x)))]$

$\therefore\; \exists x\; \neg[(a(x) \wedge a(x)]$

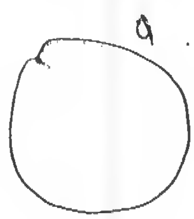
$\exists x\; \neg[(a(x))]$

$\exists x\; [\neg a(x)]$

$\forall x\,(b(x) \Rightarrow a(x)]$



$\forall x\,(a(x) \wedge (b(x) \Rightarrow a(x)))$



c) Correcto.

153.- $A \times B \neq B \times A$.

$(a,3) \neq (3,a)$

b) Falso

154.- $(a,b) = (b,a)$

Condición $a = b$.

155.- $N(A) = 2$ $\quad N(A \times B) = 2 * 4$

$N(B) = 4$ $\quad \Rightarrow 8$

$N(P(A \times B)) = 2^8$

$\Rightarrow 256$

$$\underbrace{N(A \times B)}_{8} + \underbrace{N(P(A \times B))}_{256}$$

$\Rightarrow \underline{\underline{264}}$ b) Falso

156.- $C - B = \{ \Diamond, \bullet, \nabla \}$

$N((A \times (C - B))) = 3 \times 3$

$\Rightarrow 9$ b) Falso

157.- a) verdadero

b) verdadero

c) verdadero

d) verdadero

e) $N(A \times B) = N(B \times A)$; Falso

158.- $N(A)=2$

$N(B)=3$

$N(C)=3$

$N(B\cap C)=2$

$B\times C \Rightarrow$ No toma en cuenta si hay elementos repetidos en los conjuntos

$N(B\times C)=3\times 3$

$\Rightarrow 9$

$N(A\times(B\times C))=2*9$

$\Rightarrow 18$ b) correcto

RELACIONES

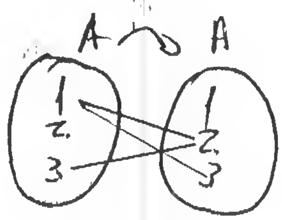
159.- En una FUNCION, el dominio debe ser el conjunto de partida b) Falso

160.- b) FALSO

161.- b) FALSO

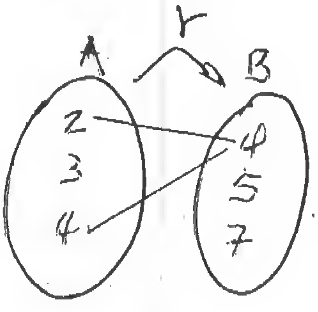
162.- a) verdadero

163.



a) verdadero

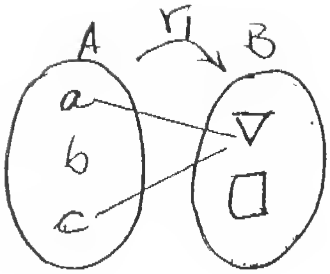
164.-

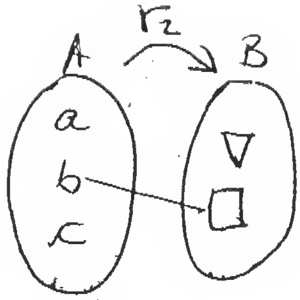


$N(r)=2$

b) Falso

65.—



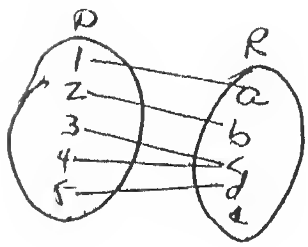


$r_1 \cup r_2$ es un función

a) verdadero; todo los elementos del dominio se relacionan 1 sola vez.

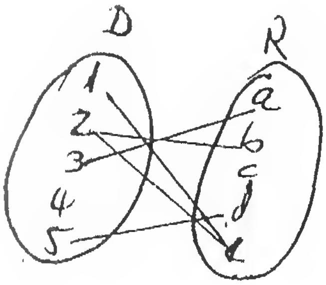
66.—

a)



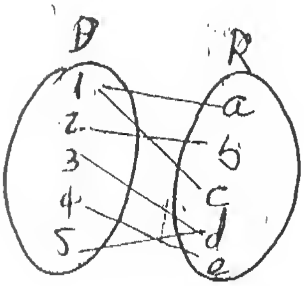
FUNCION

b)



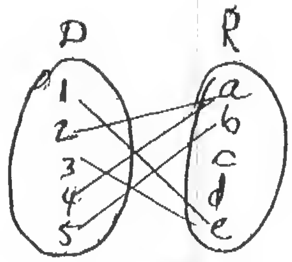
NO ES FUNCION

c)



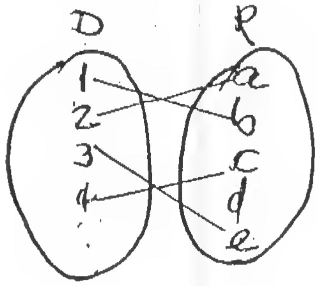
NO ES FUNCION

d)



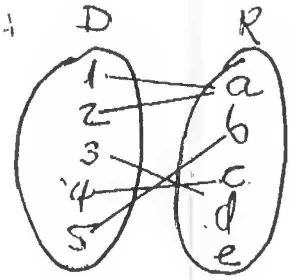
FUNCION

e)



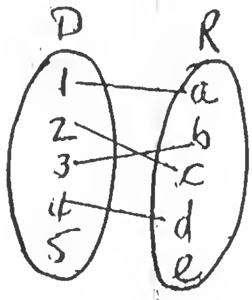
FUNCION

f)



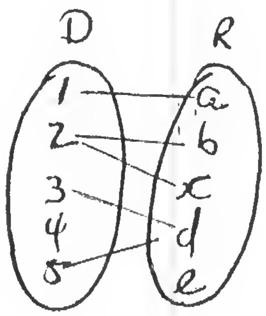
FUNCION

g)



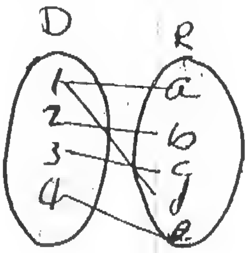
NO ES FUNCION

h)



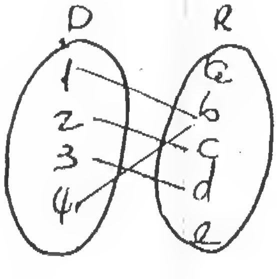
NO ES FUNCION

i)



NO ES FUNCION

j)



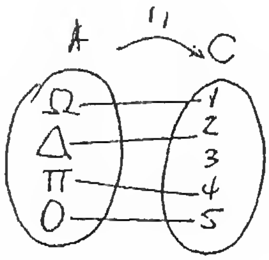
FUNCION

a) Función
d) Función
e) Función Inyectiva.
f) Función
g) Función Inyectiva.

No hay garantía que se relacionen los elementos uno a uno.

b) Falso.

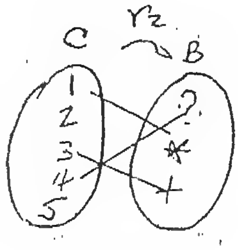
a)



$rg(r_1) = \{1, 2, 4, 5\}$

FALSO

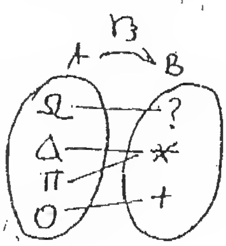
b)



$dom(r_2) = \{1, 3, 4\}$

FALSO

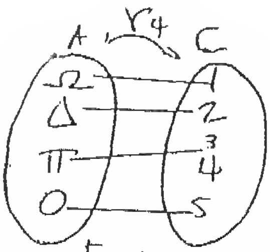
c)



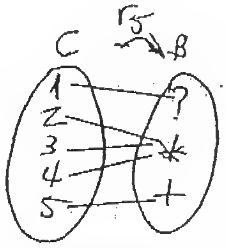
NO INYECTIVA
SÍ SOBREYECTIVA } No es biyectiva

FALSO

d)



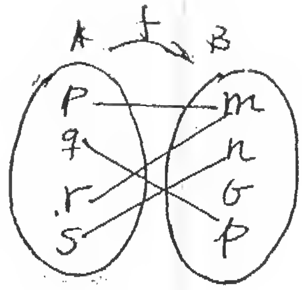
FUNCION

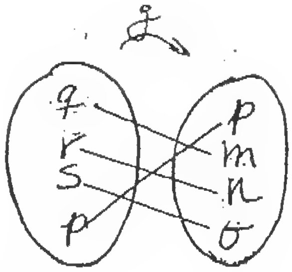


FUNCION

Si ($r_4 \wedge r_5$) son funciones, entonces ($r_4 \vee r_5$) son sobreyectivas

170.-



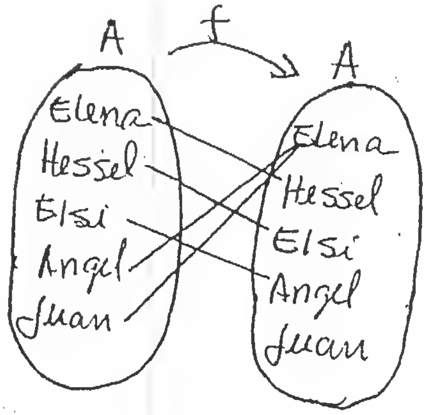


F. inyectiva

F. sobreyectiva

d) correcto

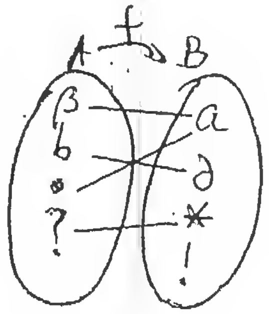
171.-

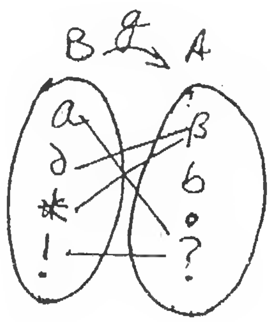


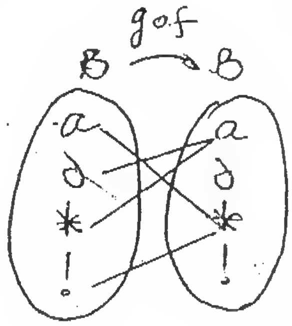
f(Juan) = Elena
f(Elena) = Hessel
} fof(Juan) = Hessel

b) correcto

172-

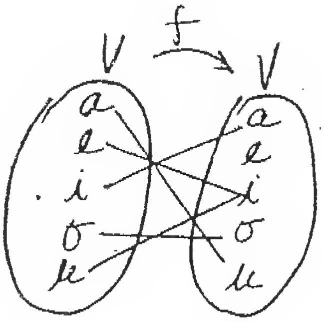


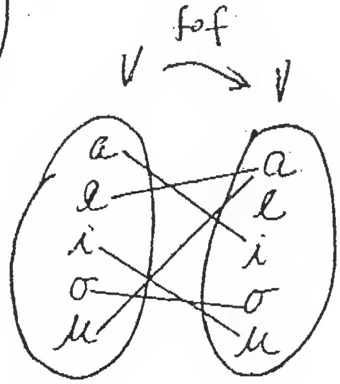




b) correcto

3.-

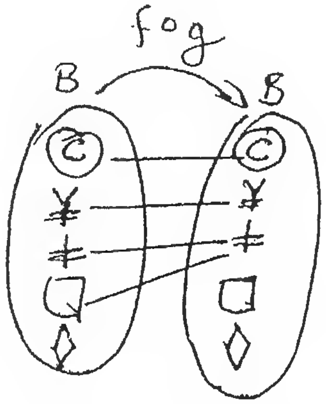


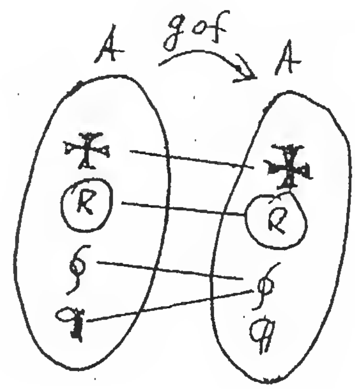


$rg(f\circ f) = \{a, i, o, u\}$

b) correcto

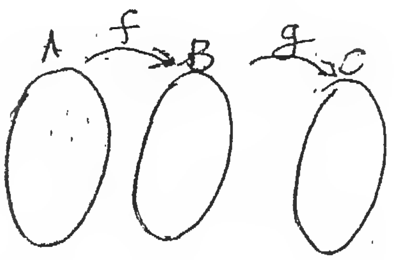
4.-





e) correcto

5.-



a) Falso

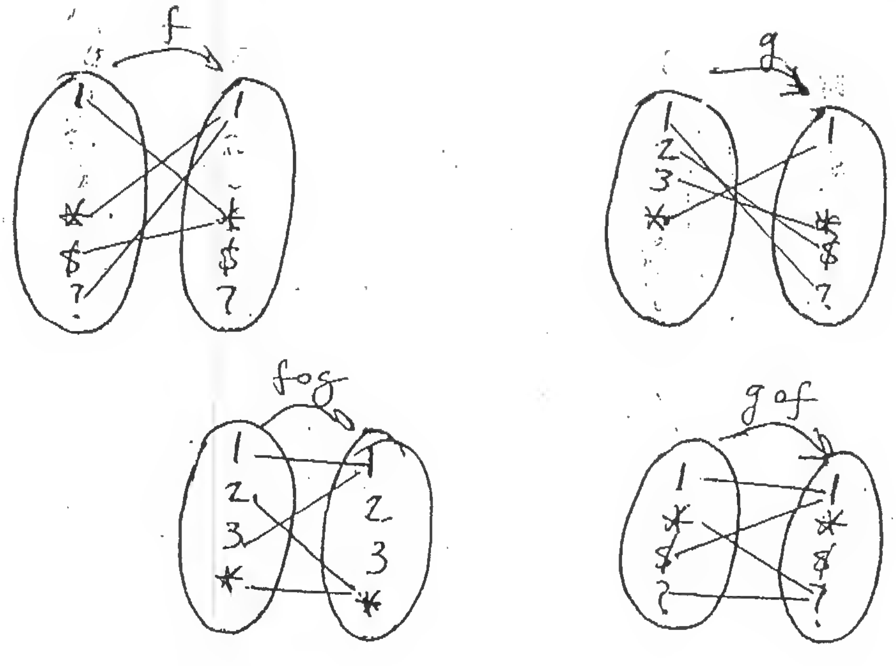
b) Falso

c) Verdadero

d) Falso

e) Falso

17.6.-

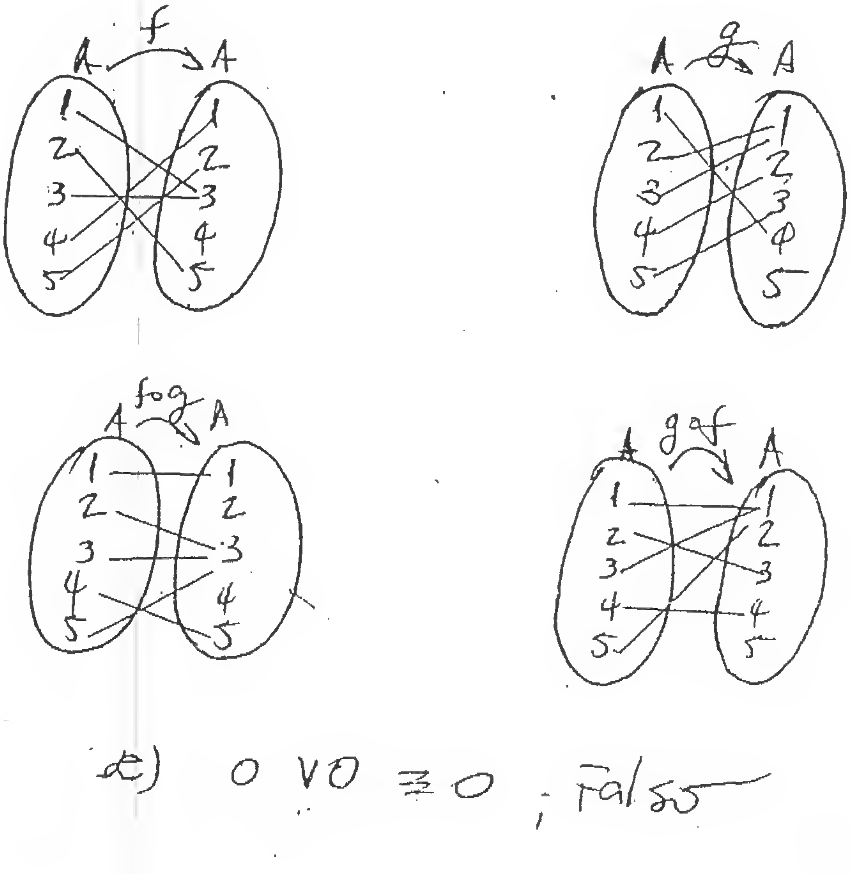


e) Falso

17.7.-

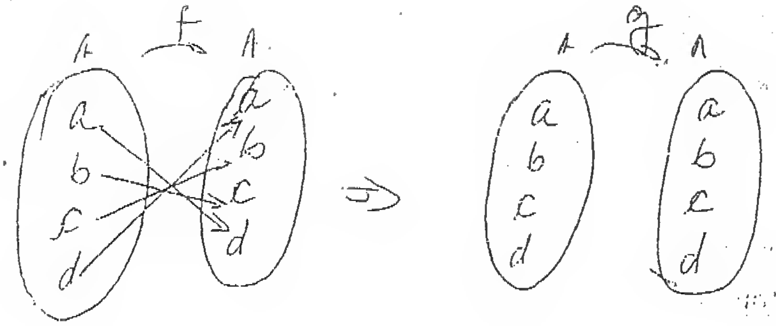
$$\left.\begin{array}{l} g(1)=2 \\ h(2)=3 \end{array}\right\} \quad hog(1)=3$$

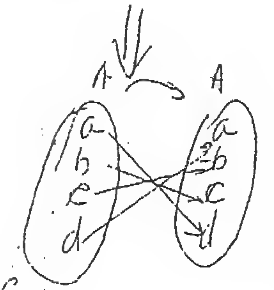
17.8.-



e) $0 \vee 0 \equiv 0$; Falso

179

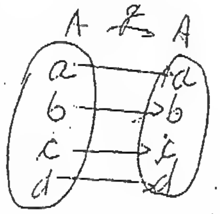




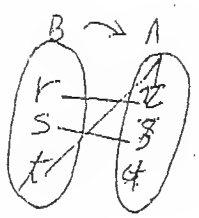
Para que se obteng gof

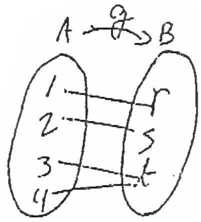
$f = g \circ f$

entonces g debe ser

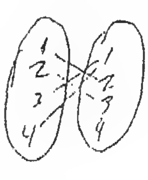


180

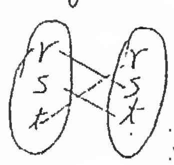




fog



gof



a) Falso; no lo es

b) Falso; rango {1,2,3}

c) Falso; (s,t)

d) Verdadero

e) Falso; es f. biyectiva

SOLUCIÓN

EJERCICIOS PROPUESTOS

CAPÍTULO DOS

3^{90}

MATEMÁTICAS

NÚMEROS REALES

1.– $\pi * \sqrt{2}$ ⇒ el resultado sigue siendo un número irracional

b) Falso

2.– $\frac{\cancel{\pi}}{2\cancel{\pi}} + 4$

⇒ $\frac{1}{2} + 4 = \frac{9}{2}$ (número racional)

b) Falso

3.–

a) $\{x / x \in \mathbb{Z} ;\ 3 < x < 4\}$ NO HAY NUMEROS ENTEROS ENTRE 3 ∧ 4.

Sol: ϕ

b) $\{x / x \in \mathbb{Q} ;\ x^2 - 2 = 0\}$

$x^2 - 2 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$ ⇒ # irracional

Sol: ϕ

c) $\{x / x \in \mathbb{I} ;\ x+1 \geq 0\}$

$x + 1 \geq 0$

$\boxed{x \geq -1}$ ⇒ # racional

Sol: ϕ

d) $\{x/x \in \mathbb{R}\ ; x^2+1 \geq 0\}$

$x^2 \geq -1$

$x = \pm i \Rightarrow$ # imaginario ; NO ES NUMERO REAL

Sol: ϕ

e) $\{x/x \in \mathbb{N}\ ;\ x^{-1}-2=0\}$ $\quad x^{-1}-2=0$

$\frac{1}{x} = 2$

$\boxed{x = \frac{1}{2}} \Rightarrow$ NO ES NUMERO NATURAL

Sol: ϕ

4.- a) $$\frac{(7-6{,}35)\div 6{,}5 + 9{,}9}{(1{,}2\div 36 + 1{,}2\div 0{,}25 - 1\frac{5}{16})\div \frac{169}{24}}$$

$$\Rightarrow \frac{0{,}65 \div 6{,}5 + 9{,}9}{\left(\frac{1{,}2}{36}+\frac{1{,}2}{0{,}25}-\frac{21}{16}\right)\times\frac{24}{169}} \Rightarrow \frac{\frac{65}{650}+9{,}9}{\left(\frac{12}{360}+\frac{120}{25}-\frac{21}{16}\right)\times}$$

$$\Rightarrow \frac{\frac{1}{10}+9{,}9}{\left(\frac{1}{30}+\frac{24}{5}-\frac{21}{16}\right)\times\frac{24}{169}} \Rightarrow \frac{0{,}1+9{,}9}{\left(\frac{8+1152-315}{240}\right)\times\frac{24}{169}}$$

$$\Rightarrow \frac{10}{\frac{845}{240}\times\frac{24}{169}} \Rightarrow \underline{20}$$

b) $3(6-1{,}333)+6(1{,}333\ldots)-16{,}666$

$\Rightarrow 3(4{,}66666\ldots)+3(2{,}666666\ldots)-16{,}6666\ldots$

$\Rightarrow 3\left(4+\frac{2}{3}\right)+3\left(2+\frac{2}{3}\right)-\left(16+\frac{2}{3}\right)$

$\Rightarrow 3\left(\frac{14}{3}\right)+3\left(\frac{8}{3}\right)-\left(\frac{50}{3}\right)$

$\frac{1}{3}\left(3(14)+3(8)-50\right)$

$\frac{1}{3}\left(42+24-50\right)$

$\boxed{\frac{16}{3}}$

$x=0{,}6666$

$10x=6{,}6666\ldots$

$-x-0{,}6666$

$9x=6$

$x=\frac{\cancel{6}^{2}}{\cancel{9}_{3}}$

$\boxed{x=\frac{2}{3}}$

c) $\dfrac{3}{1+\frac{2}{0{,}5}}-\dfrac{2}{1+\frac{3}{0{,}5}}+\dfrac{\frac{54}{3}+1{,}666\ldots}{9+2{,}6666}$

$\Rightarrow \dfrac{3}{1+\frac{\cancel{20}^{4}}{\cancel{5}_{1}}}-\dfrac{2}{1+\frac{\cancel{30}^{6}}{\cancel{5}_{1}}}+\dfrac{\frac{54}{3}+\left(1+\frac{2}{3}\right)}{9+\left(2+\frac{2}{3}\right)}$

$\Rightarrow \dfrac{3}{5}-\dfrac{2}{7}+\dfrac{\frac{54}{3}+\frac{5}{3}}{9+\frac{8}{3}} \Rightarrow \dfrac{3}{5}-\dfrac{2}{7}+\dfrac{\frac{59}{\cancel{3}}}{\frac{35}{\cancel{3}}}$

$\Rightarrow \frac{3}{5}-\frac{2}{7}+\frac{59}{35} \Rightarrow \dfrac{21-10+59}{35} \Rightarrow$

$\frac{70}{35} \Rightarrow 2$

5.- NO LO ES

$\sqrt{2}+2 \Rightarrow$ es un número irracional.

6.- $S=\{1,2,3,4\}$

$$a * b = \begin{cases} a & ; a \geq b \\ b & ; a < b \end{cases}$$

a) $1*2=2$ $1*3=3$ $1*4=4$ $2*3=3$ $3*4=4$

$2*1=2$ $3*1=3$ $4*1=4$ $3*2=3$ $4*3=4$

SI ES CONMUTATIVA. ; FALSO

b) Si $a>b>c$

$\underbrace{\underbrace{(a*b)}_{a * c} * c}_{a}$ $\underbrace{a*\underbrace{(b*c)}_{b}}_{a}$

Si es asociativa

FALSO

c) $1*2=2$

$1*3=3$

$1*4=4$

$1*5=5$

$1*1=1$

1 es el elemento neutro ; VERDADERO

d) FALSO ; $\underbrace{(4*a)}_{4}=\underbrace{(b*4)}_{4}$

e) $(1*3)*2=(2*1)*(3*4)$ FALSO

$\underbrace{\underbrace{(1*3)}_{3}*2}_{3} \neq \underbrace{\underbrace{(2*1)}_{2}*\underbrace{(3*4)}_{4}}_{4}$

7.- a) $(a \nabla b) = a$
$(b \nabla a) = b$ } NO ES CONMUTATIVA; FALSO

8.- a) Módulo 7 $\quad S = \{0, 1, 2, 3, 4, 5, 6\}$

i) $\bar{4} + \bar{5} = 2$

ii) $\bar{5} \cdot \bar{3} = 1$

iii) $\bar{2} \cdot \bar{4} = 1$

iv) $\bar{5} \cdot \bar{6} = 2$

v) $\bar{6} \cdot \bar{1} = 6$

vi) $\bar{1} + \bar{0} = 1$

vii) $\bar{2} \cdot \bar{0} = 0$

viii) $\bar{5} + \bar{3} = 1$

ix) $\bar{4} \cdot \bar{5} = 6$

x) $\bar{2} + \bar{5} = 0$

b) Módulo 6 $\quad S = \{0, 1, 2, 3, 4, 5\}$

i) $= \bar{4} + \bar{5} = 3$

ii) $\bar{5} \cdot \bar{3} = 3$

iii) $\bar{2} \cdot \bar{4} = 2$

iv) $\bar{5} \cdot \bar{6} = 0$

v) $\bar{6} \cdot \bar{1} = 0$

vi) $\bar{1} \cdot \bar{0} = 0$

vii) $\bar{2} \cdot \bar{0} = 0$

viii) $\bar{5} + \bar{5} = 2$

ix) $\bar{4} \cdot \bar{5} = 2$

x) $\bar{2} + \bar{5} = 1$

9.- $2+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{2}}}}$ $3/2$

$\Rightarrow 2+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{3/2}}}$ $\Rightarrow 2+\cfrac{1}{3+\cfrac{1}{1+\frac{2}{3}}}$ $5/3$

$\Rightarrow 2+\cfrac{1}{3+\cfrac{1}{5/3}}$ $\Rightarrow 2+\cfrac{1}{3+\frac{3}{5}}$ $\frac{18}{5}$

$\Rightarrow 2+\cfrac{1}{\frac{18}{5}}$ $\Rightarrow 2+\frac{5}{18}$ $\Rightarrow \frac{41}{18}$ c) correcto

10.- $a*0=0$; porque 0 veces se repite a.

11.- $(a+b)\cdot c = a\cdot c + b\cdot c$

(a+b) se repite c veces

$(a+b)+(a+b)+(a+b)+ \text{-----} a+b$

$\therefore a+b+a+b+a+b+ \text{-----} a+b$

$\therefore$ habrá c veces a y c veces b

$\therefore c.a+cb$

$\therefore (a+b)\cdot c = ac+bc$

a) $\dfrac{a}{b+c} \neq \dfrac{a}{b} + \dfrac{a}{c}$ FALSO

$\therefore \dfrac{ac+ab}{bc}$

$\therefore \dfrac{a}{bc}(b+c)$

b) $(a+b) \div c = \dfrac{a}{c} + \dfrac{b}{c}$ VERDADERO

$\therefore \dfrac{a+b}{c}$

$\forall a \in R \ (a+c) = a$

$a+c=a$

$c=a-a$

$\boxed{c=0}$

$\sqrt[8]{(1-\sqrt{3})^8} = \left(\sqrt[8]{1-\sqrt{3}}\right)^8$

$1-\sqrt{3} = 1-\sqrt{3}$

b) FALSO

15.- a) $e^{\pi} < 81$

$\pi \ln e < \ln 81$

$\pi < \ln 81$ verdadero

$8 < e^{\pi}$

$\ln 8 < \ln e^{\pi}$

$\ln 8 < \pi \cdot \cancel{\ln e}^{1}$

$\ln 8 < \pi$. verdadero

b) $\sqrt{3}+\sqrt{2} > \frac{1}{\sqrt{3}-\sqrt{2}} \cdot \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$; FALSO

$\Rightarrow \frac{\sqrt{3}+\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2}$

$\Rightarrow \sqrt{3}+\sqrt{2}$

$\sqrt{3}+\sqrt{2} = \sqrt{3}+\sqrt{2}$

16.-

a) $\frac{69}{200}$; $\frac{19}{100}$; 0.8 ; $\frac{1}{5}$ FALSO

0,8 ; 0,2

b) $\frac{19}{100}$; $\frac{1}{5}$; $\frac{69}{200}$; 0,8

0,19 ; 0,2 ; 0,345 ; 0,8 CORRECTO

$\frac{0,69}{2} = 0,345$

17.-

$\frac{6-1}{\frac{3}{4} \cdot 60}$

$\Rightarrow 45$

$\frac{6-2}{\frac{2}{3} \cdot 60}$

$\Rightarrow 40$

c) correcto

total : 18 cuadros

rayados : 10 cuadros

área R : $\frac{\cancel{10}^{5}}{\cancel{18}\,9}$

área R : $\frac{5}{9}$ (un poco más de la mitad)

d) correcto

a) $\frac{4}{5} = 0{,}8$

b) $\frac{3}{4} = 0{,}75$

c) $\frac{5}{8} = 0{,}625$

d) $\frac{7}{10} = 0{,}7$

e) $\frac{31}{40} = 0{,}775$

50 (8
20 0,625
40
(0)

310 (40
300 0,775
200
(00)

a) CORRECTO

Re = Naturales

$M = a^3 \cdot b^2$

$\therefore M = a \cdot a \cdot a \cdot b \cdot b$

divisores

$1, a, b, ab, a^2b, a^3b, a^2b^2, a^3b^2, ab^2, a^2, a^3, b^2$

VERDADERO

21.-

72	108	90	2
36	54	45	3
12	18	15	3
4	6	5	

MCD = 18

b) FALSO

22.- a) VERDADERO

23.-

72	2
36	2
18	2
9	3
3	3
1	

$2^3 \cdot 3^2$

$\Rightarrow 12$

24.- Divisores: 1, 2, 3, 4, 8, 9, 6, 12, 24, 18, 36

1+2+3+4+8+9+6+12+24+18+36

$\Rightarrow 123$ b) correcto

25.-

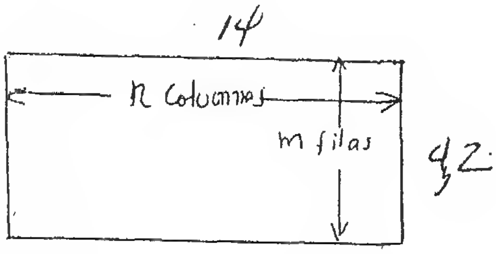
15	25	5
3	5	3
1	5	5
	1	

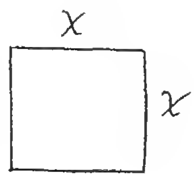
75

b) correcto

6.-

	2	3	4	5	9	11
56	V	F	V	F	F	F
261	F	V	F	F	V	F
660	V	V	V	V	F	V
1455	F	V	F	V	F	F





$nx = 14 \Rightarrow x = \frac{14}{n}$

$mx = 4,2 \Rightarrow x = \frac{4,2}{m}$

$\frac{14}{n} = \frac{4,2}{m}$

$14m = 4,2n$

$n = \frac{14}{4,2} m$

$n = \frac{\cancel{140}^{\cancel{20}\,10}}{\cancel{42}_{\cancel{6}\,3}} m$

$\boxed{n = \frac{10}{3} m}$

Si $m = 3$

$\therefore n = 10$

$área = 14 * 4,2$

$área = 58,8$

$3 * 10 * x^2 = 58,8$

$x^2 = \frac{58,8}{30}$

$x^2 = \frac{\cancel{588}^{196}}{\cancel{300}_{100}}$

$x^2 = \frac{196}{100}$

$x = \frac{14}{10}$

$\boxed{x = 1,4\ m}$

28.— Bombones: 24
caramelos: 42

$$\frac{24}{x} \quad ; \quad \frac{42}{x}$$

$x = 1, 2, 3, 6$ mayor

29.— $2x = 3y$ $\quad$ $3y = 5z$ $\quad$ $5z = 6w$

$\boxed{y = \frac{2}{3}x}$ $\quad$ $\cancel{3}\left(\frac{2}{\cancel{3}}x\right) = 5z$ $\quad$ $\cancel{5}\left(\frac{2}{\cancel{5}}x\right) = 6w$

$\boxed{z = \frac{2}{5}x}$ $\quad$ $w = \frac{2}{6}x$

$\boxed{w = \frac{1}{3}x}$

$x \Rightarrow$ debe ser entero

3	5	15, 30, 45 ......

$x = 15, 30, 45 ......$

$2x = 30$ ✗
$2x = 60$ ✓
$2x = 90$ ✗

30.— $$\frac{\frac{2}{1-a} + \frac{2}{1+a}}{\frac{2}{1+a} - \frac{2}{1-a}} \Rightarrow \frac{\frac{2(1+a)+2(1-a)}{\cancel{(1-a)(1+a)}}}{\frac{2(1-a)-2(1+a)}{\cancel{(1+a)(1-a)}}} \Rightarrow \frac{2+\cancel{2a}+2-\cancel{2a}}{\cancel{2}-2a-\cancel{2}-2a}$$

$$\Rightarrow \frac{4}{-4a} \Rightarrow -\frac{1}{a}$$

$3x^2+7x-6$

$\Rightarrow \dfrac{(\overset{1}{\cancel{3}}x+\overset{3}{\cancel{9}})(3x-2)}{\underset{1}{\cancel{3}}}$

$\Rightarrow (x+3)(3x-2).$

c) Correcto

2○ $\dfrac{x^3-1}{x^2-1} - (x+1)$

$\Rightarrow \dfrac{\cancel{(x-1)}(x^2+x+1)}{(x+1)\cancel{(x-1)}} - (x+1).$

$\Rightarrow \dfrac{(x^2+x+1)}{(x+1)} - (x+1) \Rightarrow \dfrac{x^2+x+1-(x+1)(x+1)}{(x+1)}$

$\Rightarrow \dfrac{x^2+x+1-(x^2+2x+1)}{x+1} \Rightarrow \dfrac{\cancel{x^2}+\cancel{x}+\cancel{1}-\cancel{x^2}-\cancel{2x}-\cancel{1}}{x+1}$

$\Rightarrow \boxed{-\dfrac{x}{x+1}}$

e) Correcto

3○ $\dfrac{\sqrt{1-x}-\dfrac{x}{2\sqrt{1+x}}}{1+x} \Rightarrow \dfrac{\dfrac{2(\sqrt{1+x})^2-x}{2\sqrt{1+x}}}{} \Rightarrow$

$\Rightarrow \dfrac{\dfrac{2+2x-x}{2\sqrt{1+x}}}{1+x} \Rightarrow \dfrac{x+2}{2\sqrt{1+x}\,(1+x)} \Rightarrow \dfrac{x+2}{2(x+1)^{3/2}}$

a) Correcto

4.- a) $a^4+3a^3+4a^2-6a-12$

$(a^4+4a^2-12)+(3a^3-6a)$

$(a^2+6)(a^2-2)+(3a^3-6a)$

$(a^2+6)(a^2-2)+3a(a^2-2)$

$(a^2-2)(a^2+3a+6)$

$(a^2-2)(a^2+3a+6)$

$(a+\sqrt{2})(a-\sqrt{2})(a^2+3a+6)$

b) $(ab+ac+bc)(a+b+c)-abc$

$a^2b+a^2c+3bac+ab^2+b^2c+ac^2+bc^2-abc$

$a^2b+a^2c+2abc+ab^2+b^2c+ac^2+bc^2$

$(a^2b+a^2c)+(abc+ac^2)+(abc+ab^2)+(b^2c+bc^2)$

$a^2(b+c)+ac(b+c)+ab(c+b)+bc(b+c)$

$(b+c)(a^2+ac+ab+bc)$

$(b+c)(a(a+c)+b(a+c))$

$(b+c)(a+b)(a+c)$

c) x^3-5x^2-x+5

$\Rightarrow (x^3-5x^2)-(x-5)$

$\Rightarrow x^2(x-5)-(x-5)$

$(x-5)(x^2-1)$

$(x-5)(x+1)(x-1)$

1) $x^4-3x^3+4x^2-6x+4$

$(x^4+4x^2+4)-(3x^3+6x)$

$(x^2+2)^2-3x(x^2+2)$

$(x^2+2)(x^2-3x+2)$

$(x^2+2)(x-2)(x-1)$

2) x^3-7x-6

$x^3+x^2-x^2-7x-6$

$(x^3+x^2)-(x^2+7x+6)$

$x^2(x+1)-(x+6)(x+1)$

$(x+1)(x^2-x-6)$

$(x+1)(x-3)(x+2)$

2) $\frac{4}{x^2+xy}+\frac{4}{xy+y^2}$

$\Rightarrow \frac{4}{x(x+y)}+\frac{4}{y(x+y)} \Rightarrow \frac{4y+4x}{xy(x+y)} \Rightarrow$

$\Rightarrow \frac{4\cancel{(x+y)}}{x\cancel{y}\cancel{(x+y)}} \Rightarrow \frac{4}{x}$

b) $\dfrac{2x}{x^2+3x+2} - \dfrac{x}{(x^2-4)}$

$\Rightarrow \dfrac{2x}{(x+2)(x+1)} - \dfrac{x}{(x+2)(x-2)} \Rightarrow \dfrac{2x(x-2)-x(x+1)}{(x+1)(x+2)(x-2)}$

$\Rightarrow \dfrac{2x^2-4x-x^2-x}{(x+1)(x+2)(x-2)} \Rightarrow \dfrac{x^2-5x}{(x+1)(x+2)(x-2)}$

$\Rightarrow \underline{\dfrac{x(x-5)}{(x+1)(x+2)(x-2)}}$

c) $\dfrac{3x}{x-1} + \dfrac{2}{x} - \dfrac{2}{x+1}$

$\Rightarrow \dfrac{3x(x+1)x+2(x-1)(x+1)-2x(x-1)}{x(x+1)(x-1)} \Rightarrow \dfrac{3x^3+3x^2\cancel{+2x^2}\cancel{-2}\cancel{-2x^2}\cancel{+2}}{x(x+1)(x-1)}$

$\Rightarrow \dfrac{3x^3+3x^2}{x(x+1)(x-1)} \Rightarrow \dfrac{3\cancel{x^2}\cancel{(x+1)}}{\cancel{x}\cancel{(x+1)}(x-1)}$

$\Rightarrow \dfrac{3x}{x-1}$ //

d) $\dfrac{4}{x^2-3x-4} + \dfrac{3}{x^2-16} - \dfrac{7}{x^2+5x+4}$

$\Rightarrow \dfrac{4}{(x-4)(x+1)} + \dfrac{3}{(x+4)(x-4)} - \dfrac{7}{(x+4)(x+1)}$

$\Rightarrow \dfrac{4(x+4)+3(x+1)-7(x-4)}{(x+1)(x+4)(x-4)} \Rightarrow \dfrac{\cancel{4x}+16+\cancel{3x}+3-\cancel{7x}+28}{(x+1)(x+4)(x-4)}$

$$\Rightarrow \frac{47}{(x+1)(x+4)(x-4)}$$

$$) \quad \frac{x^2+6x+9}{x^2-9} \times \frac{x-3}{4}$$

$$\Rightarrow \frac{(x+\cancel{3})(x+3)}{(x+\cancel{3})(\cancel{x-3})} \times \frac{(\cancel{x-3})}{4}$$

$$\Rightarrow \frac{x+3}{4}$$

$$) \quad \frac{xy-x}{y^2-1} \times \frac{y+1}{x+2} \times \frac{2x+4}{5x}$$

$$\Rightarrow \frac{\cancel{x(y-1)}}{\cancel{(y+1)(y-1)}} \times \frac{\cancel{(y+1)}}{\cancel{(x+2)}} \times \frac{2\cancel{(x+2)}}{5\cancel{x}}$$

$$\Rightarrow 2/5$$

$$) \quad \left(\frac{2}{x-3} - \frac{3}{x-2}\right) \cdot \left(\frac{3x}{x-5}\right)$$

$$\Rightarrow \frac{2(x-2)-3(x-3)}{(x-2)(x-3)} \times \frac{3x}{(x-5)} \Rightarrow \frac{2x-4-3x+9}{(x-2)(x-3)} \times \frac{3x}{x-5}$$

$$\Rightarrow \frac{-x+5}{(x-2)(x-3)} \times \frac{3x}{x-5} \Rightarrow \frac{-\cancel{(x-5)}}{(x-2)(x-3)} \times \frac{3x}{\cancel{(x-5)}}$$

$$\Rightarrow \frac{3x}{(x-2)(3-x)}$$

h) $\dfrac{5a^2-a-4}{a^3-1}$

$\Rightarrow \dfrac{(\cancel{5}a-\cancel{5})(5a+4)}{\cancel{5}(a-1)(a^2+a+1)} \Rightarrow \dfrac{\cancel{(a-1)}(5a+4)}{\cancel{(a-1)}(a^2+a+1)}$

$\Rightarrow \dfrac{5a+4}{a^2+a+1}$

i) $\left(\dfrac{\dfrac{x}{x+y}-\dfrac{y}{x-y}}{\dfrac{x}{x-y}+\dfrac{y}{x+y}}\right)$

$\Rightarrow \dfrac{\dfrac{x(x-y)-y(x+y)}{\cancel{(x+y)(x-y)}}}{\dfrac{x(x+y)+y(x-y)}{\cancel{(x+y)(x-y)}}} \Rightarrow \dfrac{x(x-y)-y(x+y)}{x(x+y)+y(x-y)}$

$\Rightarrow \dfrac{x^2-xy-xy-y^2}{x^2+xy+xy-y^2} \Rightarrow \dfrac{x^2-2xy-y^2}{x^2+2xy-y^2}$

j) $\dfrac{a^6+a^4+a^2+1}{a^3+a^2+a+1} \Rightarrow$

$\Rightarrow \dfrac{(a^6+a^4)+(a^2+1)}{(a^3+a^2)+(a+1)} \Rightarrow \dfrac{a^4(a^2+1)+(a^2+1)}{a^2(a+1)+(a+1)} \Rightarrow \dfrac{\cancel{(a^2+1)}(a^4+1)}{\cancel{(a^2+1)}(a+1)}$

$\Rightarrow \dfrac{a^4+1}{a+1}$

k) $\left(\frac{b}{a+b}+a\right)\left(\frac{a}{a-b}-b\right)-\left(\frac{a}{a+b}+b\right)\left(\frac{b}{a-b}-a\right)$

$\Rightarrow \left(\frac{b+a(a+b)}{a+b}\right)\left(\frac{a-b(a-b)}{a-b}\right)-\left(\frac{a+b(a+b)}{a+b}\right)\left(\frac{b-a(a-b)}{a-b}\right)$

$\Rightarrow \left(\frac{b+a^2+ab}{a+b}\right)\left(\frac{a-ab+b^2}{a-b}\right)-\left(\frac{a+ab+b^2}{a+b}\right)\left(\frac{b-a^2+ab}{a-b}\right)$

$\Rightarrow \frac{ab-ab^2+b^3+a^3-a^3b+\cancel{a^2b^2}+a^2b-\cancel{a^2b^2}+ab^3}{(a+b)(a-b)} - \frac{ab-a^3+a^2b+ab^2-a^3b+\cancel{a^2b^2}+b^3-\cancel{a^2b^2}+ab}{(a+b)(a-b)}$

$\Rightarrow \frac{\cancel{ab}-ab^2+\cancel{b^3}+a^3-\cancel{a^3b}+\cancel{a^2b}+\cancel{ab^3}-\cancel{ab}+a^3-\cancel{a^2b}-ab^2+\cancel{a^3b}-\cancel{b^3}-\cancel{ab^3}}{(a+b)(a-b)}$

$\Rightarrow \frac{2a^3-2ab^2}{a^2-b^2} \Rightarrow \frac{2a\cancel{(a^2-b^2)}}{\cancel{(a^2-b^2)}}$

$\Rightarrow 2a$

l) $\frac{1}{1-a}-\frac{1}{1+a}-\frac{2a}{1+a^2}-\frac{4a^3}{1+a^4}-\frac{8a^7}{1+a^8}$

$\Rightarrow \frac{8a^7}{(1+a^8)}\times\frac{(1-a^8)}{(1-a^8)} = \frac{8a^7(1-a^8)}{(1+a^8)(1+a^4)(1+a^2)(1+a)(1-a)}$

$\Rightarrow \frac{(1+a^8)(1+a^4)(1+a^2)(1+a)-(1+a^8)(1+a^4)(1+a^2)(1-a)-2a(1+a^8)(1+a^4)(1-a)(1+a)-4a^3}{(1+a^8)(1-a^8)}$

$\underline{(1+a^8)(1+a^2)(1+a)(1-a)-8a^7(1-a^8)}$

36.-

a) $a^2b^2(b-a)+b^2c^2(c-b)+a^2c^2(a-c)$

$\Rightarrow a^2b^3-a^3b^2+b^2c^3-b^3c^2+a^3c^2-a^2c^3$

$\Rightarrow (a^2b^3-b^3c^2)-(a^3b^2-b^2c^3)+(a^3c^2-a^2c^3)$

$\Rightarrow b^3(a^2-c^2)-b^2(a^3-c^3)+a^2c^2(a-c)$

$\Rightarrow b^3(a+c)(a-c)-b^2(a-c)(a^2+ac+c^2)+a^2c^2(a-c)$

$\Rightarrow (a-c)[b^3(a+c)-b^2(a^2+ac+c^2)+a^2c^2]$

$\Rightarrow (a-c)[ab^3+b^3c-b^2a^2-ab^2c-b^2c^2+a^2c^2]$

$\Rightarrow (a-c)[(ab^3+b^3c)-(a^2b^2+ab^2c)-(a^2c^2+b^2c^2)]$

$\Rightarrow (a-c)[b^3(a+c)-ab^2(a+c)-c^2(a^2-b^2)]$

$\Rightarrow (a-c)[(a+c)b^2\{b-a\}-c^2(a+b)(a-b)]$

$\Rightarrow (a-c)[b^2(a+c)(b-a)+(a+b)(b-a)]$

$\Rightarrow (a-c)(b-a)[b^2(a+c)+(a+b)]$

b) $(a-b)^3-(a-c)^3+(b-c)^3$

$\Rightarrow a^3-3a^2b+3ab^2-b^3-(a^3-3a^2c+3ac^2-c^3)+b^3-3b^2c+3bc^2-c^3$

$\Rightarrow \cancel{a^3}-3a^2b+3ab^2-\cancel{b^3}-\cancel{a^3}+3a^2c-3ac^2+\cancel{c^3}+\cancel{b^3}-3b^2c+3bc^2-\cancel{c^3}$

$3(-a^2b+ab^2+a^2c-ac^2-b^2c+bc^2)$

$3[-(a^2b-ab^2)+(a^2c-b^2c)-(ac^2-bc^2)]$

$3[-ab(a-b)+c(a^2-b^2)-c^2(a-b)]$

$\Rightarrow 3(a-b)[-ab + c(a+b) - c^2]$

$\Rightarrow 3(a-b)[-ab + ac + bc - c^2]$

$\Rightarrow 3(a-b)[-(ab-ac) + (bc-c^2)]$

$\Rightarrow 3(a-b)[-a(b-c) + c(b-c)]$

$\Rightarrow 3(a-b)(b-c)[c-a]$

$\Rightarrow a^4 - 18a^2 + 81$

$\Rightarrow (a^2-9)(a^2-9)$

$\Rightarrow (a+3)(a-3)(a+3)(a-3)$

$\Rightarrow (a+3)^2(a-3)^2$

$\Rightarrow 3x^4 - 10x^3 + 10x - 3$

$\Rightarrow (3x^4 - 3) - (10x^3 - 10x)$

$\Rightarrow 3(x^4-1) - 10x(x^2-1)$

$\Rightarrow 3(x^2+1)(x^2-1) - 10x(x^2-1)$

$\Rightarrow 3(x^2-1)(3(x^2+1) - 10x)$

$\Rightarrow 3(x+1)(x-1)(3x^2-10x+3)$

$\Rightarrow 3(x+1)(x-1)\underline{(\cancel{3}x-\cancel{3})(3x-3)}$

$\Rightarrow (x+1)(x-1)(x-1)(3x-3)$

$\Rightarrow 3(x+1)(x-1)(x-1)(x-1)$

$3(x+1)(x-1)^3$

e) $(3x-6)(x^2-1)-(5x-10)(x-1)^2$

$\Rightarrow 3(x-2)(x+1)(x-1)-5(x-2)(x-1)^2$

$\Rightarrow (x-2)(x-1)[3(x+1)-5(x-1)]$

$\Rightarrow (x-1)(x-2)[3x+3-5x+5]$

$\Rightarrow (x-1)(x-2)[-2x+8]$

$\Rightarrow 2(x-1)(x-2)(4-x)$

37. a) $\dfrac{1}{(a-b)(a-c)}+\dfrac{1}{(b-c)(b-a)}+\dfrac{1}{(c-a)(c-b)}$

$\Rightarrow \dfrac{1}{(a-b)(a-c)}-\dfrac{1}{(a-b)(b-c)}+\dfrac{1}{(b-c)(a-c)}$

$\Rightarrow \dfrac{b-c-(a-c)+a-b}{(a-b)(b-c)(a-c)}$

$\Rightarrow \dfrac{\cancel{b}-\cancel{c}-\cancel{a}+\cancel{c}+\cancel{a}-\cancel{b}}{(a-b)(b-c)(a-c)} \Rightarrow \underline{\underline{0}}$

b) $\dfrac{a}{a^2-1}+\dfrac{a^2+a-1}{a^3-a^2+a-1}+\dfrac{a^2-a-1}{a^3+a^2+a+1}-\dfrac{2a^2}{a^4-1}$

$\Rightarrow \dfrac{a}{(a+1)(a-1)}+\dfrac{a^2+a-1}{a^2(a-1)+(a-1)}+\dfrac{a^2-a-1}{a^2(a+1)+(a+1)}-\dfrac{2a^2}{(a^2+1)(a+1)(a-1)}$

$\Rightarrow \dfrac{a}{(a+1)(a-1)}+\dfrac{a^2+a-1}{(a-1)(a^2+1)}+\dfrac{a^2-a-1}{(a+1)(a^2+1)}-\dfrac{2a^2}{(a^2+1)(a+1)(a-1)}$

$$\Rightarrow \frac{a(a^2+1)+(a+1)(a^2+a-1)+(a+1)(a^2-a-1)-2a^2}{(a+1)(a-1)(a^2+1)}$$

$$\frac{a^3+\cancel{a}+a^3+\cancel{a^2}-\cancel{a}+\cancel{a^2}+a-\cancel{1}+a^3-\cancel{a^2}+\cancel{a}-\cancel{a^2}-\cancel{a}+\cancel{1}-2a^2}{(a+1)(a-1)(a^2+1)}$$

$$\frac{3a^3-2a^2+a}{(a+1)(a-1)(a^2+1)} \Rightarrow \frac{a(3a^2-2a+1)}{(a+1)(a-1)(a^2+1)}$$

$$\frac{a-c}{a^2+ac+c^2} \times \frac{a^3-c^3}{a^2b-bc^2}\left(1+\frac{c}{a-c}-\frac{1+c}{c}\right) \div \frac{c(1+c)-a}{bc}$$

$$\frac{a-c}{\cancel{(a^2+ac+c^2)}} \times \frac{(a-c)\cancel{(a^2+ac+c^2)}}{b(a^2-c^2)}\left(\frac{c(a-c)+c^2-(1+c)(a-c)}{c(a-c)}\right) \times \frac{bc}{c+c^2-a}$$

$$\frac{\cancel{a-c}}{b(a+c)\cancel{(a-c)}}\left(\frac{ac-\cancel{c^2}+\cancel{c^2}-(a-c+ac-c^2}{\cancel{c}(a-c)}\right)\frac{b\cancel{c}}{c+c^2-a}$$

$$\frac{1}{\cancel{b}(a+c)}\left(\frac{\cancel{ac}-a+c-\cancel{ac}+c^2}{a-c}\right) \times \frac{\cancel{b}}{c+c^2-a}$$

$$\frac{1}{a+c} \times \frac{\cancel{c+c^2-a}}{a-c} \times \frac{1}{\cancel{c+c^2-a}}$$

$$\Rightarrow \frac{1}{a^2-c^2}$$

d) $\dfrac{a+b}{(b-c)(c-a)} + \dfrac{b+c}{(c-a)(a-b)} + \dfrac{c+a}{(a-c)(b-c)}$

$\Rightarrow -\dfrac{a+b}{(b-c)(a-c)} - \dfrac{b+c}{(a-b)(a-c)} + \dfrac{c+a}{(a-c)(b-c)}$

$\Rightarrow \dfrac{-(a+b)(a-b) - (b+c)(b-c) + (c+a)(a-b)}{(a-b)(a-c)(b-c)}$

$\Rightarrow \dfrac{-(a^2-b^2) - (b^2-c^2) + ac - bc + a^2 - ab}{(a-b)(a-c)(b-c)}$

$\Rightarrow \dfrac{\cancel{-a^2} \cancel{+b^2} \cancel{-b^2} + c^2 + ac - bc \cancel{+a^2} - ab}{(a-b)(a-c)(b-c)} \Rightarrow \dfrac{c^2+ac-bc-ab}{(a-b)(a-c)(b-c)}$

$\Rightarrow \dfrac{c(c+a)-(bc+ab)}{(a-b)(a-c)(b-c)} \Rightarrow \dfrac{c(c+a)-b(c+a)}{(a-b)(a-c)(b-c)}$

$\Rightarrow \dfrac{(c+a)(c-b)}{(a-b)(a-c)(b-c)} \Rightarrow -\dfrac{(c+a)\cancel{(c-b)}}{(a-b)(a-c)\cancel{(c-b)}} \Rightarrow \dfrac{c+a}{(a-b)(c-a)}$

e) $a^3+b^3+c^3 = 3abc \quad \therefore \quad a+b+c=0$

$(a+b+c)^3 = 0^3$

$a^3+b^3+c^3+3a^2b+3a^2c+3b^2c+3ab^2+3ac^2+3bc^2+6abc=0$

$\quad + \quad +3a^3-3a^3 \quad +3b^3-3b^3 + 3c^3-3c^3$

$a^3+b^3+c^3+3a^2\underbrace{(a+b+c)}_{0}-3a^3+3b^2\underbrace{(a+b+c)}_{0}-3b^3+3c^2\underbrace{(a+b+c)}_{0}-3c^3+6abc=0$

$-2a^3-2b^3-2c^3 = -6abc$

$-2(a^3+b^3+c^3) = -6abc$

$\}\ a^3+b^3+c^3 = 3abc$

38.- $\dfrac{5x+2}{x+1}$ NO DEFINIDA

$x+1=0$

$x=-1$

39.- $\dfrac{(x^3-y^3)(x^2+2xy+y^2)}{(x^2-y^2)(x^2+xy+y^2)}-x-y$

$\Rightarrow \dfrac{\cancel{(x-y)}\cancel{(x^2+xy+y^2)}(x+y)^{\cancel{2}}}{\cancel{(x+y)}\cancel{(x-y)}\cancel{(x^2+xy+y^2)}}-x-y$

$\Rightarrow x+y-x-y$

$\Rightarrow \underline{\underline{0}}$

$\boxed{\text{Valor Absoluto}}$

40.- $S=\{-2,-1,0,1,2\}$

$a\#b=|a-b|-2$

a) $1\#0=-1$

$|1-0|=2$

$1-2=-1$ Correcto

b) $\underbrace{(-2 \# -1)}$ $\# 1 = 0$

$\underbrace{|-2-(-1)|}$ $- 2$

$\underbrace{1 - 2}$

$\underbrace{-1 \quad \# 1}$

$\underbrace{|-1-1|-2}$

$2 - 2 = 0$

Correcto

c) Verdadero ; $|a-b| = |b-a|$

d) $a \# 0 = |a| - 2$

$|a-0| - 2$

$|a| - 2$

Verdadero

e) $a \# a = a$

$|a-a| - 2$

$\underbrace{0 - 2}_{-2} \neq a$

Falso

41.

a) $\underbrace{(\sqrt{4} = -2)}_{0} \Rightarrow (-2\pi \in I)$

$\underbrace{\Rightarrow}_{1}$

26

b) $\underbrace{6\div(10\div5)=3}_{1} \vee \underbrace{(-15)^{-2}\in \text{Negativo}}_{0}$; VERDADERO

1

c) $\underbrace{\frac{2e}{e}\in I}_{0} \wedge \underbrace{|x-e|=|e-x|}_{1}$; FALSO

$\underbrace{0 \wedge 1}_{0}$

d) $\underbrace{(\sqrt{2}\in I)}_{1} \Rightarrow \underbrace{(-3=|-4|)}_{1}$; VERDADERO

$1 \Rightarrow 1 \equiv 1$

e) VERDADERO

a) $\underbrace{|-7|+|3|}_{10}-|-5| = 5$

$\underbrace{10-5}_{5}$

b) $\underbrace{|6-9|}_{3}+\underbrace{|10-4|}_{6}+\underbrace{|5|}-\underbrace{|-5|} = 9$

$3 + 6 + \cancel{5} - \cancel{5}$

$\Rightarrow 9$

c) $\underbrace{|4-8|}-\underbrace{|-6|}+\underbrace{|14-11|}-\underbrace{|-8|}_{8} = -7$

$4 - 6 + 3 - 8$

$7 - 14$

$\Rightarrow -7$

d) $|3(-1)-(-1)| - |2(-1)-(-1)| - |3-(-1)|$

$\Rightarrow |-3+1| - |-2+1| - |3+1|$

$2 - 1 - 4 \Rightarrow -3$ //

e) $2(1)-3-|3(1)-2(2)|+|4-5(-2)|$

$2-3-|3-4|+|4+10|$

$-1 \quad -1 \quad +14$

$\Rightarrow 12$

f) $|3(0)-1|-[3(-4)+6]-|-3-2(-4)|$

$1-[-12+6]-|-3+8|$

$1+6-5$

$\Rightarrow \underline{\underline{2}}$

43.-

a) $|x-a| \Rightarrow x-a \geq 0 \ \wedge -x+a<0 \Rightarrow \sqrt{(x-a)^2}$

b) $|1-x| \Rightarrow 1-x \geq 0 \ \wedge \ -1+x<0 \Rightarrow \sqrt{(1-x)^2}$

c) $|x-a+b| \Rightarrow x-a+b \geq 0 \ \wedge - x+a-b<0 \Rightarrow \sqrt{(x-a+b)^2}$

4.— $x \rightarrow$ número de canicas original

	regala	sobra
1	$\frac{x}{2}$	$\frac{x}{2}$
2	$\frac{1}{3}\left(\frac{x}{2}\right)$	$\frac{2}{3}\left(\frac{x}{2}\right) = 6$

$$\Rightarrow \frac{x}{3} = 6$$

$$\boxed{x = 18 \text{ canicas}}$$ a) correcto

5.—

$x \Rightarrow$ pastel

Elena: $\frac{x}{2}$

Antonio: $\frac{x}{4}$

mamá: $\frac{x}{4}$

$$\frac{x}{2} + \frac{x}{4} + \frac{x}{4} \Rightarrow x \text{ (todo el pastel)}$$

sobra: $\underline{\underline{0}}$

d) correcto

6.—

1er número: x

2do número: $3x+1$

3er número: $2(3x+1)-1$

$$x + (3x+1) + (2(3x+1)-1) = 12$$

$$x + 3x + 1 + 6x + 2 - 1 = 12$$

$$10x = 12 - 2$$

$$10x = 10$$

$$\boxed{x = 1}$$

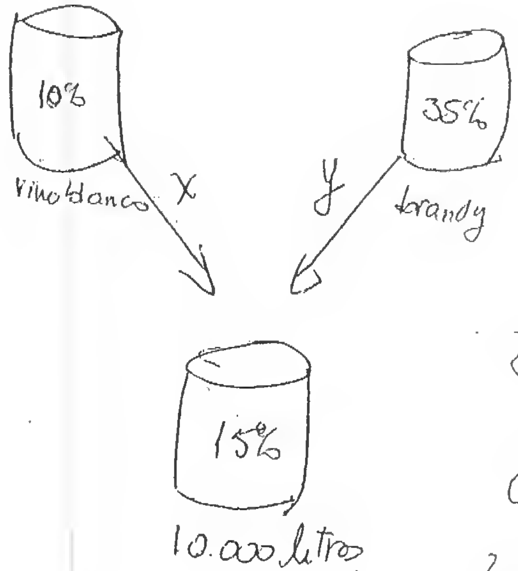
1º número $\Rightarrow 1$

2º núm. $\Rightarrow 4$

3º número $\Rightarrow 7$

d) correcto

47.-



$$\left\{ \boxed{x+y=10.000} \right. (-0,1)$$

$$0,1x+0,35x=0,15(10.000)$$

$$\left\{ \boxed{0,1x+0,35x=1500} \right.$$

$$\begin{array}{l} -0,1x-0,1y=-1000 \\ \underline{0,1x+0,35y=1500} \\ // \quad 0,25y=500 \end{array}$$

$$y=\frac{500}{0,25}$$

$$\boxed{y=2000 \text{ litros}} \quad \boxed{x=8000 \text{ litros}}$$

a) correcto

48.- $\frac{2}{x+5}+\frac{1}{x-5}+\frac{20}{(x+5)(x-5)}=0$

$$\Rightarrow \frac{2(x-5)+x+5+20}{(x+5)(x-5)}=0 \Rightarrow 2x-10+x+25=0$$

$$3x=-15$$

$$x=-\frac{15}{3}$$

$$\boxed{x=-5}$$

$Ap(x): \{-5\}$

49.– $kx^2+4kx+3=x^2$

Suma de raíces $=10$
$(x_1+x_2=10)$

Suma de raíces: $-\frac{b}{a}=10$

$kx^2-x^2+4kx+3=0$

$x^2\underbrace{(k-1)}_{a}+\underbrace{4k}_{b}x+3=0$

$-\frac{4k}{k-1}=10$

$\frac{-4k}{k-1}-10=0$

$\frac{-4k-10(k-1)}{k-1}=0$

$\underbrace{-4k-10k}+10=0$

$-14k=-10$

$k=\frac{-10}{-14}$

$\boxed{k=\frac{5}{7}}$ c) Correcto

50.– $p(x): \sqrt{x}+\sqrt{x+2}=3$

$(\sqrt{x}+\sqrt{x+2})^2=3^2$

$(\sqrt{x})^2+2\sqrt{x}\sqrt{x+2}+(\sqrt{x+2})^2=9$

$x+2\sqrt{x(x+2)}+x+2=9$

$2\sqrt{x^2+2x}=9-2-2x$

$(2\sqrt{x^2+2x})^2=(7-2x)^2$

$4(x^2+2x)=49-28x+4x^2$

$4x^2+8x=49-28x+4x^2$

$\cancel{4x^2}+8x+28x-\cancel{4x^2}=49$

$36x=49$

$\boxed{x=\frac{49}{36}}$

51.- $p(x): \frac{\cancel{x}}{x-2} + \frac{x}{x^2-4} = 0$

$\Rightarrow \frac{x(x+2)+x}{x^2-4} = 0 \quad \Rightarrow x^2+2x+x=0$

$\Rightarrow x^2+3x=0$

$\Rightarrow x(x+3)=0$

$\Rightarrow \boxed{x=0} \qquad x+3=0$

$\boxed{x=-3}$

Suma: $0-3$

$\Rightarrow -3$

b) correcto

$x_1+x_2 = -\frac{b}{a}$

$\Rightarrow -\frac{3}{1}$

$\Rightarrow -3$

52.- $|9-x^2| - \frac{7x}{22} - \frac{48}{11} = 0$

$22\left(9-x^2 - \frac{7x}{22} - \frac{48}{11}\right) = 0 \cdot 22 \wedge 22\left(-9+x^2 - \frac{7x}{22} - \frac{48}{11}\right) = 0 \cdot 22$

$198 - 22x^2 - 7x - 96 = 0 \quad \wedge \quad -198+22x^2-7x-96=0$

$-22x^2-7x+102=0 \qquad 22x^2-7x-294=0$

$22x^2+7x-102=0$

Suma de soluciones: $-\frac{b}{a}$

$x_1+x_2 = \frac{-7}{22} \qquad x_3+x_4 = -\frac{-7}{22}$

$\Rightarrow \frac{7}{22}$

Suma: $-\frac{7}{22} + \frac{7}{22}$

$\Rightarrow 0$

a) correcto

3.- $x \Rightarrow$ costo x hora normal

dinero $\Rightarrow$ precio * cantidad

$$435 = 40x + 12 * 1{,}5x$$

$$435 = 40x + 18x$$

$$58x = 435$$

$$x = \frac{435}{58}$$

$$\boxed{x = \$\,7{,}5}$$ d) correcto

4.-

a) $$\frac{x^2+17}{x^2-1} = \frac{x-2}{x+1} - \frac{5}{1-x}$$

$$\Rightarrow \frac{x^2+17}{x^2-1} - \frac{x-2}{x+1} - \frac{5}{x-1} = 0 \Rightarrow \frac{x^2+17-(x-2)(x-1)-5(x+1)}{x^2-1} = 0$$

$$\Rightarrow x^2+17-(x^2-x-2x+2)-5x-5=0 \Rightarrow \cancel{x^2}+17-\cancel{x^2}+3x-\cancel{2}-5x-\cancel{5}=0$$

$$-2x+10=0 \Rightarrow -2x=-10$$

$$\boxed{x=5}$$

b) $$\frac{ax+b}{ax-b} - \frac{ax-b}{ax+b} = \frac{4b}{a^2x^2-b^2}$$

$$\Rightarrow \frac{ax+b}{ax-b} - \frac{ax-b}{ax+b} - \frac{4b}{(ax+b)(ax-b)} = 0 \Rightarrow \frac{(ax+b)^2-(ax-b)^2-4b}{(ax+b)(ax-b)} = 0$$

$$\Rightarrow a^2x^2+2abx+b^2-(a^2x^2-2abx+b^2)-4b=0 \Rightarrow \cancel{a^2x^2}+2abx+\cancel{b^2}-\cancel{a^2x^2}+2abx-\cancel{b^2}-4b=0$$

$$4abx-4b=0 \Rightarrow 4b(ax-1)=0 \quad ax=1 \Rightarrow \boxed{x=1/a}$$

c) $1-|x|=5$

$-|x|=5-1$

$-|x|=4$

$|x|=-4$

$\boxed{x=-4} \wedge -x=-4$

$\boxed{x=4}$

d) $|5-x|=13-x$

$5-x=13-x \quad \wedge \quad -5+x=13-x$

$-x+x=13-5 \qquad x+x=13+5$

$0=8 \qquad 2x=18$

FALSO $\qquad \boxed{x=9}$

e) $|3-x|-|x+2|=5$

$3-x-|x+2|=5 \quad \wedge \quad -3+x-|x+2|=5$

$3-x-(x+2)=5 \wedge 3-x-(-x-2)=5$

$3-x-x-2=5 \wedge 3-\cancel{x}+\cancel{x}+2=5$

$-2x=5+2-3 \qquad 5=5$

$-2x=4 \qquad$ Todo los números

$\boxed{x=-2}$

$-3+x-(x+2)=5 \wedge -3+x-(-x-2)=5$

$-3+\cancel{x}-\cancel{x}-2=5 \wedge -3+x+x+2=5$

$-5\neq 5 \qquad 2x=5-2+3$

FALSO; NINGÚN NÚMERO $\qquad 2x=6$

$\boxed{x=3}$

$\boxed{x=-3}$

$|3-(-3)|-|-3+2|=5$

$5=5$

SI

$\boxed{x=0}$

$|3-0|-|0+2|=5$

$1\neq 5$

NO

$x=4$

$|3-4|-|4+2|=5$

$-5\neq 5$

NO

Sol: $(-\infty,-2]$

) $|2x-3| = |x+7|$

$2x-3 = |x+7| \quad \wedge \quad -2x+3 = |x+7|$

$x-3 = x+7 \wedge 2x-3 = -x-7 \quad \wedge \quad -2x+3 = x+7 \wedge -2x+3 = -x-7$

$x-x = 7+3 \quad \wedge \quad 2x+x = -7+3 \qquad -2x-x = 7-3 \qquad -2x+x = -7-3$

$x = 10 \qquad 3x = -4 \qquad -3x = 4 \qquad -x = -10$

$\boxed{x = -4/3} \qquad \boxed{x = -4/3} \qquad \boxed{x = 10}$

Sol: $\{-4/3, 10\}$

) $|2x-3| = 5$

$2x-3 = 5 \wedge -2x+3 = 5$

$2x = 5+3 \quad \wedge \quad -2x = 5-3$

$2x = 8 \qquad -2x = 2$

$\boxed{x = 4} \qquad \boxed{x = -1}$

Sol: $\{-1, 4\}$

) $|x-2| + |x-1| = x-3$

$x-2+|x-1| = x-3 \quad \wedge \quad \wedge \quad -x+2+|x-1| = x-3$

$x-2+x-1 = x-3 \wedge \cancel{x}-2-\cancel{x}+1 = x-3 \quad \wedge \quad -\cancel{x}+2+\cancel{x}-1 = x-3 \wedge -x+2-x+1 = x-3$

$x-x = -3+3 \quad \wedge \quad -x = -3-1+2 \quad \wedge \quad -x = -3+1-2 \quad \wedge \quad -2x-x = -3-3$

$\boxed{x = 0} \qquad -x = -2 \qquad -x = -4 \qquad -3x = -6$

$\boxed{x = 2} \qquad \boxed{x = 4} \qquad \boxed{x = 2}$

$|0-2|+|0-1| \neq 0-3$ NO $\qquad |2-2|+|2-1| \neq 2-3$ NO $\qquad |4-2|+|4-1| \neq 4-3$ NO $\qquad$ NO

Sol: $\varnothing$

i) $4x + 6x = 9x^2 - 15x$

$\underbrace{4x + 6x}_{10x}$

$9x^2 - 15x - 10x = 0$

$9x^2 - 25x = 0$

$x(9x - 25) = 0$

$\boxed{x = 0}$

$9x - 25 = 0$

$9x = 25$

$\boxed{x = \frac{25}{9}}$

j) $(x+1)^2 |x+1| + 1 = 0$

$(x^2 + 2x + 1)(x+1) + 1 = 0 \quad \wedge \quad (x^2 + 2x + 1)(-x - 1) + 1 = 0$

$x^3 + x^2 + 2x^2 + 2x + x + 1 + 1 = 0 \quad \wedge \quad -x^3 - x^2 - 2x^2 - 2x - x - \cancel{1} + \cancel{1} = 0$

$x^3 + 3x^2 + 3x + 2 = 0 \quad \wedge \quad -x^3 - 3x^2 - 3x = 0$

$(x+1)^3 + 1 = 0$

$-x(x^2 + 3x + 3) = 0$

$\sqrt[3]{(x+1)^3} = \sqrt[3]{-1}$

$\boxed{x = 0}$

$x + 1 = -1$

No.

$\boxed{x = -2}$

No.

$x^2 + 3x + 3 = 0$

Tiene raices imaginarias

Solución : ϕ

5)

1) $x \in \mathbb{N}$

2) $n-1$

3) $n+1$

4) $2n$; $n \in \mathbb{N}$

5) $2n+1$; $n \in \mathbb{N}$

6) $(n+1)^2$

7) $(n+1)^2$

8) $2n+1$, $2(n+1)+1$; $n \in \mathbb{N}$

9) $(n+1)^2 - (n)^2$; $n \in \mathbb{N}$

10) $(2(n+1))^3 - (2n)^3$; $n \in \mathbb{N}$

11) $n - r = 0$; n es inverso aditivo de r

12) $n \times r = 1$; n es inverso multiplicativo de r

13) $\frac{1}{5+1}$

14) $3x$

15) $(a+b)^2$

16) a^2+b^2

17) a b c

18) $100c+10d+u$

19) $\frac{p}{q}$

20) $\frac{m}{2}$

$\frac{n}{4}$

22) $|x|$

23) $\frac{m+n}{2}$

24) $\sqrt{x}$

25) $\sqrt{ab}$

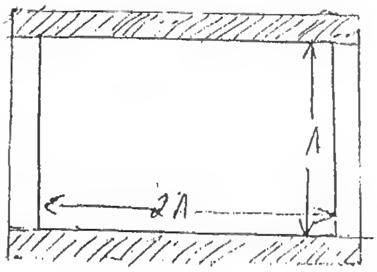
26) $MG = \sqrt{ab}$

27) $x \propto y$

28) $x \infty y$

29) $\overline{ab} < \delta$

56.– $L = 2A$



área marco $= 2(2A+4)2 + 2A \cdot 2$

$$244 = 4(2A+4) + 4A$$

$$244 = 8A + 16 + 4A$$

$$228 = 12A$$

$$A = \frac{228}{12} = 19$$

$$L = 2(19) = 38$$

57.– $x \to$ número

$$x - 4 + 2\tfrac{1}{2} = \tfrac{1}{3}x$$

$$x - \tfrac{1}{3}x = 4 - \tfrac{5}{2}$$

$$\frac{3x - x}{3} = \frac{8-5}{2}$$

$$\frac{2x}{3} = \frac{3}{2}$$

$$x = \frac{9}{4}$$

58.-

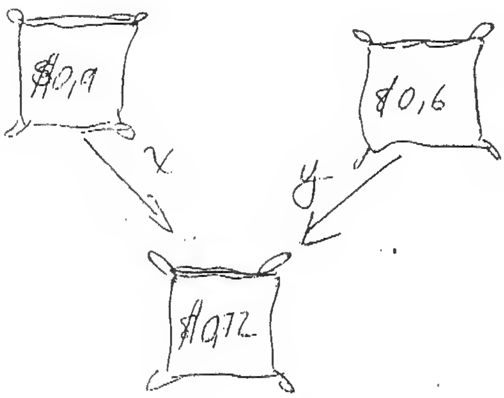
$$\frac{1}{t} = \frac{1}{15} + \frac{1}{20} + \frac{1}{30}$$

$$\frac{1}{t} = \frac{4+3+2}{60}$$

$$\frac{1}{t} = \frac{\cancel{9}^{3}}{\cancel{60}_{20}}$$

$$\boxed{t = \frac{20}{3}\ h}$$

59.-



$$0,9x + 0,6y = 0,72(x+y)$$

$$\cancel{90}x + \cancel{60}y = \cancel{72}(x+y)$$

$$\cancel{30}x + \cancel{20}y = \cancel{24}(x+y)$$

$$15x + 10y = 12(x+y)$$

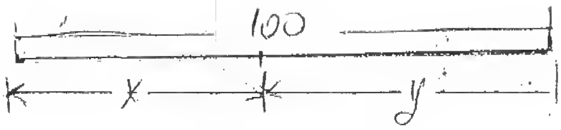
$$15x + 10y = 12x + 12y$$

$$15x - 12x = 12y - 10y$$

$$3x = 2y$$

$$\boxed{x = \frac{2}{3}y}$$

60.-



$$(x+y=100) \Rightarrow x=100-y$$

$$\boxed{\left(\frac{x}{4}\right)^2}\, x/4 + \boxed{\left(\frac{y}{4}\right)^2}\, y/4 = 397$$

$\frac{x}{4}$ $\frac{y}{4}$

$$\left(\frac{x}{4}\right)^2+\left(\frac{y}{4}\right)^2=397$$

$$\frac{x^2}{16}+\frac{y^2}{16}=397$$

$$x^2+y^2=397 \times 16$$

$$\boxed{x^2+y^2=6352}$$

$$(100-y)^2+y^2=6352$$

$$10.000-200y+y^2+y^2=6352$$

$$2y^2-200y+3648=0$$

$$y^2-100y+1824=0$$

$$y=\frac{100 \pm \sqrt{100^2-4(1)(1824)}}{2(1)}$$

$$y=\frac{100 \pm 52}{2} \rightarrow 76,\ 24$$

61.-

$$\text{Ingreso} = 120 \times 40$$

$$\Rightarrow \$4800$$

$$(120+5x)\cdot(40-x)=4800$$

$$\cancel{4800}-120x+200x-5x^2=\cancel{4800}$$

$$5x^2-80x=0$$

$$5x^2(x-16)=0$$

$$x=0 \quad \boxed{x=16}$$

$$\text{Precio} = 120+5x$$

$$\Rightarrow 120+5(16)$$

$$\Rightarrow \$200$$ c) correct

2.-

Ingreso = Precio * cantidad

$11475 = (180+5x)(60-x)$

$11475 = 10800 - 180x + 300x - 5x^2$

$5x^2 - 120x + 11475 - 10800 = 0$

$5x^2 - 120x + 675 = 0$

$5(x^2 - 24x + 135) = 0$

$x^2 - 24x + 135 = 0$

$(x-15)(x-9) = 0$

$x = 15 \wedge x = 9$

$Precio_1 = 180 + 5(15)$

$= 180 + 75$

$P_1 = \$255$

$P_2 = 180 + 5(9)$

$P_2 = 180 + 45$

$P_2 = \$225$

3.- $I \rightarrow$ tasa de interés en 1er año

$2I \rightarrow$ tasa de interés en 2do año

al final 1er año $= C(1 + \frac{I}{100})$

$= 100(1 + \frac{I}{100})$

$= 100(\frac{100+I}{100})$

al final 2do año $= C(1 + \frac{2I}{100})$

$11232 = (100+I)(\frac{100+2I}{100})$

$11232 = (100+I)(100+2I)$

$11232 = 10000 + 200I + 100I + 2I^2$

$2I^2 + 300I - 1232 = 0$

$\rightarrow I = \frac{-300 \pm \sqrt{300^2 - 4(2)(-1232)}}{2(2)}$

$I = \frac{-300 \pm 316}{4} \rightarrow 4$

$I_1 = 4\%$

$I_2 = 8\%$

41

64°

a) $17.500 = (600-5x)\cdot x$

$17.500 = 600x - 5x^2$

$5x^2 - 600x + 17500 = 0$

$5(x^2 - 120x + 3500) = 0$

$x^2 - 120x + 3500 = 0$

$(x-70)(x-50) = 0$

$x = 70 \vee x = 50$

b) $18.000 = (600-5x)\cdot x$

$18000 = 600x - 5x^2$

$5x^2 - 600x + 18000 = 0$

$5(x^2 - 120x + 3600) = 0$

$x^2 - 120x + 3600 = 0$ Precio $= 600 - 5(60)$

$(x-60)(x-60) = 0$ $\Rightarrow \$300$

$\boxed{x = 60}$

c) $U = I - C$

$5500 = (600-5x)\cdot x - (8000 + 75x)$

$5500 = 600x - 5x^2 - 8000 - 75x$

$5x^2 - 525x + 13500 = 0$

$5(x^2 - 105x + 2700) = 0$

$(x-60)(x-45) = 0$

$\boxed{x = 60} \vee \boxed{x = 45}$

d) $U = I - C$

$$5750 = (600 - 5x) \cdot x - (8000 + 75x)$$

$$5750 = 600x - 5x^2 - 8000 - 75x$$

$$5x^2 - 525x + 13750 = 0$$

$$5(x^2 - 105x + 2750) = 0$$

$$x^2 - 105x + 2750 = 0$$

$$(x - 55)(x - 50) = 0$$

$$\boxed{x = 55} \lor \boxed{x = 50}$$

Precio $= 600 - 5x$

$$P_1 = 600 - 5(55)$$

$$P_1 = \$325$$

$$P_2 = 600 - 5(50)$$

$$P_2 = 600 - 250$$

$$P_2 = \$350$$

5.- $\underbrace{L = A + 2}$

área $= A \cdot L$

$= A \cdot (A + 2)$

$\Rightarrow A^2 + 2A$.

incremento

$$L' = A + 2 + 3$$

$$\left.\begin{array}{l} L' = A + 5 \\ A' = A + 3 \end{array}\right\} \text{área}' = L' \cdot A'$$

$$\Rightarrow (A+5) * (A+3) = A^2 + 2A + 51$$

$$\cancel{A^2} + 3A + 5A + 15 = \cancel{A^2} + 2A + 51$$

$$8A - 2A = 51 - 15$$

$$6A = 36$$

$$\boxed{A = 6}$$

$$\boxed{L = 8}$$

66.—

$$\frac{1}{T}=\frac{1}{2\frac{1}{2}}+\frac{1}{3\frac{1}{3}}+\frac{1}{5}$$

$$\Rightarrow \frac{1}{T}=\frac{1}{\frac{5}{2}}+\frac{1}{\frac{10}{3}}+\frac{1}{5} \Rightarrow \frac{1}{T}=\frac{2}{5}+\frac{3}{10}+\frac{1}{5}$$

$$\Rightarrow \frac{1}{T}=\frac{4+3+2}{10} \Rightarrow \frac{1}{T}=\frac{9}{10} \Rightarrow \boxed{T=\frac{10}{9}h}$$

67.—

$$\frac{1}{T}=\frac{1}{3}+\frac{1}{4}+\frac{1}{6}$$

$$\Rightarrow \frac{1}{T}=\frac{4+3+2}{12} \Rightarrow \frac{1}{T}=\frac{\cancel{9}^{3}}{\cancel{12}_{4}}$$

$$\boxed{T=\frac{4}{3}\text{ día}}$$

68.— $T_y=8h$

$T_p=10h$

$T_c=12h$

$X \Rightarrow$ obra

$$X=V_1(1)+V_2T$$

$$X=(V_y+V_p)(1)+(V_y+V_c)T$$

$$X=\left(\frac{X}{8}+\frac{X}{10}\right)(1)+\left(\frac{X}{8}+\frac{X}{12}\right)T$$

$$X-\frac{X}{8}-\frac{X}{10}=\left(\frac{3X+2X}{24}\right)T$$

$$\frac{40X-5X-4X}{40}=\frac{5X}{24}T$$

$$\frac{31X}{\cancel{40}_{5}}=\frac{5X}{\cancel{24}_{3}}T$$

$$T=\frac{93}{25}\text{ horas}$$

$$T_{total}=T+1h.$$

$$T_{total}=\frac{93}{25}+1$$

$$\boxed{T_{total}=\frac{118}{25}\text{ horas}}$$

1°

$$I_2 - I_1 = 110$$

$$5000\left(\frac{i+1}{100}\right) - 4000\left(\frac{i}{100}\right) = 110$$

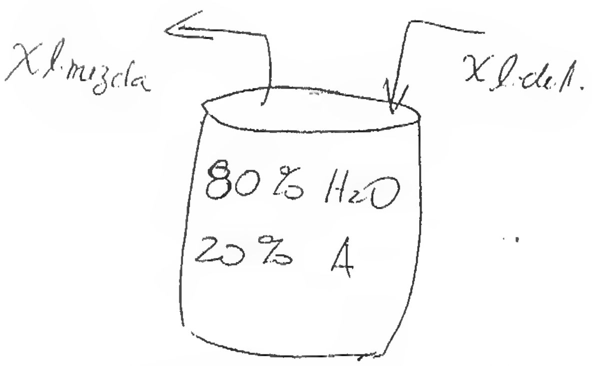
$$50\cancel{00}\left(\frac{i}{1\cancel{00}}\right) - 4\cancel{000}\left(\frac{i}{1\cancel{00}}\right) = 11\cancel{0}$$

$$5i + 5 - 4i = 11$$

$$5i - 4i = 11 - 5$$

$$\boxed{i = 6\%} \quad \boxed{i = 7\%}$$

1°



Concentración: $0{,}2(10-x) + x = 10(0{,}5)$

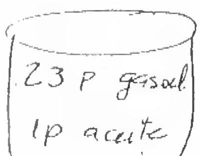
$$2 - 0{,}2x + x = 5$$

$$0{,}8x = 3$$

$$x = \frac{3}{0{,}8} \times \frac{5}{5} = \boxed{\frac{15}{4}\ l}$$

1°

combustible



1 litro (6 partes)

5P. gasolina

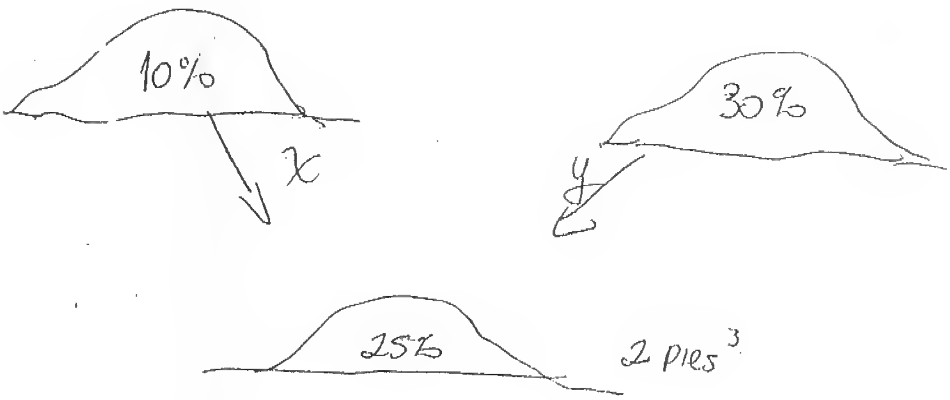
1p aceite $\Rightarrow$

$$5 + x = 23$$

$$x = 18\ p. \times \frac{1\ \text{litro}}{6p}$$

$$\boxed{x = 3\ \text{litros gasolina}}$$

72.-



$$\begin{cases} x+y=2 \quad (-1) \\ 0{,}1x+0{,}3y=2(0{,}25) \end{cases}$$

$0{,}1x+0{,}3y=0{,}5$

$\boxed{x+3y=5}$

$$\begin{array}{r} -x-y=-2 \\ x+3y=5 \\ \hline 2y=3 \end{array}$$

$\boxed{y=\frac{3}{2} \text{ pies}^3}$

$\boxed{x=\frac{1}{2} \text{ pies}^3}$

73.-

$x \to$ galones de extra

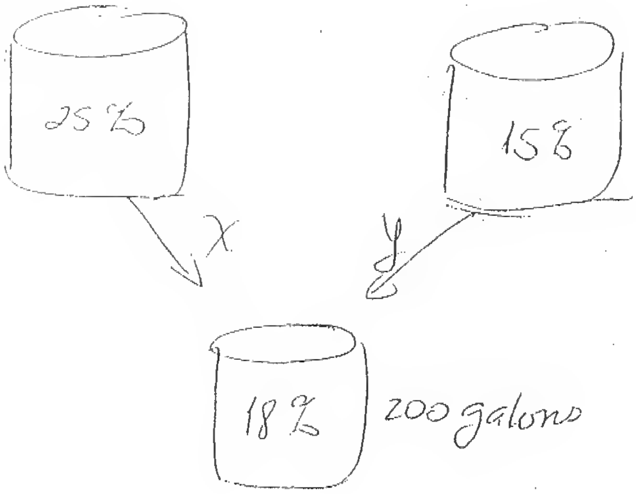
$y \to$ galones de super $\begin{cases} x+y=15000 \quad (-11) \\ 0{,}55x+0{,}6y=8550 \end{cases}$

$55x+60y=855000$

$$\begin{array}{r} 11x+12y=171000 \\ -11x-11y=-165000 \\ \hline \end{array}$$

$\boxed{y=6000 \text{ galones}}$ $\qquad x=15000-6000$

$\boxed{x=9000 \text{ galones}}$



$\boxed{x+y=200}\ (-3)$

$0{,}25x+0{,}15y=0{,}18(200)$

$0{,}25x+0{,}15y=36$

$25x+15y=3600$

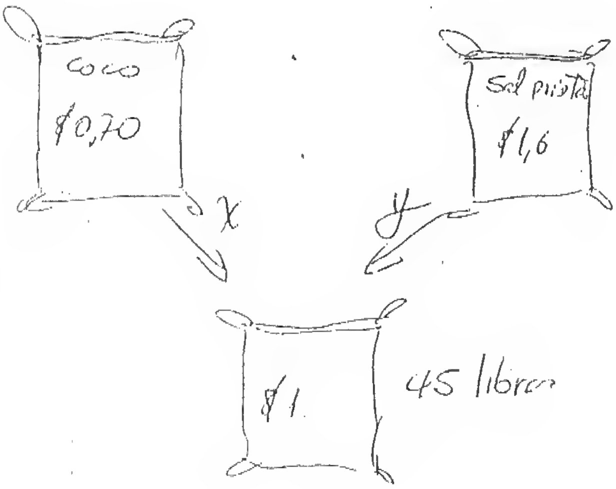
$\boxed{5x+3y=720}$

$$\begin{aligned}-3x-3y&=-600\\ 5x+3y&=720\\ \hline 2x\quad&=120\end{aligned}$$

$\boxed{x=60 \text{ galons}}$

$y=200-60$

$\boxed{y=140 \text{ galo}}$



$\boxed{x+y=45}\ (-7)$

$0{,}7x+1{,}6y=1(45)$

$\boxed{7x+16y=450}$

$$\begin{aligned}-7x-7y&=-315\\ 7x+16y&=450\\ \hline 9y&=135\end{aligned}$$

$\boxed{y=15 \text{ libras}}$

$x=45-15$

$\boxed{x=30 \text{ libras}}$

76.-

$x \to$ capital al 9%

$y \to$ capital al 6%

$I_{total} = 18000(0{,}08)$

$I_{total} = \$1440$

$\boxed{x + y = 18000}\ (-2)$

$0{,}09x + 0{,}06y = 1440$

$9x + 6y = 144000$

$\boxed{3x + 2y = 48.000}$

$-2x - 2y = -36000$

$3x + 2y = 48.000$

$\boxed{x = 12000}$

$y = 18.000 - 12000$

$\boxed{y = 6000}$

77.- $4x^2 - 4xy - y^2 = 1$

(x) $4x^2 - 4xy - (y^2 + 1) = 0$

$$x = \frac{4y \pm \sqrt{(4y)^2 - 4(4)(-(y^2+1))}}{2(4)}$$

$$x = \frac{4y \pm \sqrt{16y^2 + 16y^2 + 16}}{8}$$

$$x = \frac{4y \pm \sqrt{16(2y^2+1)}}{8}$$

$$x = \frac{\cancel{4}y \pm \cancel{4}\sqrt{2y^2+1}}{\cancel{8}_2}$$

$$\boxed{x = \frac{y \pm \sqrt{2y^2+1}}{2}}$$

(y) $y^2 + 4xy + (1 - 4x^2) = 0$

$$y = \frac{-4x \pm \sqrt{(-4x)^2 - 4(1)(1-4x^2)}}{2(1)}$$

$$y = \frac{-4x \pm \sqrt{16x^2 - 4 + 16x^2}}{2}$$

$$y = \frac{-4x \pm \sqrt{4(8x^2-1)}}{2}$$

$$y = \frac{-\cancel{4}^{2}x \pm \cancel{2}\sqrt{8x^2-1}}{\cancel{2}_1}$$

$$\boxed{y = -2x \pm \sqrt{8x^2-1}}$$

78.-

a) $p(x) = (x+1)(x+2)(x+3) = x(x+4)(x+5)$

$(x^2+2x+x+2)(x+3) = x(x^2+5x+4x+20)$

$(x^2+3x+2)(x+3) = x^3+9x^2+20x$

$\cancel{x^3}+3x^2+2x+3x^2+9x+6 = x^3+9x^2+20x$

$\cancel{x^3}+6x^2+11x+6-\cancel{x^3}-9x^2-20x=0$

$-3x^2-9x+6=0$

$-3(x^2+3x-2)=0$

$x^2+3x-2=0$

$x = \dfrac{-3 \pm \sqrt{3^2-4(1)(-2)}}{2(1)}$

$x = \dfrac{-3 \pm \sqrt{17}}{2}$

b) $(x+1)^3+(x-1)^2 = x^3+x+1$

$x^3+3x^2+3x+1+x^2-2x+1 = x^3+x+1$

$\cancel{x^3}+4x^2+\cancel{x}+2-\cancel{x^3}-\cancel{x}-1=0$

$4x^2+1=0$

$4x^2=-1$

$\sqrt{x^2} = \frac{-1}{4}$ No hay raices reales, sino imaginarias

$\boxed{x = \phi}$

c) $p(x): x^2-4ax+4a^2-c^2=0$

$x = \dfrac{4a \pm \sqrt{(4a)^2-4(1)(4a^2-c^2)}}{2(1)}$

$x = \dfrac{4a \pm \sqrt{16a^2-16a^2+4c^2}}{2}$

$x = \dfrac{\cancel{4}^{2}a \pm \cancel{2}^{1}c}{\cancel{2}_{1}}$

$\boxed{x = 2a \pm c}$

d) $q(x) = a^2x^2 + a(b-c)x - bc = 0$

$$x = \frac{-a(b-c) \pm \sqrt{[a(b-c)]^2 - 4a^2(-bc)}}{2a^2}$$

$$x = \frac{-a(b-c) \pm \sqrt{a^2b^2 - 2a^2bc + a^2c^2 + 4a^2bc}}{2a^2}$$

$$x = \frac{-a(b-c) \pm \sqrt{a^2b^2 + 2a^2bc + a^2c^2}}{2a^2}$$

$$x = \frac{-a(b-c) \pm \sqrt{(a^2(b+c)^2)}}{2a^2} \Rightarrow \frac{-\cancel{a}(b-c) \pm \cancel{a}(b+c)}{2a^{\cancel{2}}}$$

$$\therefore x = \frac{-(b-c) \pm (b+c)}{2a} \Rightarrow \frac{-\cancel{b}+c+\cancel{b}+c}{2a} \Rightarrow \frac{\cancel{2}c}{\cancel{2}a} \Rightarrow \frac{c}{a}$$

$$\frac{-b\cancel{+c}-b\cancel{-c}}{2a} \Rightarrow -\frac{\cancel{2}b}{\cancel{2}a} \Rightarrow -\frac{b}{a}$$

e) $$\frac{2x+a}{b} - \frac{x-b}{a} = \frac{3ax+(a-b)^2}{ab}$$

$$\frac{a(2x+a)-b(x-b)}{\cancel{ab}} = \frac{3ax+a^2-2ab+b^2}{\cancel{ab}}$$

$$2ax+a^2-bx+b^2 = 3ax+a^2-2ab+b^2$$

$$2ax-3ax-bx = \cancel{a^2}-2ab\cancel{+b^2}\cancel{-a^2}\cancel{-b^2}$$

$$-ax-bx = -2ab$$

$$-x(a+b) = -2ab$$

$$x = \frac{2ab}{a+b}$$

f) $p(x): \frac{x+a}{a-b} + \frac{x+a}{a+b} = \frac{x+b}{a+b} + \frac{2(x-b)}{a-b}$

$$\Rightarrow \frac{(x+a)(a+b)+(x+a)(a-b)}{\cancel{(a+b)(a-b)}} = \frac{(x+b)(a-b)+2(x-b)(a+b)}{\cancel{(a+b)(a-b)}}$$

$$ax+\cancel{bx}+a^2+\cancel{ab}+ax-\cancel{bx}+a^2-\cancel{ab} = ax-\cancel{bx}+ab-b^2+2ax+2bx-2ab-2b^2$$

$$2ax+2a^2 = 3ax+bx-ab-3b^2$$

$$-3ax+2ax-bx = -ab-3b^2-2a^2$$

$$-ax-bx = -ab-2a^2-3b^2$$

$$-x(a+b) = -(2a^2+3b^2+ab)$$

$$x = \frac{2a^2+3b^2+ab}{a+b}.$$

g) $p(x): \frac{(a+b)^2(x+1)-(a+b)(x+1)+(x+1)}{a+b+1} = (a+b)^2-(a+b)+1$

$$\Rightarrow \frac{(x+1)\cancel{((a+b)^2-(a+b)+1)}}{a+b+1} = \cancel{[(a+b)^2-(a+b)+1]}$$

$$\Rightarrow (x+1) = a+b+1 \Rightarrow x+1 = a+b+1 \Rightarrow x = a+b+\cancel{1}-\cancel{1}$$

$$\Rightarrow \boxed{x = a+b}$$

h) $p(x): (a+x)(b+x)-a(b+c) = \frac{a^2c}{b} + x^2$

$$\Rightarrow \cancel{ab}+ax+bx+\cancel{x^2}-\cancel{ab}-ac = \frac{a^2c}{b}+\cancel{x^2}$$

$$\Rightarrow ax+bx = \frac{a^2c}{b}+ac \Rightarrow x(a+b) = \frac{a^2c+abc}{b}.$$

$$\Rightarrow x\cancel{(a+b)} = \frac{ac\cancel{(a+b)}}{b} \Rightarrow x = \frac{ac}{b}$$

i) $\frac{x}{ab} + \frac{x}{bc} + \frac{x}{ac} - 1 = abc - x(a+b+c)$

$$\Rightarrow \frac{cx + ax + bx - abc}{abc} = abc - x(a+b+c)$$

$$\Rightarrow \frac{x(a+b+c) - abc}{abc} = abc - x(a+b+c)$$

$$\Rightarrow x(a+b+c) - abc = abc(abc - x(a+b+c))$$

$$\Rightarrow \underline{x(a+b+c)} - abc = a^2b^2c^2 - \underline{xabc(a+b+c)}$$

$$\Rightarrow x(a+b+c) + xabc(a+b+c) = a^2b^2c^2 + abc$$

$$\Rightarrow x(a+b+c)(1+abc) = abc(abc+1)$$

$$\Rightarrow x = \frac{abc\cancel{(abc+1)}}{(a+b+c)\cancel{(abc+1)}} \Rightarrow \frac{abc}{a+b+c}.$$

79.– $q(x)$: $\frac{4}{x+4} + \frac{1}{x+3} + \frac{3}{x+1} = 0$

$$\Rightarrow \frac{4(x+3)(x+1) + (x+4)(x+1) + 3(x+4)(x+3)}{(x+4)(x+3)(x+1)} = 0$$

$$\Rightarrow 4(x^2 + x + 3x + 3) + x^2 + x + 4x + 4 + 3(x^2 + 3x + 4x + 12) = 0$$

$$\Rightarrow 4x^2 + 16x + 12 + x^2 + 5x + 4 + 3x^2 + 21x + 36 = 0$$

$$\Rightarrow 8x^2 + 42x + 52 = 0 \qquad 4x^2 + 21x + 26 = 0$$

$$(4x + 13)(4x + 8) = 0$$

$4x - 13 = 0$ | $4x + 8 = 0$

$4x = 13$ | $4x = -8$

$x = 13/4$ | $x = -2$

$$p(x): \frac{3x}{2x+1} + \frac{x+5}{x+1} = \frac{x-19}{2x^2+3x+1}$$

$$\Rightarrow \frac{3x(x+1)+(x+5)(2x+1)}{(2x+1)(x+1)} = \frac{x-19}{\frac{(\cancel{2}x+\cancel{2})(2x+1)}{\cancel{2}}}$$

$$\Rightarrow \frac{3x^2+3x+2x^2+x+10x+5}{\cancel{(2x+1)(x+1)}} = \frac{x-19}{\cancel{(x+1)(2x+1)}}$$

$$\Rightarrow 5x^2+14x+5-x+19=0$$

$$\Rightarrow 5x^2+13x+24=0$$

$$x = \frac{-13 \pm \sqrt{13^2-4(5)(24)}}{2(5)} \Rightarrow \cancel{\frac{-13 \pm \sqrt{169-480}}{10}}$$ tiene soluciones imaginarias

$$m(x): \frac{3a+x}{3a-x} + \frac{2a-3x}{2a+3x} = \frac{9}{2}$$

$$\Rightarrow \frac{(3a+x)(2a+3x)+(2a-3x)(3a-x)}{(3a-x)(2a+3x)} - \frac{9}{2} = 0$$

$$\Rightarrow \frac{6a^2+9ax+2ax+3x^2+6a^2-2ax-9ax+3x^2}{(3a-x)(2a+3x)} - \frac{9}{2} = 0$$

$$\Rightarrow \frac{6x^2+12a^2}{(3a-x)(2a+3x)} - \frac{9}{2} = 0 \Rightarrow \frac{2(6x^2+12a^2)-9(3a-x)(2a+3x)}{2(3a-x)(2a+3x)} = 0$$

$$\Rightarrow 12x^2+24a^2-9(6a^2+9ax-2ax-3x^2)=0$$

$$\Rightarrow 12x^2+24a^2-54a^2-63ax+27x^2=0 \Rightarrow 39x^2-30a^2-63ax=0$$

$$\Rightarrow 3(13x^2-21ax-10a^2)=0$$

$$(13x-26a^2)(13x+5a^2)=0$$

$$x = 2a^2 \quad x = -\frac{5a^2}{13}$$

d) $p(x): \dfrac{1}{x-a} + \dfrac{1}{x-b} = \dfrac{1}{x-c}$

$$\Rightarrow \frac{1}{x-a} + \frac{1}{x-b} - \frac{1}{x-c} = 0 \Rightarrow \frac{(x-b)(x-c)+(x-a)(x-c)-(x-b)(x-a)}{(x-a)(x-b)(x-c)} = 0$$

$$\Rightarrow x^2-cx-bx+bc+x^2-cx-ax+ac-(x^2-ax-bx+ab)=0$$

$$\Rightarrow \cancel{2}x^2-2cx-\cancel{ax}-\cancel{bx}+ac+bc-\cancel{x^2}+\cancel{ax}+\cancel{bx}-ab=0$$

$$\Rightarrow x^2-2cx-ab+ac+bc=0$$

$$x=\frac{2c\pm\sqrt{(2c)^2-4(1)(-ab+ac+bc)}}{2(1)} \Rightarrow \frac{2c\pm\sqrt{4c^2+4ab-4ac-4bc}}{2}$$

$$\Rightarrow \frac{2c\pm\sqrt{4[(c^2-ac)-(bc-ab)]}}{2} \Rightarrow \frac{\cancel{2}c\pm\cancel{2}\sqrt{c(c-a)-b(c-a)}}{\cancel{2}}$$

$$\Rightarrow c\pm\sqrt{(c-a)(c-b)}\ //$$

e) $\dfrac{1}{a}+\dfrac{1}{b}-\dfrac{1}{x}=\dfrac{1}{a+b-x}$

$$\Rightarrow \frac{bx+ax-ab}{abx}-\frac{1}{a+b-x}=0 \Rightarrow \frac{bx(a+b-x)+ax(a+b-x)-ab(a+b-x)-abx}{abx(a+b-x)}$$

$$\Rightarrow abx+b^2x-bx^2+a^2x+abx-ax^2-a^2b-ab^2+\cancel{abx}-\cancel{abx}=0$$

$$-ax^2-bx^2+a^2x+2abx+b^2x-a^2b-ab^2=0$$

$$-x^2(a+b)+x(a+b)^2-ab(a+b)=0 \Rightarrow -(a+b)(x^2-(a+b)x+ab)=0$$

$$\Rightarrow (x-a)(x-b)=0 \Rightarrow \boxed{x=a} \quad \boxed{x=b}$$

$$r(x):\ \frac{6}{x+4}-\frac{x+4}{x-4}+\frac{7x^2+50}{3(x^2-16)}=\frac{4}{3}$$

$$\Rightarrow \frac{6}{x+4}-\frac{x+4}{x-4}+\frac{7x^2+50}{3(x+4)(x-4)}-\frac{4}{3}=0$$

$$\Rightarrow \frac{18(x-4)-3(x+4)^2+7x^2+50-4(x+4)(x-4)}{3(x+4)(x-4)}=0$$

$$\Rightarrow 18x-72-3(x^2+8x+16)+7x^2+50-4(x^2-16)=0$$

$$18x-72-\cancel{3x^2}-24x-48+\cancel{7x^2}+50-\cancel{4x^2}+64=0$$

$$-6x=72+48-50-64$$

$$-6x=6$$

$$\boxed{x=-1}$$

$$\frac{x+a+b}{x+a}=\frac{x+a-b}{x-a}-\frac{a^2+b^2}{x^2-a^2}$$

$$\Rightarrow \frac{x+a+b}{x+a}-\frac{x+a-b}{x-a}+\frac{a^2+b^2}{(x+a)(x-a)}=0$$

$$\Rightarrow \frac{(x+a+b)(x-a)-(x+a-b)(x+b)+a^2+b^2}{(x+a)(x-a)}=0$$

$$\Rightarrow x^2-\cancel{ax}+\cancel{ax}-a^2+bx-ab-(x^2+\cancel{bx}+ax+ab-\cancel{bx}-b^2)+a^2+b^2=0$$

$$\Rightarrow \cancel{x^2}-\cancel{a^2}+bx-ab-\cancel{x^2}-ax-ab+b^2+\cancel{a^2}+b^2=0$$

$$\Rightarrow bx-ax=+2ab-2b^2$$

$$\Rightarrow x(b-a)=2b(a-b)\Rightarrow x=\frac{-2b\cancel{(b-a)}}{\cancel{b-a}}\Rightarrow x=-2b$$

h) $p(x): \dfrac{\frac{x+1}{x-1}}{1+\frac{x+x}{x-1}} = \dfrac{1}{2}$

$$\Rightarrow \dfrac{\frac{x+1}{\cancel{x-1}}}{\frac{x-1+2x}{\cancel{x-1}}} = \dfrac{1}{2} \Rightarrow \dfrac{x+1}{3x-1} - \dfrac{1}{2} = 0$$

$$\Rightarrow \dfrac{2(x+1)-(3x-1)}{2(3x-1)} = 0 \Rightarrow 2x+2-3x+1=0$$

$$-x+3=0$$

$$x=3$$

80.- a) $r(x): \sqrt{x^2-7} = 3$

$$\left(\sqrt{x^2-7}\right)^2 = 3^2$$

$$x^2-7=9$$

$$x^2 = 9+7$$

$$x^2=16$$

$$x=\pm 4$$

b) $m(x): \sqrt{x+1} = x-1$

$$\left(\sqrt{x+1}\right)^2 = (x-1)^2$$

$$x+1 = x^2-2x+1$$

$$x^2-2x+\cancel{1}-x-\cancel{1}=0$$

$$x^2-3x=0$$

$$x(x-3)=0$$

$$x=0 \quad x=3$$

c) $p(x): \sqrt{x} + \sqrt{3x+4} = \sqrt{4-x}$

$$\left(\sqrt{x}+\sqrt{3x+4}\right)^2 = \left(\sqrt{4-x}\right)^2$$

$$\left(\sqrt{x}\right)^2 + 2\sqrt{x}\sqrt{3x+4} + \left(\sqrt{3x+4}\right)^2 = 4-x$$

$$\Rightarrow x + 2\sqrt{x(3x+4)} + 3x+4 = 4-x$$

$$\Rightarrow 2\sqrt{3x^2+4x} = \cancel{4}-x-3x-\cancel{4} = -5x$$

$$\Rightarrow \left(2\sqrt{3x^2+4x}\right)^2 = (-5x)^2$$

$$4(3x^2+4x) = 25x^2$$

$$12x^2+16x-25x^2=0$$

$$-13x^2+16x=0$$

$$-x(13x-16)=0$$

$$x=0$$

$$x=16/13$$

d.- $q(x)$: $\sqrt{a+x} + \sqrt{a-x} = \sqrt{2a}$

$$\left(\cancel{\sqrt{a+x}}\right)^{\cancel{2}} = \left(\sqrt{2a} - \sqrt{a-x}\right)^2$$

$$a+x = \left(\cancel{\sqrt{2a}}\right)^{\cancel{2}} - 2\sqrt{2a}\sqrt{a-x} + \left(\cancel{\sqrt{a-x}}\right)^{\cancel{2}}$$

$$\cancel{a}+x = 2a - 2\sqrt{2a(a-x)} + \cancel{a}-x$$

$$-2a+x+x = -2\sqrt{2a^2-2ax}$$

$$-2a+2x = -2\sqrt{2a^2-2ax}$$

$$-\cancel{2}(a-x) = -\cancel{2}\sqrt{2a^2-2ax}$$

$$(a-x)^2 = \left(\cancel{\sqrt{2a^2-2ax}}\right)^{\cancel{2}}$$

$$a^2 - \cancel{2ax} + x^2 = 2a^2 - \cancel{2ax}$$

$$x^2 = 2a^2 - a^2$$

$$\sqrt{x^{\cancel{2}}} = \sqrt{a^{\cancel{2}}}$$

$$x = \pm a$$

e).- $r(x)$: $\dfrac{\sqrt{x+4} + \sqrt{x-4}}{\sqrt{x+4} - \sqrt{x-4}} = x-3$

$$\frac{\sqrt{x+4} + \sqrt{x-4}}{\sqrt{x+4} - \sqrt{x-4}} \times \frac{\sqrt{x+4} + \sqrt{x-4}}{\sqrt{x+4} + \sqrt{x-4}} = x-3$$

$$\frac{\left(\sqrt{x+4} + \sqrt{x-4}\right)^2}{\left(\cancel{\sqrt{x+4}}\right)^{\cancel{2}} - \left(\cancel{\sqrt{x-4}}\right)^{\cancel{2}}} = x-3 \Rightarrow \frac{\left(\cancel{\sqrt{x+4}}\right)^{\cancel{2}} + 2\sqrt{x+4}\sqrt{x-4} + \left(\cancel{\sqrt{x-4}}\right)^{\cancel{2}}}{x+4-(x-4)}$$

$$\frac{x\cancel{+4}+2\sqrt{(x+4)(x-4)}+x\cancel{-4}}{x+4-(x-4)}=x-3$$

$$\frac{2x+2\sqrt{x^2-16}}{\cancel{x}+4\cancel{-x}+4}=x-3$$

$$\frac{\cancel{2}^{1}(x+\sqrt{x^2-16})}{\cancel{8}\,4}=x-3$$

$$x+\sqrt{x^2-16}=4(x-3)$$

$$\sqrt{x^2-16}=4x-12-x$$

$$(\sqrt{x^2-16})^2=(3x-12)^2$$

$$x^2-16=9x^2-72x+144$$

$$9x^2-x^2-72x+144+16=0$$

$$8x^2-72x+160=0$$

$$8(x^2-9x+20)=0$$

$$x^2-9x+20=0$$

$$(x-5)(x-4)=0$$

$$x-5=0 \qquad x-4=0$$

$$x=5 \qquad x=4$$

f.-

$$\frac{1}{\sqrt{3a+x}-\sqrt{a-x}}+\frac{1}{\sqrt{3a+x}+\sqrt{a-x}}=\frac{1}{\sqrt{a}}$$

$$\frac{\sqrt{3a+x}+\cancel{\sqrt{a-x}}+\sqrt{3a+x}-\cancel{\sqrt{a-x}}}{(\sqrt{3a+x}-\sqrt{a-x})(\sqrt{3a+x}+\sqrt{a-x})}=\frac{1}{\sqrt{a}}$$

$$\frac{2\sqrt{3a+x}}{(\cancel{\sqrt{}}3a-x)^{\cancel{2}}-(\cancel{\sqrt{}}a-x)^{\cancel{2}}}=\frac{1}{\sqrt{a}}$$

$$\Rightarrow \frac{2\sqrt{3a+x}}{3a-x-(a-x)}=\frac{1}{\sqrt{a}} \Rightarrow \frac{2\sqrt{3a+x}}{3a\cancel{-x}-a\cancel{+x}}=\frac{1}{\sqrt{a}}$$

$$\Rightarrow \frac{\cancel{2}\sqrt{3a+x}}{\cancel{2}a}=\frac{1}{\sqrt{a}}. \Rightarrow \sqrt{3a+x}=\frac{a}{\sqrt{a}}$$

$$\Rightarrow (\cancel{\sqrt{}}3a+x)^{\cancel{2}}=\left(\frac{a}{\sqrt{a}}\right)^2 \Rightarrow 3a+x=\frac{a^{\cancel{2}}}{\cancel{a}}$$

$$\Rightarrow 3a+x=a \Rightarrow x=a-3a$$

$$x=-2a$$

g.-

$$\frac{1}{\sqrt{4a+x}+\sqrt{a}}+\frac{1}{\sqrt{4a+x}-\sqrt{a}}=\frac{4}{3\sqrt{a}}$$

$$\frac{\sqrt{4a+x}-\cancel{\sqrt{a}}+\sqrt{4a+x}+\cancel{\sqrt{a}}}{(\sqrt{4a+x}+\sqrt{a})(\sqrt{4a+x}-\sqrt{a})}=\frac{4}{3\sqrt{a}}$$

$$\frac{2\sqrt{4a+x}}{(\sqrt{4a+x})^{2}-(\sqrt{a})^{2}}=\frac{4}{3\sqrt{a}}$$

$$\frac{\cancel{2}^{1}\sqrt{4a+x}}{4a+x-a}=\frac{\cancel{4}^{2}}{3\sqrt{a}}$$

$$\frac{\sqrt{4a+x}}{4a+x-a}=\frac{2}{3\sqrt{a}} \Rightarrow \frac{\sqrt{4a+x}}{3a+x}=\frac{2}{3\sqrt{a}}$$

$$3\sqrt{a}\sqrt{4a+x}=2(3a+x)$$

$$\left(3\sqrt{a(4a+x)}\right)^{2}=(6a+2x)^{2}$$

$$9(a(4a+x))=36a^{2}+24ax+4x^{2}$$

$$\cancel{36a^{2}}+9ax=\cancel{36a^{2}}+24ax+4x^{2}$$

$$4x^{2}+24ax-9ax=0$$

$$4x^{2}+15ax=0$$

$$x(4x+15a)=0$$

$$x=0 \qquad 4x+15a=0$$

$$4x=-15a$$

$$x=-\frac{15a}{4}$$

h.-) $\varphi(x): \sqrt{x+3} - \sqrt{x-1} = \sqrt{2x+2}$

$$(\sqrt{x+3} - \sqrt{x-1})^2 = (\sqrt{2x+2})^2$$

$$(\sqrt{x+3})^2 - 2\sqrt{x+3}\sqrt{x-1} + (\sqrt{x-1})^2 = 2x+2$$

$$x+3 - 2\sqrt{(x+3)(x-1)} + x-1 = 2x+2$$

$$\cancel{2x+2} - 2\sqrt{x^2 - x + 3x -} = \cancel{2x+2}$$

$$-2\sqrt{x^2 - x + 3x} = 0$$

$$(\sqrt{x^2 - x + 3x})^2 = 0^2$$

$$x^2 - x + 3x = 0.$$

$$(x+3)(x-1) = 0$$

$$(x+3) = 0 \qquad x-1 = 0$$

$$\cancel{x = -3} \qquad x = 1$$

Sol $x = 1$ //

i.-) $(\sqrt{4x+3} + 1)^2 = (\sqrt{2x-2})^2$

$$(\sqrt{4x+3})^2 + 2\sqrt{4x+3} + 1 = 2x-2$$

$$4x+3 + 2\sqrt{4x+3} + 1 = 2x-2$$

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$$2\sqrt{4x+3} = 2x-2-4x-4$$

$$2\sqrt{4x+3} = -2x-6$$

$$\cancel{2}\sqrt{4x+3} = \cancel{2}(-x-3) \Rightarrow (\sqrt{4x+3})^2 = (-x-3)^2$$

$$4x+3 = x^2+6x+9 \Rightarrow x^2+6x-4x+9-3$$

$$\Rightarrow x^2+2x+6=0$$

Sol: ϕ.

d.- $(\sqrt{x+4})^2 = (1+\sqrt{2x-2})^2$

$$x+4 = 1+2\sqrt{2x-2}+(\sqrt{2x-2})^2$$

$$x+4 = 1+2\sqrt{2x-2}+2x-2$$

$$x+4-1-2x+2 = 2\sqrt{2x-2}$$

$$(-x+5)^2 = (2\sqrt{2x-2})^2$$

$$x^2-10x+25 = 4(2x-2)$$

$$x^2-10x+25 = 8x-8$$

$$x^2-10x-8x+25+8=0$$

$$x^2-18x+33=0$$

$$x = \frac{18 \pm \sqrt{18^2-4(1)(33)}}{2(1)}$$

$$x = \begin{cases} 2{,}07 \\ 15{,}92 \end{cases}$$

k.- $\sqrt{x}+\sqrt{x+2}=3$

$$(\sqrt{x+2})^2 = (3-\sqrt{x})^2$$

$$x+2 = 9-6\sqrt{x}+(\sqrt{x})^2$$

$$\cancel{x}+2 = 9-6\sqrt{x}+\cancel{x}$$

$$6\sqrt{x} = 9-2$$

$$(6\sqrt{x})^2 = (7)^2$$

$$36x = 49$$

$$x = \frac{49}{36}$$

1.- $$\frac{x-2}{\sqrt{2x-7}}=\sqrt{x-4}$$

$$\Rightarrow \frac{x-2}{\sqrt{2x-7}}-\sqrt{x-4}=0 \Rightarrow \frac{x-2-\sqrt{x-4}\,\sqrt{2x-7}}{\sqrt{2x-7}}=0$$

$$\Rightarrow x-2-\sqrt{(x-4)(2x-7)}=0 \Rightarrow (x-2)^2=\left(\sqrt{2x^2-7x-8x+28}\right)^2$$

$$\Rightarrow x^2-4x+4=2x^2-15x+28$$

$$\Rightarrow 2x^2-15x+28-x^2+4x-4=0$$

$$\Rightarrow x^2-11x+24=0 \Rightarrow (x-8)(x-3)=0$$

$$x=8 \qquad \cancel{x=3}$$

2.- $P(x)$: $kx^2+3x+1=0$

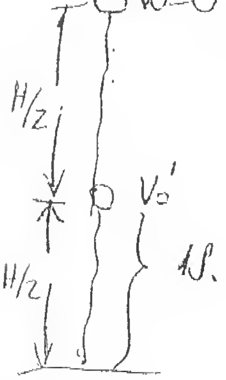
Solución única: $b^2-4ac=0$

$$3^2-4k(1)=0$$

$$-4k=9 \Rightarrow \boxed{k=-9/4}$$

3.- H, $\qquad y=V_0t+\frac{1}{2}gt^2 \qquad V^2=V_0^2+2gy$

$\frac{H}{2}$; $t=1\,s.$ $\qquad \frac{H}{2}=V_0'(1)+\frac{1}{2}g(1)^2 \qquad (V_0')^2=0+\cancel{2}g\left(\frac{H}{\cancel{2}}\right)$



$$H=2V_0'+g \qquad (V_0')^2=gH$$

$$\frac{(V_0')^2}{g}=2V_0'+g \qquad H=\frac{(V_0')^2}{g}$$

$$(V_0')^2=2gV_0'+g^2$$

$$(V_0')^2-2gV_0'-g^2=0 \Rightarrow V_0'=\frac{2g\pm\sqrt{(2g)^2-4(1)(-g^2)}}{2(1)}$$

$$V_0'=\frac{2g\pm\sqrt{8}\,g}{2} \Rightarrow g\left(\frac{2\pm2\sqrt{2}}{2}\right) \Rightarrow g(1+\sqrt{2})$$

$V_0' = g(1+\sqrt{2})$

$H = V_0' + g$

$H = g(1+\sqrt{2}) + g$

$\boxed{H = 33{,}46\,m.}$

$V = V_0 + gt$

$V_0' = 0 + gt$

$t = \frac{V_0'}{g}$

$t = \frac{g(1+\sqrt{2})}{g}$

$\boxed{t = 1+\sqrt{2}\,s}$

$T = 1+\sqrt{2}+1 = (2+\sqrt{2})\,s.$

84.- $a = 4\,m/s^2$

$V = 3\,m/s$

$x = 20\,m$

$t = ??$

$x = V_0 t + \frac{1}{2} a t^2$

$20 = 3t + \frac{1}{2}(4)t^2$

$20 = 3t + 2t^2$

$2t^2 + 3t - 20 = 0$

$(2t+8)(2t-5) = 0$

$2t = -8$ $\qquad$ $2t = 5$

$\cancel{t = -4\,s.}$ $\qquad$ $\boxed{t = \frac{5}{2}\,s.}$

85.- $P(x): \underbrace{\left||x-1|+1\right|}_{} \leq 0$

siempre será positivo

$\underbrace{|x-1|}_{\downarrow} + 1$

también siempre
será positivo, por lo tanto nunca será 0

$A_{P(x)} = \phi$

a) Verdadero.

36.- $P(x): |x-a| < \delta$

$x-a<\delta \wedge -x+a<\delta$

$\boxed{x<a+\delta} \wedge -x<\delta-a$

$\boxed{x>a-\delta}$

a) verdadero

7.- $q(x): \dfrac{|x-2|-1}{x^2+1} \geq 0$

$|x-2|-1\geq 0$

P. crítico

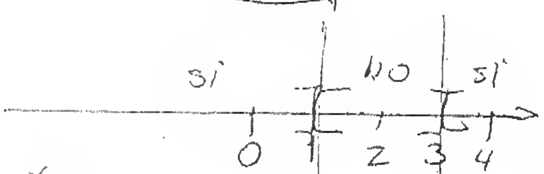
$x^2+1=0$

NO HAY

$x-2-1\geq 0 \vee -x+2-1\geq 0$

$\boxed{x\geq 3} \vee -x\geq -1$

$\boxed{x\leq 1}$



$x=0$

$\dfrac{|-2|-1}{1} \geq 0$

$x=2$

$\dfrac{0-1}{5} \geq 0$

NO

$x=4$

$\dfrac{2-1}{17} \geq 0$

si

$A_{q(x)}: (-\infty, 1] \cup [3, +\infty)$

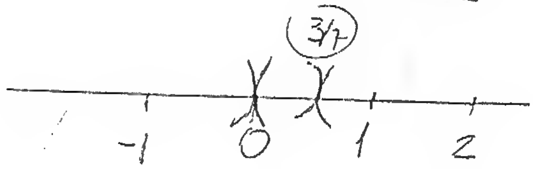
$P(x): 7x^2 < 3x$

$7x^2-3x<0$

$x(7x-3)<0$

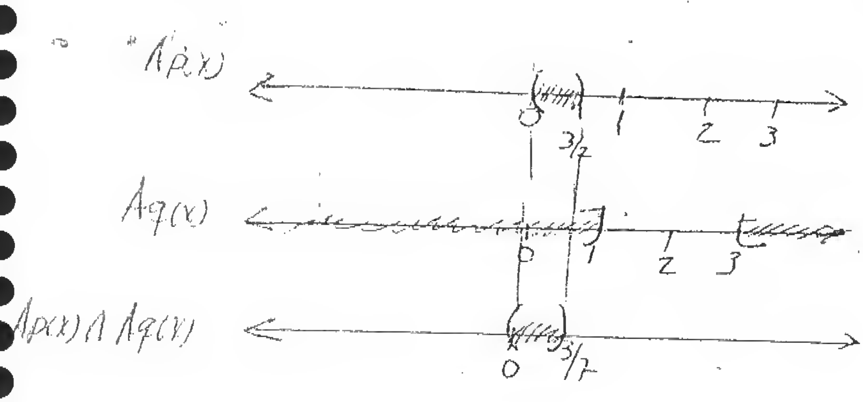
$\boxed{x<0} \wedge 7x<3$

$\boxed{x<3/7}$



$x=-1$	$x=0,25$	$x=1$
$7(-1)^2-3(-1)<0$	$7(0,25)^2-3(0,25)<0$	$7(1)-3(1)<0$
NO	si	NO

$A_{p(x)}: (0, 3/7)$



d) correcto

88.- $p(x): |7x+4| \leq 10$

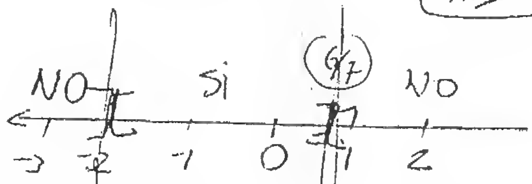
$7x+4 \leq 10 \wedge -7x-4 \leq 10$

$7x \leq 10-4$ $\quad -7x \leq 10+4$

$7x \leq 6$ $\quad -7x \leq 14$

$\boxed{x \leq \frac{6}{7}}$ $\quad 7x \geq -14$

$\boxed{x \geq -2}$



$x=-3$ $\quad x=0$ $\quad x=2$

$17 \leq 10$ $\quad 4 \leq 10$ $\quad 18 \leq 10$

NO $\quad$ Si $\quad$ NO

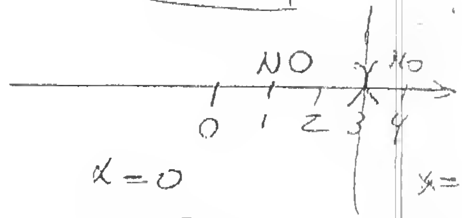
$A p(x): [-2, 6/7]$.

$q(x): x^2 - 6x+9 < 0$

$\sqrt{(x-3)^2} < \sqrt{0}$

$x-3 < 0$

$\boxed{x < 3}$



$x=0$ $\quad x=4$

$0+0+9<0$ $\quad 16-24+9<0$

NO $\quad$ NO

$\boxed{A q(x): \phi}$

a) verdadera

89.- $1-x \geq 2x+6$

$-x-2x \geq 6-1$

$-3x \geq 5$

$3x \leq -5 \Rightarrow \boxed{x \leq -5/3}$ d) correcto

90.- $p(n): |2n-5| \leq 7$

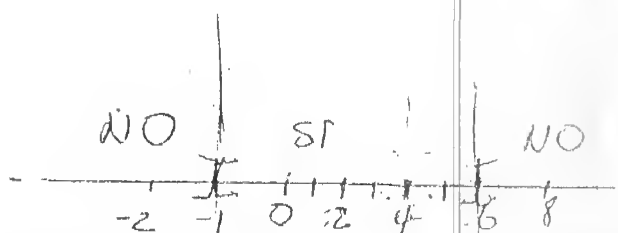
$2n-5 \leq 7 \wedge -2n+5 \leq 7$

$2n \leq 12$ $\quad \wedge -2n \leq 7-5$

$\boxed{n \leq 6}$ $\quad \wedge -2n \leq 2$

$2n \geq -2$

$\boxed{n \geq -1}$



$n=-2$ $\quad n=0$ $\quad n=8$

$9 \leq 7$ $\quad 5 \leq 7$ $\quad 11 \leq 7$

NO $\quad$ Si $\quad$ NO

$A p(x): [-1, 6]$

6 elementos: $\{1, 2, 3, 4, 5, 6\}$

91.- $[0, +\infty)$

92.- a) $(x \leq 14) \wedge (x > 2)$

$[2, 14]$

b) $[(x \leq -3) \wedge (x > 2)] \vee [(x \geq 3 \wedge (x \leq 5)]$

$\phi \cup [3,5]$

$\therefore [3,5]$

93.-

a) $5(x-1) - x(7-x) > x^2$

$5x - 5 - 7x + \cancel{x^2} - \cancel{x^2} > 0$

$2x > 5$

$\boxed{x > 5/2}$

b) $|2x+4| < 10$

$2x+4 < 10 \wedge -2x-4 < 10$

$2x < 6 \quad \wedge \quad -2x < 14$

$\boxed{x < 3} \qquad 2x > -14$

$\boxed{x > -7}$

Sol: $(-7, 3)$

c) $\left| \frac{x-3}{x-4} \right| < \frac{5}{2}$

$\Rightarrow \frac{x-3}{x-4} - \frac{5}{2} < 0 \wedge \frac{3-x}{x-4} - \frac{5}{2} < 0$

$\Rightarrow \frac{2(x-3) - 5(x-4)}{2(x-4)} < 0 \wedge \frac{2(3-x) - 5(x-4)}{2(x-4)} < 0$

$\Rightarrow \frac{2x - 6 - 5x + 20}{2(x-4)} < 0 \wedge \frac{6 - 2x - 5x + 20}{2(x-4)} < 0$

$\Rightarrow \dfrac{-3x+14}{2(x-4)} > 0 \quad \wedge \quad \dfrac{-7x+26}{2(x-4)} > 0$

P. crítico

$x-4=0$

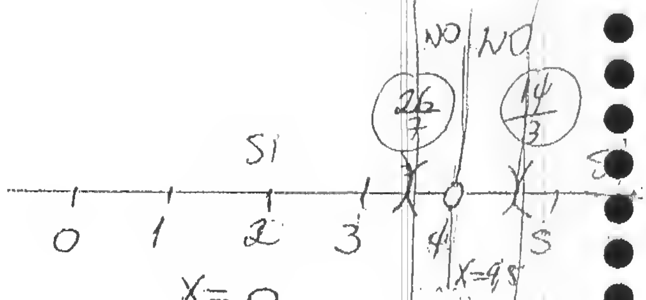
$\boxed{x=4}$

$-3x+14>0 \quad \wedge \quad -7x+26>0$

$-3x>-14 \qquad -7x>-26$

$3x<14 \qquad 7x<26$

$\boxed{x<\frac{14}{3}} \qquad \boxed{x<\frac{26}{7}}$



$x=0$

$\frac{3}{4}<\frac{5}{2}$

SI

$x=4,5$

$\frac{1,5}{0,5}<2,5$

NO

$x=5$

$\frac{2}{1}<2,5$

$x=3,8$

$\left|\frac{9,8}{0,2}\right|<2,5$

NO

Sol: $(-\infty, \frac{26}{7}) \cup (\frac{14}{3}, +\infty)$

d) $\dfrac{4}{x+1} - \dfrac{3}{x+2} > 1$

$\Rightarrow \dfrac{4}{x+1} - \dfrac{3}{x+2} - 1 > 0 \Rightarrow \dfrac{4(x+2)-3(x+1)-(x+1)(x+2)}{(x+1)(x+2)} > 0$

$\Rightarrow \dfrac{4x+8-3x-3-x^2-2x-x-2}{(x+1)(x+2)} > 0 \Rightarrow \dfrac{-x^2-2x+3}{(x+1)(x+2)} > 0$

$-x^2-2x+3>0$

$x^2+2x-3<0$

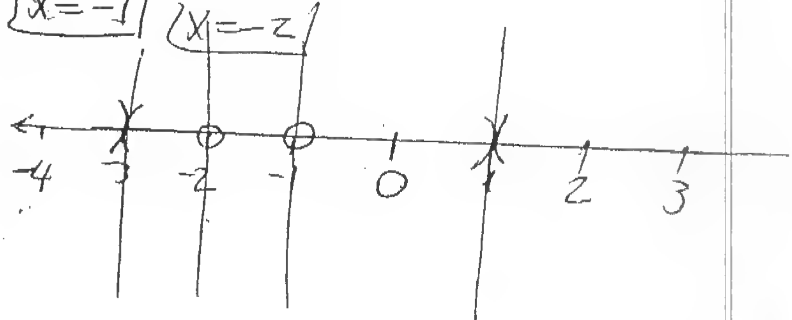
$(x+3)(x-1)<0$

$\boxed{x<-3} \quad \boxed{x<1}$

P. crítico

$x+1=0 \qquad x+2=0$

$\boxed{x=-1} \quad \boxed{x=-2}$



$X=-4$

$-\frac{4}{3}+\frac{3}{2}>1$

$-1{,}33+1{,}5>1$

NO

$x=-2{,}5$

$-\frac{4}{1{,}5}+\frac{3}{0{,}5}>1$

$-2{,}67+6>1$

Si

$x=-1{,}5$

$-\frac{4}{0{,}5}-\frac{3}{0{,}5}>1$

NO

$X=0$

$4-\frac{3}{2}>1$

Si

$x=2$

$\frac{4}{3}-\frac{3}{4}>1$

$1{,}33-0{,}75>1$

NO

Sol: $(-3,-2)\cup(-1,1)$

) $|x-1|\geq\frac{x+1}{2}$

$x-1\geq\frac{x+1}{2} \quad \vee \quad -x+1\geq\frac{x+1}{2}$

$2(x-1)\geq x+1 \quad \vee \quad 2(-x+1)\geq x+1$

$2x-2\geq x+1 \quad\quad -2x+2\geq x+1$

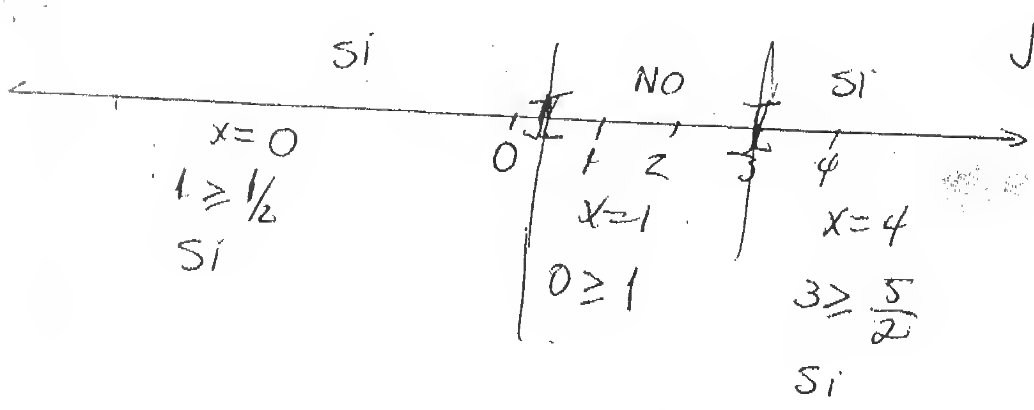
$2x-x\geq 2+1 \quad\quad -2x-x\geq 1-2$

$\boxed{x\geq 3}$

$-3x\geq -1$

$3x\geq 1$

$\boxed{x\geq 1/3}$



Sol: $(-\infty,\frac{1}{3}]\cup[3,+\infty)$

69

f) $\frac{3x}{2} + 3|x-2| \leq 3$

$\Rightarrow 3x + 6|x-2| \leq 6$

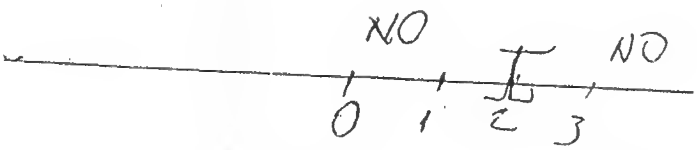
$3x + 6(x-2) \leq 6 \quad \wedge \quad 3x + 6(-x+2) \leq 6$

$3x + 6x - 12 \leq 6 \quad \wedge \quad 3x - 6x + 12 \leq 6$

$9x \leq 6 + 12 \quad \wedge \quad -3x \leq 6 - 12$

$x \leq \frac{18}{9} \qquad +3x \geq 6$

$\boxed{x \leq 2} \qquad \boxed{x \geq 2}$



Sol: ϕ

$x = 0 \qquad x = 3$

$6 \leq 3 \qquad \frac{9}{2} + 3 \leq 3$

NO

94.- a) $p(x)$: $2 + 4x < 6x + 7$

$-6x + 4x < 7 - 2$

$-2\cancel{x} < 5$

$2x > -5$

$\boxed{x > -5/2}$

$A p(x)$: $(-5/2, +\infty)$

b) $q(x)$: $2 < 2x - 2 \leq 12$

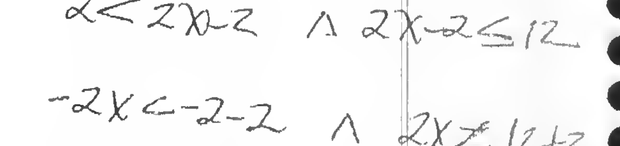
$2 < 2x - 2 \quad \wedge \quad 2x - 2 \leq 12$

$-2x < -2 - 2 \quad \wedge \quad 2x \leq 12 + 2$

$-2x < -4$

$2x > 4 \qquad x < \frac{14}{2}$

$\boxed{x > 2} \qquad \boxed{x < 7}$



$x = 0$	$x = 3$	$x = 8$
$2 < -2 \leq 12$	$2 < 4 < 12$	$2 < 14 < 12$
NO	SI	NO

Sol $(2, 7)$

c) $r(x):\ 8-3x \le 2x-7 < x-13$

$8-3x \le 2x-7 \ \wedge\ 2x-7 < x-13$

$-3x-2x \le -7-8 \ \wedge\ 2x-x < -13+7$

$-5x \le -15$

$5x \ge 15$ $\quad \boxed{x < -6}$

$\boxed{x \ge 3}$

$-7 \quad -6 \quad -5 \quad 0 \quad 2 \quad 3 \quad 4$

$x=-7$ $\quad x=0$ $\quad x=4$

$29 \le -21$ $\quad 8 \le 7$ $\quad -4 \le 1 \le -7$

NO $\quad$ NO $\quad$ NO

Sol: ϕ

d) $m(x):\ \dfrac{8}{x} \ge 3$

$\dfrac{8}{x} - 3 \ge 0 \Rightarrow \dfrac{8-3x}{x} \ge 0$

P. crítico $\boxed{x=0}$

$8-3x \ge 0$

$-3x \ge -8$

$\boxed{x \le 8/3}$

$-1 \quad 0 \quad 1 \quad 2 \quad 3$ $\quad (8/3)$

$x=-1$ $\quad x=1$ $\quad x=3$

NO $\quad$ Si $\quad$ NO

Sol: $(0, \frac{8}{3}]$

2) $p(x):\ \dfrac{2x}{x-4} \le 8$

$\Rightarrow \dfrac{2x}{x-4} - 8 \le 0 \Rightarrow \dfrac{2x-8(x-4)}{x-4} \le 0 \Rightarrow \dfrac{2x-8x+32}{x-4} \le 0$

P. crítico $\boxed{x=4}$

$\dfrac{-6x+32}{x-4} \le 0$

$-6x+32 \le 0$

$-6x \le -32$

$6x \ge 32$

$\boxed{x \ge \frac{16}{3}}$

Si $\quad$ NO $\quad (16/3)$ $\quad$ Si

$0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

Sol: $(-\infty, 4) \cup (\frac{16}{3}, +\infty)$

$x=0$ $\quad x=5$ $\quad x=6$

$0 \le 8$ (Si) $\quad 10 \le 8$ $\quad$ Si

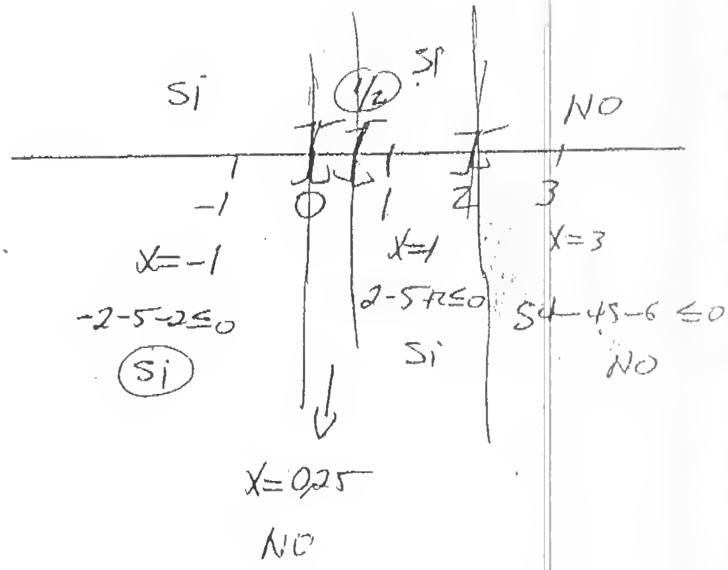
f) $q(x): 2x^3 - 5x^2 + 2x \leq 0$

$x(2x^2 - 5x + 2) \leq 0$

$x(2x-4)(2x-1) \leq 0$

$\boxed{x \leq 0}$ $\quad 2x \leq 4 \quad 2x \leq 1$

$\boxed{x \leq 2}$ $\quad \boxed{x \leq 1/2}$



Sol: $(-\infty, 0] \cup [1/2, 2]$.

g) $p(x): \dfrac{x^2 - 3x - 18}{13x - x^2 - 42} \geq 0$

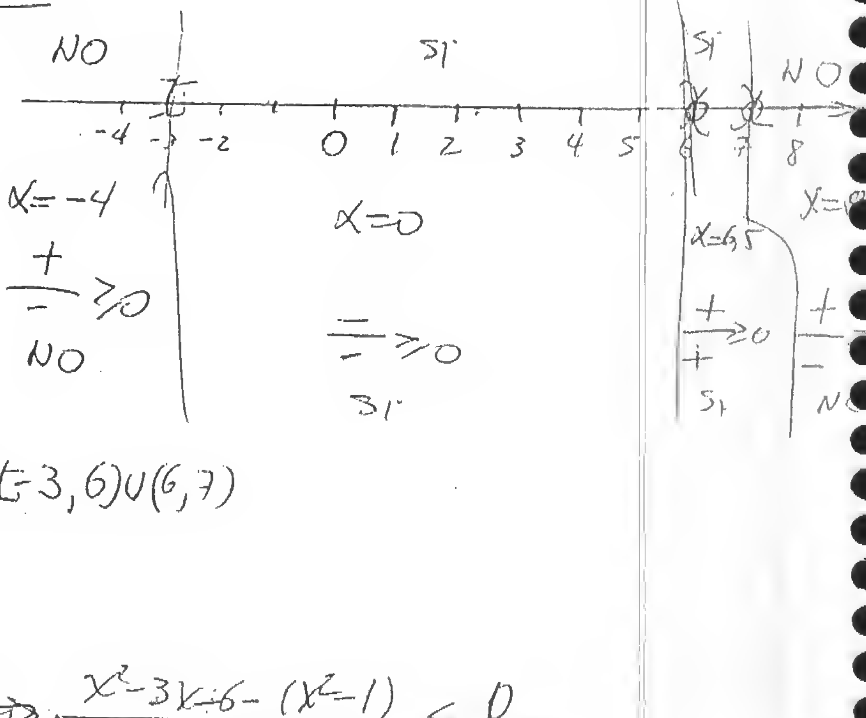
$\Rightarrow \dfrac{(x-6)(x+3)}{-(x^2 - 13x + 42)} \geq 0 \Rightarrow \dfrac{(x-6)(x+3)}{-(x-7)(x-6)} \geq 0$

P. Critic

$\boxed{x = 7}$ $\boxed{x = 6}$

$(x-6)(x+3) \geq 0$

$\boxed{x \geq 6}$ $\boxed{x \geq -3}$



Sol: $[-3, 6) \cup (6, 7)$

h) $q(x): \dfrac{x^2 - 3x - 6}{x^2 - 1} \leq 1$

$\Rightarrow \dfrac{(x^2 - 3x - 6)}{(x+1)(x-1)} - 1 \leq 0 \Rightarrow \dfrac{x^2 - 3x - 6 - (x^2 - 1)}{(x+1)(x-1)} \leq 0$

$\Rightarrow \dfrac{\cancel{x^2} - 3x - 6 - \cancel{x^2} + 1}{x^2 - 1} \leq 0 \Rightarrow \dfrac{-3x - 5}{x^2 - 1} \leq 0$

P. Critic

$x^2 = 1$

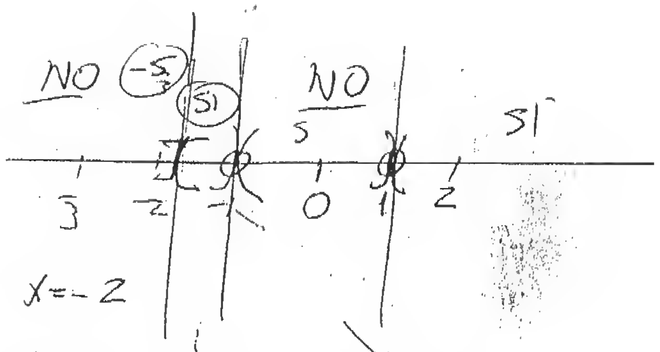
$\boxed{x = \pm 1}$

$-3x-5 \le 0$

$-3x \le 5$

$3x \ge -5$

$\boxed{x \ge -5/3}$



$x=-2$ $\quad$ $x=-1{,}5$ $\quad$ $x=0$ $\quad$ $x=2$

$\frac{4}{3} \le 1$ NO $\quad$ $0{,}6 \le 1$ SI $\quad$ $\frac{-6}{-1} \le 1$ NO $\quad$ $\frac{-}{+} \le 1$

Sol: $[-\frac{5}{3}; -1) \cup (1, +\infty)$

95.- $a \ge 0 \wedge b \ge 0$

$$\left(\frac{a+b}{2}\right)^2 \ge (\sqrt{ab})^2$$

$$\frac{a^2+2ab+b^2}{4} \ge ab \Rightarrow a^2+2ab+b^2 \ge 4ab$$

$$\Rightarrow a^2+2ab-4ab+b^2 \ge 0 \Rightarrow a^2-2ab+b^2 \ge 0$$

$$\Rightarrow (a-b)^2 \ge 0$$ el cuadrado siempre dá como resultado un número positivo

96.- $ab > 0$

$$\frac{a}{b}+\frac{b}{a} \ge 2$$

$$\Rightarrow \frac{a^2+b^2}{ab} \ge 2$$

$$\Rightarrow a^2+b^2 \ge 2ab$$

$$a^2-2ab+b^2 \ge 0$$

$$\underbrace{(a-b)^2}_{} \ge 0$$

siempre sera un número positivo

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97.- $a^2+4b^2+3c^2+14>2a+12b+6c$

$(a^2-2a)+(4b^2-12b)+(3c^2+6c)>-14$

$a(a-2)+4b(b-3)+3c(c+2)>-14$

$(-)(-)+4(-)(-)+3(-)(-)>-14$

$(+)+(+)+(+)>-14$

$(+)>(-)$

98.- $a+b+c\geq 0 \therefore a^3+b^3+c^3\geq 3abc$

$\therefore (a+b+c)^3\geq 0^3$

$\therefore \underbrace{a^3+b^3+c^3}_{3abc}+3ab^2+3ac^2+3a^2b+3a^2c+3bc^2+3b^2c+6abc\geq 0$

$\therefore (3ab^2+3abc+3a^2b)+(3ac^2+3abc+3a^2c)+(3b^2c+3abc+3bc^2)\geq 0$

$\therefore 3ab(b+c+a)+3ac(c+b+a)+3bc(b+a+c)\geq 0$

$\therefore \underbrace{3(a+b+c)}_{\geq 0}\underbrace{(ab+ac+bc)}_{\text{debe ser también positivo}}\geq 0$

99.- $a>0;\ b>0;\ c>0$

$(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\geq 9$

$\Rightarrow (a+b+c)\left(\frac{bc+ac+ba}{abc}\right)\geq 9 \Rightarrow \frac{abc+a^2c+a^2b+b^2c+abc+ab^2+b^2a+ac^2}{abc}$

$\Rightarrow \frac{3abc+a^2c+a^2b+b^2c+b^2a+bc^2+ac^2}{abc}\geq 9 \Rightarrow 3abc+a^2c+a^2b+b^2c+ab^2+bc^2+ac^2\geq 9abc$

$\Rightarrow a^2c+a^2b+ab^2+b^2c+ac^2+bc^2-6abc\geq 0$

100.- $a \geq 0;\ b \geq 0,\ c \geq 0;\ d \geq 0$

$$\left(\frac{a+b+c+d}{4}\right)^4 - \left(\sqrt[4]{abcd}\right)^4$$

$$\Rightarrow (a+b+c+d)^4 \geq 256\,abcd$$

$$[(a+b)+(c+d)]^4 \geq 4a \times 4b \times 4c \times 4d$$

$$\Rightarrow \underbrace{(a+b)^4}_{\substack{5\text{ término} \\ (a^4 - b^4)}} + \underbrace{4(a+b)^3(c+d)}_{4(a^3 \ldots b^3)} + \underbrace{6(a+b)^2(c+d)^2}_{\substack{6(a^2+b^2+c^2+d^2) \\ 2+abcd}} + 4(a+b)(c+d)^3 + \underbrace{(c+d)^4}_{\substack{5\text{ término} \\ (c^4 \cdot d^4)}} \geq 4a \cdot 4b \cdot 4c \cdot 4d$$

101.-

① $P(1) = \dfrac{(1)^2(1+1)^2}{4} = 1^3$

$\dfrac{1(2)^2}{4} = 1 \qquad \dfrac{4}{4} = 1$

** Necesitamos llegar a.

$$\frac{(N+1)^2\,(N+2)^2}{4}$$

* Hay que demostrar que la fórmula se cumple para. $N+1$

$$1^3 + 2^3 + 3^3 + \ldots\ldots N^3 + (n+1)^3 = \frac{N^2(N+1)^2}{4} + (N+1)^3$$

$$\frac{N^2(N^2+2N+1)}{4} + \frac{4(N+1)^3}{4}$$

$$\frac{N^4 + 2N^3 + N^2 + 4(N^3 + 3N^2 + 3N + 1)}{4}$$

$$\frac{N^4 + 2N^3 + N^2 + 4N^3 + 12N^2 + 12N + 4}{4}$$

$$\frac{N^4 + 6N^3 + 13N^2 + 12N + 4}{4}$$

Se ha reemplazado N por (N+1)

$$\frac{(N+1)^2\,(N+2)^2}{4}$$

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102. a) $2^{2n}+5$ divisible 3

$n=1 \Rightarrow \frac{9}{3}=3$

$n=2 \Rightarrow \frac{21}{3}=7$ 2^2+3

$n=3 \Rightarrow \frac{69}{3}=23$ $2^4 3$

$n=4 \Rightarrow \frac{261}{3}=87$ $2^6 3$

$n \Rightarrow$ $2^{2(n-1)}+k$

$$\frac{2^{2n}+5}{3}=k+2^{2(n-1)}$$

$$\frac{2^{2(n+1)}+5}{3}=k+2^{2(n+1-1)}$$

$$\frac{2^{2n+2}+5}{3}=k+2^{2n}\cdot\frac{2^2}{2^2}$$

$$\frac{2^{2n}\cdot 4+5}{3}=k+4\cdot 2^{2n-2}$$

$$\frac{4\cdot 2^{2n}+5}{3}=k+4\cdot 2^{2(n-1)}$$

b) $\underbrace{2^{2n}\ 13n-1}$ divisible 9

$n=1$ $\frac{6}{9}$ NO ES DIVISIBLE

NO CUMPLE PARA $n\in N$

$n=2$ $\frac{21}{9}$ NO ES DIVISIBLE

$n=3$ $\frac{72}{9}=8$

c) a^n-b^n es divisible $(a-b)$

$n \rightarrow$ impar

$$(a^n-b^n)=(a-b)(a^{n-1}+a^{n-2}b+a^{n-3}b^2+\dots ab^{n-2}+b^{n-1})$$

$n \rightarrow$ par

$$(a^n-b^n)=(a^{\frac{n}{2}}-\overbrace{b^{\frac{n}{2}}}^{\text{impar}})(a^{\frac{n}{2}}+b^{\frac{n}{2}})$$

$$\Rightarrow (a^{\frac{n}{2}}+b^{\frac{n}{2}})\cdot(a-b)(a^{\frac{n}{2}-1}+a^{\frac{n}{2}-2}b+\dots ab^{\frac{n}{2}-2}+b^{n-1})$$

) $a^{2n}-1$ es divisible $(a+1)$

$a^{2n}-1$ → garantiza que sea siempre una diferencia de cuadrado

$(a^{2n}-1)=(a^n-1)(a^n+1)$

$\Rightarrow (a^n-1)(a^n+1)\begin{cases} n \text{ es par} \Rightarrow (a^n-1) \text{ es nuevamente diferencia de cuadrado, obteniéndose } n \text{ impar.} \\ n \text{ es impar} \Rightarrow \underline{(a^n+1)}=(a+1)(a^{n-1}-a^{n-2}+a^{n-3}+\dots 1) \end{cases}$

e) $a+ar+ar^2+\dots ar^{n-2}+ar^{n-1}=\dfrac{a(1-r^n)}{1-r}$

$+ar^{n-1}+ar^n=\dfrac{a(1-r^n)}{1-r}+ar^n$

$\Rightarrow \dfrac{a(1-r^n)+ar^n(1-r)}{1-r}$

$\Rightarrow \dfrac{a-\cancel{ar^n}+\cancel{ar^n}-ar^n r}{1-r}$

$\Rightarrow \dfrac{a-ar^{n+1}}{1-r} \Rightarrow \dfrac{a(1-r^{n+1})}{1-r}$

Forma original.

$\dfrac{a(1-r^n)}{1-r} \Rightarrow \dfrac{a(1-r^{n+1})}{1-r}$ $n=n+1$

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f.) $a_1 + (a_1+d) + (a_1+2d) + (a_1+3d) + \cdots (a_1+(n-1)d) = \frac{n}{2}(2a_1 + (n-1)d)$

$$+(a_1+(n+1-1)d) + (a_1+nd) = \frac{(n+1)}{2}(2a_1+(n+1-1)d)$$

$$\Rightarrow \frac{(n+1)}{2}(2a_1+nd)$$

$$\Rightarrow \frac{(2a_1 n + n^2 d + 2a_1 + nd)}{2}$$

$$\Rightarrow \frac{(2a_1 n + n^2 d + 2a_1 + nd - nd + nd)}{2}$$

$$\Rightarrow \frac{(2a_1 + 2nd) + (2a_1 n + n^2 d - nd)}{2}$$

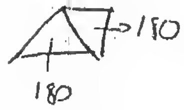
$$\Rightarrow \frac{\cancel{2}(a_1+nd)}{\cancel{2}} + \frac{n(2a_1+nd-d)}{2}$$

$$\Rightarrow (a_1+nd) + \frac{n}{2}(2a_1+(n-1)d)$$

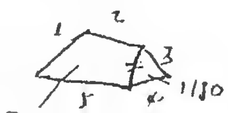
103.— n

3 

$\Rightarrow 180°$ $\quad a_1 = 180°$ $\quad S = 2R(n-2)$

4 

$\Rightarrow 360°$ $\quad d = 180°$ $\quad S = 180(n-2)$

5 

$\Rightarrow 540°$

$$a_n = S$$
$$a_1 + (n-3)d = 2R(n-2)$$
$$180 + (n-3)180 = 180(n-2)$$
$$\cancel{180}(1+n-3) = \cancel{180}(n-2)$$
$$\cancel{n}\cancel{-2} = \cancel{n}\cancel{-2}$$
$$0 = 0$$ Son iguales

104.— $1+3+5+\cdots+(2n-1) = n^2$

$$\cdots + (2n-1) + (2n+2-1) = (n+1)^2$$
$$\cdots + (2n-1) + (2n+1) = n^2+2n+1$$
$$\Rightarrow n^2 + (2n+1)$$

105.- $2^1 + 2^2 + 2^3 + \ldots\ldots + 2^n = 2(2^n - 1)$.

$\ldots\ldots + 2^n + 2^{n+1} = 2(2^{n+1} - 1)$

$\Rightarrow 2 \cdot 2^{n+1} - 2$.

$\Rightarrow 2^{n+1} + 2^{n+1} - 2$.

$\Rightarrow 2^{n+1} + 2 \cdot 2^n - 2$.

$\Rightarrow 2^{n+1} + 2(2^n - 1)$

106.-

① $p(1) = \dfrac{1(1+1)(6+9+1-1)}{30} = (1)^4$

$\dfrac{1(2)(15)}{30} = 1 \qquad \dfrac{30}{30} = 1$

* Reemplazamos N por N+1.

$$\frac{(N+1)(N+2)\left[6(N+1)^3 + 9(N+1)^2 + N+1-1\right]}{30}$$

$$\frac{(N+1)(N+2)(6N^3 + 18N^2 + 18N + 6 + 9N^2 + 18N + 9 + N)}{30}$$

$$\frac{(N+1)(N+2)(6N^3 + 27N^2 + 37N + 15)}{30}$$

$$1^4 + 2^4 + 3^4 + \ldots\ldots N^4 + (N+1)^4 = \frac{N(N+1)(6N^3 + 9N^2 + N - 1)}{30} + (N+1)^4$$

$$\frac{(N^2+N)(6N^3 + 9N^2 + N - 1) + 30N^4 + 120N^3 + 180N^2 + 120N + 30}{30}$$

$$\frac{6N^5 + 6N^4 + 9N^4 + 9N^3 + N^2 + N^2 - N^2 - N + 30N^4 + 120N^3 + 180N^2 + 120N + 30}{30}$$

$$\frac{6N^5 + 45N^4 + 130N^3 + 180N^2 + 119N + 30}{30}$$

$$\boxed{\frac{(N+1)(N+2)(6N^3 + 27N^2 + 37N + 15)}{30}}$$

107.- $1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots\ldots + \frac{1}{2^n} = 2 - \frac{1}{2^n}$.

$$+\frac{1}{2^n} + \frac{1}{2^{n+1}} = 2 - \frac{1}{2^{n+1}}$$

$$\Rightarrow 2 - \frac{1}{2 \cdot 2^n} \Rightarrow 2 - \frac{1}{2} \cdot \left(\frac{1}{2^n}\right)$$

$$\Rightarrow 2 - \left(1 - \frac{1}{2}\right)\frac{1}{2^n} \Rightarrow 2 - \frac{1}{2^n} + \frac{1}{2 \cdot 2^n}$$

$$\Rightarrow 2 - \frac{1}{2^n} + \frac{1}{2^{n+1}}$$

108.- $2+6+10+ \dots + (4n-2) = 2n^2$

$$\dots + (4n-2) + (4(n+1)-2) = 2(n+1)^2$$

$$\dots + (4n-2) + (4n+4-2) = 2(n^2+2n+1)$$

$$\dots + (4n-2) + (4n+2) = 2n^2+4n+2$$

$$\Rightarrow 2n^2 + (4n+2)$$

109.- $K+K^2+K^3+ \dots + K^n = \frac{K}{K-1}(K^n-1)$

$$\dots + K^n + K^{n+1} = \frac{K(K^n-1)}{K-1} + K^{n+1}$$

$$\Rightarrow \frac{K \cdot K^n - K + K^{n+1}(K-1)}{K-1}$$

(original) $\frac{K}{K-1}(K^n-1)$

$$\Rightarrow \frac{K^{n+1} - K + K \cdot K^{n+1} - K^{n+1}}{K-1}$$

$n = n+1$ $\frac{K}{K-1}(K^{n+1}-1)$

$$\Rightarrow \frac{K \cdot K^{n+1} - K}{K-1} \Rightarrow \frac{K(K^{n+1}-1)}{K-1}$$

110.- 1, 2, 3, 4, 5

5	4

1,2,3,4,5 — 4 números ya está en el 1er Casillero

$\Rightarrow 5 * 4$

$\Rightarrow 20$ a) Verdadero

111.-

C	A
3	4

3	2	1

$\Rightarrow 3*4*3*2*1$

$\Rightarrow$ 72 formas.

112.-

a) $n = 6$

$x = 4$

$$_6P_4 \Rightarrow \frac{6!}{2!} \Rightarrow \frac{6*5*4*3*\cancel{2!}}{\cancel{2!}}$$

$\Rightarrow$ 360 palabras

b) $n=6$

$x=6$

$$_6P_6 = \frac{6!}{0!} \Rightarrow 6\times5\times4\times3\times2\times1$$

$\Rightarrow$ 720 palabras

c)

2	5	4	3	2	1

$\Rightarrow 2\times5\times4\times3\times2\times1$

0, A

$\Rightarrow$ 240 palabras

113.- $n=18$

a) $X=3$

$$_{18}C_3 = \frac{18!}{3!\,15!} \Rightarrow \frac{\cancel{18}^{3}\times17\times16\times\cancel{15!}^{1}}{\cancel{3\times2}_{1}\times\cancel{15!}_{1}}$$

$\Rightarrow 816$

114.- 10 V

7 M

a) $\left.\begin{matrix}3V\\4M\end{matrix}\right\} 7 \Rightarrow {}_{10}C_3 * {}_7C_4$

$$\Rightarrow \frac{10!}{3!\,7!} * \frac{7!}{4!\,3!}$$

$$\Rightarrow \frac{10\times9\times\cancel{8}^{3\times4}\times\cancel{7!}}{\cancel{3}\times\cancel{2}\times1\times\cancel{7!}} * \frac{7\times6\times5\times\cancel{4!}^{1}}{\cancel{4!}_{1}\,3\times2\times1}$$

$\Rightarrow$ 4200 comités

b)

$\left.\begin{matrix}3V\\4M\end{matrix}\right\} + \left.\begin{matrix}4V\\3M\end{matrix}\right\} \Rightarrow 10C_4 * 7C_3$

$$\frac{10!}{4!\,6!} * \frac{7!}{4!\,3!} \Rightarrow \frac{10\times9\times8\times7\times\cancel{6!}}{\cancel{6!}\,4\times3\times2\times1} \times \frac{7\times6\times5\times\cancel{4!}}{\cancel{4!}\,3\times2\times1}$$

$\Rightarrow 7350$

$4200 + 7350$

$\Rightarrow$ 11550

81

115.- $n = 21$
$x = 3$

$$_{21}C_3 \Rightarrow \frac{21!}{3!\,18!}$$

$$\Rightarrow \frac{\cancel{21}^{7} \times \cancel{20}^{10} \times 19 \times \cancel{18!}}{\cancel{18!}\,\cancel{3}\times\cancel{2}\times 1}$$

$$\Rightarrow 1330$$

e) Correcto

116.- $(x+k)^5$

$$\binom{5}{i}(x)^{5-i}(k)^i = x^2$$

$$\cancel{x}^{5-i} = \cancel{x}^2$$

$$5-i=2$$

$$-i = 2-5$$

$$\boxed{i=3}$$

$$\binom{5}{3}\cdot \cancel{x^2}(k)^3 = 80$$

$$\frac{5!}{3!\,2!}\cancel{x^2}k^3 = 80$$

$$\frac{5\times 4\times \cancel{3!}}{\cancel{3!}\,2!}k^3 = 80$$

$$10k^3 = 80$$

$$k^3 = 8$$

$$\boxed{k=2}$$ i) correcto

117.-

a) $$(1-2a)^3 = \binom{3}{0}(1)^3 - \binom{3}{1}(1)^2(2a) + \binom{3}{2}(1)(2a)^2 - \binom{3}{3}(2a)^3$$

$$\Rightarrow 1 - 2a + 12a^2 - 8a^3$$

b) $$\left(a-\frac{1}{a}\right)^5 = \binom{5}{0}a^5 - \binom{5}{1}a^4\left(\frac{1}{a}\right) + \binom{5}{2}a^3\left(\frac{1}{a}\right)^2 - \binom{5}{3}a^2\left(\frac{1}{a}\right)^3$$

$$+\binom{5}{4}a\left(\frac{1}{a}\right)^4 - \binom{5}{5}\left(\frac{1}{a}\right)^5$$

$$\Rightarrow a^5 - 5a^3 + 10a - 10\frac{1}{a} + 5\frac{1}{a^3} - \frac{1}{a^5}$$

$$\Rightarrow \boxed{a^5 - 5a^3 + 10a - \frac{10}{a} + \frac{5}{a^3} - \frac{1}{a^5}}$$

118.- $\left(\frac{1}{2}u - 2v\right)^{10}$ 7mo T

$i = 7-1$

$i = 6$

$$\binom{10}{6}\left(\frac{1}{2}u\right)^{10-6} \cdot \left(-2v\right)^{4} \Rightarrow \frac{10!}{6! \cdot 4!} \cdot \frac{u^4}{\cancel{2^4}} \cdot \frac{\cancel{2^6}v^6}{}$$

$$\Rightarrow \frac{\cancel{10 \times 9 \times 8 \times 7 \times 6!}}{\cancel{6! \times 4 \times 3 \times 2 \times 1}} \Rightarrow 210 \cdot 2^2 u^4 v^6$$

$$\Rightarrow 840\, u^4 v^6$$

119.- Encontrar el Término medio en : $(2x+3y^{-1})^6$

$a = 2x$ $b = 3y^{-1}$ $N = 6$ $R = 4$ $R-1 = 3$

Hay 7 términos

$$T.C. = \frac{7+1}{2} = \frac{8}{2} = 4.$$

$$\binom{N}{R-1}(a)^{N-(R-1)}(b)^{R-1} = \binom{6}{3}(2x)^{6-3}(3y^{-1})^{3}.$$

$$\frac{\cancel{6} \times 5 \times 4 \times \cancel{3!}}{\cancel{3!} \times \cancel{3} \times \cancel{2} \times 1}(2x)^3(2+y^{-3}) = 20(8x^3)(2+y^{-3}) =$$

$$\boxed{= 4320\, x^3 y^{-3}.}$$

120.- $\left(6x - \frac{1}{2x}\right)$ independiente de $x \Rightarrow x^0$

$$\binom{10}{i}(6x)^{10-i}\left(\frac{1}{2x}\right)^{i} = x^0$$

$$x^{10-i} \cdot x^{-i} = x^0 \Rightarrow \cancel{x}^{10-2i} = \cancel{x}^0$$

$10 - 2i = 0$

$2i = 10 \Rightarrow i = 5$ $T = 6^{10}$

$$\Rightarrow \frac{10!}{5!\,5!}$$

$$\Rightarrow \frac{\cancel{10 \times 9 \times 8} \times 7 \times 6 \times \cancel{5!}}{\cancel{5!} \times \cancel{5 \times 4 \times 3} \times 2 \times 1}$$

$$\Rightarrow 252$$

83

121.— $\left(x^2+\frac{3}{x^3}\right)^7$

$\binom{7}{i}\cdot(x^2)^{7-i}\cdot\left(\frac{3}{x^2}\right)^i=x^1 \Rightarrow (x^2)^{7-i}\cdot(x^{-2})^i=x^1$

$\Rightarrow x^{2(7-i)}\cdot x^{-2i}=x^1 \Rightarrow x^{14-2i-2i}=x^1$

$14-4i=1 \Rightarrow -4i=1-14$

$i=\frac{13}{4}$

NO EXISTE
i debe ser un número entero, y no lo es

122.— $\left(x^2+\frac{1}{x}\right)^9$

Término $\Rightarrow x^0$

$\binom{9}{i}(x^2)^{9-i}\cdot\left(\frac{1}{x}\right)^i=x^0 \Rightarrow x^{2(9-i)}\cdot(x^{-1})^i=x^0$

$x^{18-2i}\cdot x^{-i}=x^0 \Rightarrow x^{18-3i}=x^0 \Rightarrow 18-3i=0$

$3i=18 \Rightarrow \underline{i=6}$ 7mo Término

$\binom{9}{6}\Rightarrow\frac{9!}{3!\,6!}\Rightarrow\frac{9\times8\times7\times6!}{3\times2\times1\times6!}$

$\Rightarrow 84$

$123.- \ (5+2x^2)^7$

$$\binom{7}{i}(5)^{7-i}(2x^2)^i = x^{10}$$

$$\Rightarrow \cancel{x}^{2i} = \cancel{x}^{10} \quad \Rightarrow 2i = 10$$

$$\boxed{i = 5}$$

$$\binom{7}{5}(5)^2(2)^5 x^{10} \Rightarrow \frac{7!}{5!\,2!} * 25 \times 32 x^{10}$$

$$\Rightarrow \frac{7 \times \overset{3}{\cancel{6}} \times \cancel{5!}}{\cancel{5!}\ \cancel{2} \times 1} * 25 \times 32\ x^{10} \Rightarrow 16800x^{10}$$

$124.- \ \left(x^2 - \frac{y}{x}\right)^5$ Término que contiene x^3

$$\binom{5}{i}(x^2)^{5-i} \cdot \left(-\frac{y}{x}\right)^i = x^3$$

$$\Rightarrow (x^2)^{5-i} \cdot (x^{-1})^i = x^3 \Rightarrow x^{10-2i} \cdot x^{-i} = x^3$$

$$\Rightarrow \cancel{x}^{10-3i} = \cancel{x}^3 \quad \Rightarrow \quad 10 - 3i = 3$$

$$-3i = 3 - 10$$

$$-3i = -7$$

$i = 7/3$ No existe el término; i debe ser un número entero ≥ 0

e) correcto

125.- $S_n = 168$

$a_1 = 30$

$d = -2$

a) $S_n = \frac{n}{2}(2a_1 + (n-1)d$

$168 = \frac{n}{2}(2(30) + (n-1)(-2))$

$336 = n(60 + 2 - 2n)$

$336 = 62n - 2n^2$

$2n^2 - 62n - 336 = 0$

$2(n^2 - 31n - 168) = 0 \Rightarrow n^2 - 31n - 168$

$n = \frac{31 \pm \sqrt{31^2 - 4(1)(-168)}}{2(1)} \Rightarrow \frac{31 \pm 40{,}44}{2}$

$n = 35{,}7$

$a_n = a_1 + (n-1)d$

$a_n = 30 + (35{,}7 - 1)(-2)$

$a_n = -39{,}4$

126.- $S_{10} = 35$

$a_1 = 10$

$a_{10} = ¿?$

$n = 10$

$S_n = \frac{n}{2}(a_1 + a_n)$

$35 = \frac{10}{2}(10 + a_{10})$

$\frac{35}{5} = 10 + a_{10}$

$7 = 10 + a_{10}$

$a_{10} = 7 - 10$

$a_{10} = -3$

127 —

$n+2(n+1)=44$

$n+2n+2=44$

$3n=44-2$

$3n=42$

$n=14$

$\Rightarrow n+3$

$\Rightarrow 17$ c) correto

128.— a) $S_n=\frac{n}{2}(a_1+a_n)$ VERDADERO

$S_n=\frac{100}{2}(1+100)$

$S_n=5050$

b) $\dfrac{\binom{20}{10}}{\binom{21}{10}} \Rightarrow \dfrac{\frac{20!}{\cancel{10!}\,10!}}{\frac{21!}{\cancel{10!}\,11!}} \Rightarrow \dfrac{\frac{\cancel{20!}}{\cancel{10!}}}{\frac{21\times\cancel{20!}}{11\times\cancel{10!}}}=\frac{11}{21}$

VERDADERO

c) $\binom{n}{0}=\binom{n}{n}$

VERDADERO

$\frac{\cancel{n!}}{0!\,n!}=\frac{\cancel{n!}}{n!\,0!}$

d) $n!=n(n-1)!$ VERDADERO

e) $2\binom{1000}{2} \Rightarrow \cancel{2}\times\frac{1000!}{998!\,\cancel{2!}}=\frac{1000\times999\times\cancel{998!}}{\cancel{998!}}=999000$; FALSO

87

129.— $a_1=2$

$$a_1+a_1r+a_1r^2=86$$

$$2+2r+2r^2=86$$

$$2r^2+2r+2-86=0$$

$$2(r^2+r-42)=0$$

$$(r+7)(r-6)=0$$

$$r=-7 \quad \boxed{r=6}$$ b) correcto

130.—

a) $f(n)=-1,1,-1,1,-1,1,\ldots$

P. geométrica, $r=-1$

b) $f(n)=\dfrac{1}{3^n}$

$f(n)=\dfrac{1}{3},\dfrac{1}{9},\dfrac{1}{27},\ldots$ P. geométrica

$$r=\frac{\frac{1}{9}}{\frac{1}{3}} \Rightarrow r=\frac{1}{3}$$

c) $f(n)=\dfrac{1}{n^2}$

$\Rightarrow 1,\dfrac{1}{4},\dfrac{1}{9},\dfrac{1}{16},\ldots$ La sucesión NO es ni geométrica o aritmética; la razón no es constante

$r_1=\dfrac{\frac{1}{4}}{1}$ $\quad$ $r_2=\dfrac{\frac{1}{9}}{\frac{1}{4}}$

$\boxed{r_1=\frac{1}{4}}$ $\quad$ $\boxed{r_2=\frac{4}{9}}$

d) $f(n)=\dfrac{1}{n!} \Rightarrow \dfrac{1}{1},\dfrac{1}{2},\dfrac{1}{6},\dfrac{1}{24},\ldots$ NO es geométrica o aritmética

131.— $\sqrt{2\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}}$

$\Rightarrow \left(2\left(2\left(2\left(2(2)^{1/2}\right)^{1/2}\right)^{1/2}\right)^{1/2}\right)^{1/2} \Rightarrow 2^{1/2}\cdot 2^{1/4}\cdot 2^{1/8}\cdot 2^{1/16}\cdot 2^{1/32}$

$\Rightarrow 2^{1/2+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}}$

$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}=\frac{16+8+4+2+1}{32}$

$\Rightarrow \frac{31}{32}$

$\Rightarrow 2^{\frac{31}{32}}$

a) correcto

132.—

a) $\frac{1}{1001}, \frac{1}{2001}, \frac{1}{3001}$

$\Rightarrow \frac{1}{1000n+1}$

b) $1, \frac{1}{9}, \frac{1}{25}, \frac{1}{49}$

$\Rightarrow \frac{1}{(2n-1)^2}$

133.— $a_1 = 3$
$r = 2$

$f(4) = a_1 r^3$
$\Rightarrow 3(2)^3$
$\Rightarrow 24$

$f(6) = a_1 r^5$
$\Rightarrow 3(2)^5$
$\Rightarrow 96$

$f(11) = a_1 r^{10}$
$= 3(2)^{10}$
$\Rightarrow 3072$

134.- $a_5 = 162$ $\quad a_5 = a_1 r^4$ $\quad a_8 = a_1 r^7$

$a_8 = 432$ $\quad 162 = a_1 r^4$ $\quad 432 = a_1 r^7$

$a_3 = ¿?$ $\quad a_1 = \frac{162}{r^4}$ $\quad a_1 = \frac{432}{r^7}$

$a_3 = a_1 r^2$

$\frac{162}{r^4} = \frac{432}{r^7}$

$a_3 = \frac{243}{4}\sqrt[3]{3} \times \left(\frac{2}{\sqrt[3]{3}}\right)^2$

$\frac{r^7}{r^4} = \frac{432}{162}$

$a_3 = \frac{243 \times \cancel{4}}{\cancel{4}} \times \frac{\sqrt[3]{3}}{\sqrt[3]{9}}$

$r^3 = \frac{216}{81}$; $r = \sqrt[3]{\frac{216}{81}} \Rightarrow \frac{\sqrt[3]{216}}{\sqrt[3]{81}}$

$a_3 = \frac{243}{1}\sqrt[3]{\frac{\cancel{81}}{\cancel{9}\,3}}$

$r = \frac{6}{\sqrt[3]{27}\sqrt[3]{3}} \Rightarrow \frac{\cancel{6}\,2}{\cancel{3}\sqrt[3]{3}} \Rightarrow \boxed{\frac{2}{\sqrt[3]{3}}}$

$a_3 = \frac{243}{\sqrt[3]{3}} \cdot \frac{\sqrt[3]{9}}{\sqrt[3]{9}}$

$a_1 = \frac{162}{\left(\frac{2}{\sqrt[3]{3}}\right)^4} \Rightarrow \frac{162}{\frac{16}{(\sqrt[3]{3})^3\sqrt[3]{3}}} \Rightarrow \frac{81 \times 3\sqrt[3]{3}}{4}$

$a_3 = \frac{243\sqrt[3]{9}}{3}$

$a_3 = 81\sqrt[3]{9}$

$\boxed{a_1 = \frac{243}{4}\sqrt[3]{3}}$

135.- $S_3 = 9$

$S_6 = -63$

$S_{10} = ¿?$

$9 = \frac{a_1(1-r^3)}{1-r}$

$9(1-r) = a_1(1-r^3)$

$9(1-r) = a_1(1-r^3)$

$9 = \frac{a_1(1-r^3)}{1-r}$

$-63 = \frac{a_1(1-r^6)}{1-r}$

$-63(1-r) = a_1(1-r^6)$

$-63(1-r) = a_1(1+r^3)(1-r^3)$

$-\frac{63}{1+r^3} = \frac{a_1(1-r^3)}{1-r}$

$S_{10} = \frac{a_1(1-r^{10})}{1-r}$

$\cancel{9}^{1} = -\frac{\cancel{63}^{7}}{1+r^3}$

$1+r^3 = -7$

$+r^3 = -7-1$

$r^3 = -8$

$\boxed{r = -2}$

$S_{10} = \frac{3(1-(-2)^{10})}{1-(-2)}$

$S_{10} = \frac{\cancel{3}(1-1024)}{\cancel{3}}$

$S_{10} = -1023$

$a_1 = \frac{9(1-r)}{1-r^3} \Rightarrow \frac{9(1-(-2))}{1-(-2)^3}$

$a_1 = \frac{27}{9}$

$\boxed{a_1 = 3}$

136.—

$n=10$ $\qquad a_{10}=a_1 r^9$

$r=\frac{1}{2}$ $\qquad a_1=\frac{a_{10}}{r^9} \Rightarrow \frac{10}{\left(\frac{1}{2}\right)^9}$

$a_{10}=10$

$a_1=$¿? $\qquad a_1=10 \times 2^9$

$\boxed{a_1=5120}$

137.—

$r=\frac{2}{5}$ $\qquad a_{10}=a_1 r^{n-1}$

$a_{10}=150$ $\qquad a_{10}=a_1 r^9$

$S_{10}=$¿? $\qquad a_1=\frac{a_{10}}{r^9} \Rightarrow \frac{150}{\left(\frac{2}{5}\right)^9} \Rightarrow \frac{150 \times 5^9}{2^9}$

$$S_{10}=a_1\frac{(1-r^{10})}{1-r} \Rightarrow \frac{150 \times 5^9}{2^9} \times \frac{\left(1-\left(\frac{2}{5}\right)^{10}\right)}{1-\frac{2}{5}}$$

$$S_{10}=\frac{150 \times 5^9}{2^9} \times \frac{1-\frac{2^{10}}{5^{10}}}{\frac{5-2}{5}} \Rightarrow \frac{150 \times 5^9}{2^9} \times \frac{\frac{5^{10}-2^{10}}{5^{10\,9}}}{\frac{3}{\cancel{5}}}$$

$$S_{10}=\frac{150 \times 5^9}{2^9} \times \frac{\frac{5^{10}-2^{10}}{5^9}}{\frac{3}{1}} \Rightarrow \frac{\overset{25}{\cancel{150}} \times \cancel{5^9}}{2^{9}\,8} \times \frac{5^{10}-2^{10}}{\cancel{3} \times \cancel{5^9}}$$

$$\boxed{S_{10}=\frac{25}{256}\left(5^{10}-2^{10}\right)}$$

138.—

a) $0,675675675\ldots$

$$\Rightarrow \frac{675}{1000}+\frac{675}{1000000}+\frac{675}{1000000000}+\ldots$$

$$r=\frac{\frac{675}{1000000}}{\frac{675}{1000}}=\frac{1}{1000} \qquad N=\frac{a_1}{1-r}$$

$$N = \frac{\frac{675}{1000}}{1 - \frac{1}{1000}} \Rightarrow \frac{\frac{675}{\cancel{1000}}}{\frac{1000-1}{\cancel{1000}}} \Rightarrow \frac{\overset{75}{\cancel{675}}}{\underset{111}{\cancel{999}}}$$

$$N = \frac{75}{111}$$

b) 3,4738247382...

$$\Rightarrow 3 + \underbrace{\frac{47382}{100000} + \frac{47382}{10000000000}}_{} + \dots$$

$$r = \frac{1}{100000} \qquad N = \frac{\frac{47382}{100.000}}{1 - \frac{1}{100.000}} \Rightarrow \frac{\frac{47382}{\cancel{100.000}}}{\frac{100.000-1}{\cancel{100.000}}}$$

$$N = \frac{\overset{15794}{\cancel{47382}}}{\underset{33.333}{\cancel{99999}}} \Rightarrow \frac{15794}{33.333} + 3$$

$$N = \frac{15794 + 99999}{33.333} = \frac{115793}{33.333}$$

c) 0,3754337543...

$$\Rightarrow \underbrace{\frac{37543}{100000} + \frac{37543}{10000000000}}_{} + \dots$$

$$r = \frac{1}{100.000} \qquad N = \frac{\frac{37543}{100.000}}{1 - \frac{1}{100000}} \Rightarrow \frac{\frac{37543}{\cancel{100.000}}}{\frac{100000-1}{\cancel{100.000}}}$$

$$N = \frac{37543}{99.999}$$

d) 12,213333213333...

$$\Rightarrow 12 + \frac{213333}{1000000} + \frac{213333}{10^{12}} + \dots$$

$$r = \frac{1}{1000000}$$

$$N = 12 + \frac{\frac{213333}{1000000}}{1 - \frac{1}{1000000}} \Rightarrow$$

$$N = 12 + \frac{\frac{\cancel{213333}}{\cancel{1000000}}}{\frac{\cancel{999999}}{\cancel{1000000}}} \quad \frac{71111}{333333}$$

$$N = 12 + \frac{71111}{333333} \Rightarrow \frac{3999996 + 71111}{333333}$$

$$N = \frac{4071107}{333333}$$

139.— $a_1 + a_1 r = 15$ $\qquad 27 = \frac{a_1}{1-r}$

$$27\left[a_1 + a_1\frac{(27-a_1)}{27}\right] = (15)*27 \qquad 1 - r = \frac{a_1}{27}$$

$$27a_1 + a_1(27 - a_1) = 405 \qquad -r = \frac{a_1}{27} - 1 \Rightarrow \frac{a_1 - 27}{27}$$

$$27a_1 + 27a_1 - a_1^2 - 405 = 0$$

$$-a_1^2 + 54a_1 - 405 = 0 \qquad \boxed{r = \frac{27 - a_1}{27}}$$

$$a_1^2 - 54a_1 + 405 = 0$$

$$(a_1 - 45)(a_1 - 9) = 0 \qquad r = \frac{27-9}{27}$$

$$\cancel{a_1 = 45} \qquad r = \frac{\cancel{18}\,2}{\cancel{27}\,3}$$

$$\boxed{a_1 = 9} \qquad \boxed{r = \frac{2}{3}}$$

140.— $a_1 = 100$

$d = 20$

$n = ¿?$

$S_n = 5800$

$$S_n = \frac{n}{2}(2a_1 + (n-1)d)$$

$$5800 = \frac{n}{2}(2(100) + (n-1)20)$$

$$11600 = n(200 + 20n - 20)$$

$$1160\cancel{0} = 18\cancel{0}n + 2\cancel{0}n$$

$$2n + 18n - 1160 = 0$$

$$2(n^2+9n-580)=0$$

$$(n+29)(n-20)=0$$

$$n=20$$

$$a_n=100+19(20)$$

$$a_n=480$$

141.– $[200+0{,}02(5000)]+200+0{,}02(4800)]+[200+0{,}02(4600)]+\cdots$

$$300+296+292+288+\cdots$$

a) $\frac{500\cancel{0}}{20\cancel{0}}=$ 25 pagos

b) \$ 200

c) $S_{25}=\frac{n}{2}(2a_1+(n-1)d)$

$$\Rightarrow \frac{25}{2}(2(300)+24(-4))$$

$$S_{25}\Rightarrow \$\,6300$$

d) $I=6300-5000$

$$I=\$\,1300$$

142.– $a_1=1700$

$a_n=\$\,200$

$d=-150$

$n=¿?$

DEPRECIACION LINEAL

$$a_n=a_1+(n-1)d$$

$$200=1700+(n-1)(-150)$$

$$-150\cancel{0}=-15\cancel{0}n+15\cancel{0}$$

$$15n=150+15$$

$$n=11$$

c) $a_{20} = a_1 + 19d$

$a_{20} = 125 + 19(4)$

$a_{20} = \$\ 201$

143.- $S_n = 5490$

$d = 4$

$a_1 = 125$

a) $n = ¿?$

b) $a_n = ¿?$

$S_n = \frac{n}{2}(2a_1 + (n-1)d)$

$5490 = \frac{n}{2}(2(125) + (n-1)4)$

$10980 = n(250 + 4n - 4)$

$10980 = 246n + 4n^2$

$4n^2 + 246n - 10980 = 0$

$2n^2 + 123n - 5490 = 0$

$n = \frac{-123 \pm \sqrt{123^2 - 4(2)(-5490)}}{2(2)}$

$n = \frac{-123 \pm 243}{4} = \$\ 30$

$a_n = a_1 + 29(4)$

$a_n = 125 + 116$

$a_n = \$\ 241$

144.- Deuda $= \$\ 1800$

$= [150 + 0,01(1800)] + [150 + 0,01(1650)] + [150 + 0,01(1500)] + \ldots$

$168 + 166,5 + 165 + \ldots$

145.-

$a_1 = 1500$

$n = 9$

$a_9 = 420$

$d = ¿?$

$a_n = a_1 + (n-1)d$

$420 = 1500 + 8d$

$8d = 420 - 1500$

$d = \frac{-1080}{8}$

$d = -135$

depreciación: $135

146.- $a_1 = \$2000$

$d = -160$

$n = ¿?$

$a_n = \$400$

Depreciación lineal

$a_n = a_1 + (n-1)d$

$400 = 2000 + (n-1)(-160)$

$400 - 2000 = -160n + 160$

$16\cancel{0}n = 160\cancel{0} + 16\cancel{0}$

$n = \frac{176}{16}$

$n = 11$

147.-

$a_8 = \$153$

$a_{15} = \$181$

a) $d = ¿?$

b) $a_1 = ¿?$

c) $a_{20} ¿?$

$a_n = a_1 + (n-1)d$

$(153 = a_1 + 7d)(-2)$ $\quad 181 = a_1 + 14d$

$-306 = -2a_1 - 14d$

$181 = a_1 + 14d$

$-125 = -a_1$

$a_1 = 125$

$153 = a_1 + 7d$

$7d = 153 - a_1$

$d = \frac{153 - 125}{7}$

$d = 4$

a) $a_1 = \$ 168$

b) $d = 1{,}5$

$a_n = ?$

$a_n = a_1 + (n-1)d$

$a_n = 168 + (11)(-1{,}5)$

$a_n = \$ 151{,}5$

$n = \frac{180\cancel{0}}{15\cancel{0}} = 12$ pagos

c) $S_n = \frac{n}{2}(a_1 + a_n)$

$S_n = \frac{12}{2}(168 + 151{,}5)$

$S_n = \$ 1917$

d) Intereses: $1917 - 1800$

$\$ 117$

1.13. 9, 12, 15

$a_1 = 9$

$d = 3$

$S_n = 306$

$S_n = \frac{n}{2}(2a_1 + (n-1)d)$

$306 = \frac{n}{2}(2(9) + (n-1)3)$

$612 = n(18 + 3n - 3)$

$612 = 15n + 3n^2$

$3n^2 + 15n - 612 = 0$

$3(n^2 + 5n - 204) = 0$

$n^2 + 5n - 204 = 0$

$(n+17)(n-12) = 0$

$\cancel{n = -17}$

$n = 12$

149.- , -12, -7, -2, 3, 8, ...

$a_1 = -12$

$d = 5$

$n = ¿?$

$S_n = 105$

$S_n = \frac{n}{2}(2a_1 + (n-1)d)$

$105 = \frac{n}{2}(2(-12) + (n-1)5)$

$210 = n(-24 + 5n - 5)$

$210 = -29n + 5n^2$

$5n^2 - 29n - 210 = 0$

$(5n - 50)(5n + 21) = 0$

$5n - 50 = 0$ $\quad \cancel{5n = -21}$

$5n = 50$

$n = 10$

150.- $n = 50$
$d = 3$
$a_1 = 8$
$S_{50} = ¿?$

$S_n = \frac{n}{2}(2a_1 + (n-1)d)$

$S_{50} = \frac{50}{2}(2(8) + 49(3))$

$S_{50} = 25(16 + 147)$

$S_{50} = 25(163)$

$\boxed{S_{50} = 4075}$

d) Correcto

151.- $a_1 = 105$
$a_n = 994$
$d = 7$

$a_n = a_1 + (n-1)d$

$994 = 105 + (n-1)7$

$994 - 105 = 7n - 7$

$7n = 889 + 7$

$n = 128$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_n = \frac{128}{2}(105 + 994) \Rightarrow \boxed{70336}$

e) Correcto

Solución

Ejercicios Propuestos

CAPÍTULO TRES

MATEMÁTICAS

FUNCIONES DE UNA VARIABLE REAL

1.- FALSO; de ser así, deja de ser FUNCION

2.- $f(x) = \frac{2x+1}{x-3}$

Punto crítico

$x-3=0$

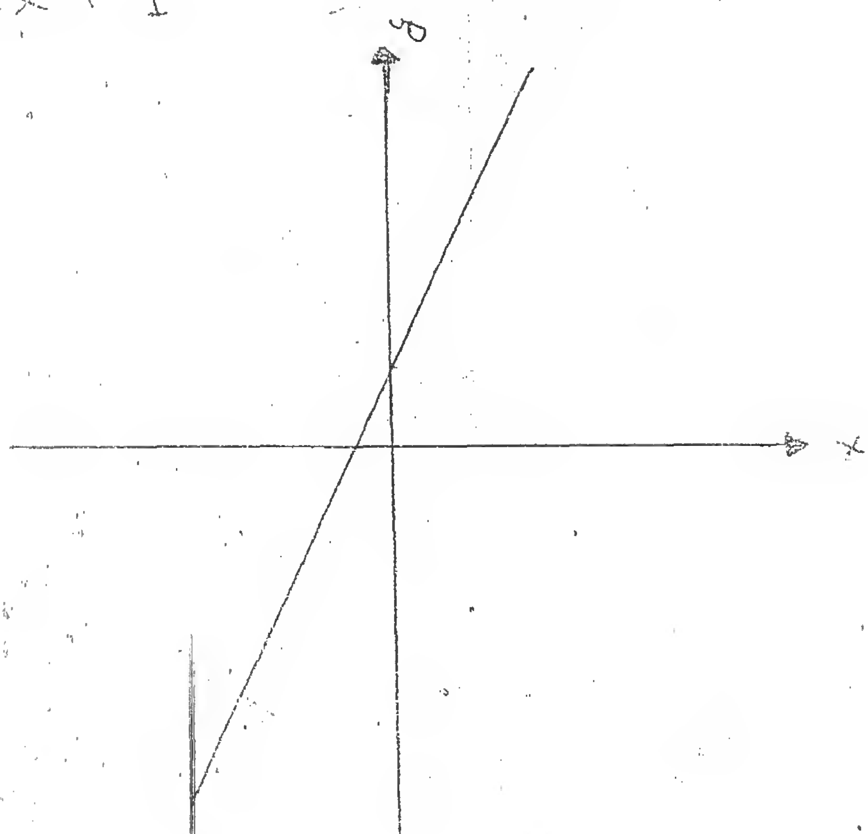
$\boxed{x=3}$

Máximo dominio: $(-\infty, 3) \cup (3, +\infty)$

a) CORRECTO

3) $f(x) = 2x + 1$

$(2, \infty)$



El rango de la función es $(-\infty, \infty)$

a) Verdadero X Falso

4.-

a) $f(x)=\sqrt{\sqrt{x}-1}$ dom: $[1,+\infty)$ VERDADERO

$\sqrt{x}-1\geq 0$

$(\sqrt{x})^2\geq 1^2$

$\boxed{x\geq 1}\Rightarrow$ dom: $[1,+\infty)$

b) $f(x)=\dfrac{x^8-x^3+x-\sqrt{2}}{\sqrt{3}-1}$ dom: $\mathbb{R}$ VERDADERO

NO HAY RESTRICCIONES

POR LO TANTO dom $f=\mathbb{R}$

c) $f(x)=\dfrac{1}{x-1}$; dom $f=\mathbb{R}-\{1\}$ VERDADERO

Punto crítico

$x-1=0$

$\boxed{x=1}\Rightarrow$ dom $f=\mathbb{R}-\{1\}$

d) $f(x)=\dfrac{\sqrt{x-1}}{x^2-4}$ FALSO

Punto crítico

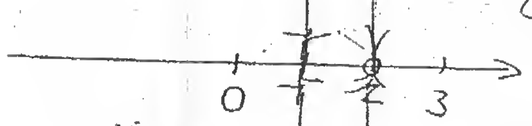
$x-1\geq 0$

$x\geq 1$

$x^2-4=0$

$\sqrt{x^2}=\sqrt{4}$

$\boxed{x=\pm 2}$



dom f: $[1,2)\cup(2,+\infty)$

$x=0$ $\sqrt{x-1}$ NO

$x=1{,}5$ $\sqrt{x-1}$ SI

$x=3$ $\sqrt{x-1}$ SI

5.- a) $g(x) = \dfrac{x}{x-1}$

Ponto crítico

$x - 1 = 0$

$\boxed{x = 1}$

$\text{dom } g = \mathbb{R} - \{1\}$

b) $h(x) = \dfrac{2x}{x+3}$

Ponto crítico

$x + 3 = 0$

$\boxed{x = -3}$

$\text{dom } h = \mathbb{R} - \{-3\}$

c) $f(x) = \sqrt{1 - x^2}$

$1 - x^2 \geq 0$

$-x^2 \geq -1$

$x^2 \leq 1$

$\boxed{x \leq \pm 1}$

-2	-1	0	1	2
$x = -2$		$x = 0$		$x = 2$
$\sqrt{1-x^2}$		$\sqrt{1-x^2}$		$\sqrt{1-x^2}$
NO		Si		NO

$\text{dom } f: [-1, 1]$

d) $r(x) = \sqrt{x^2 - 1}$

$x^2 - 1 \geq 0$

$x^2 \geq 1$

$\boxed{x \geq \pm 1}$

-2	-1	0	1	2
$x = -2$		$x = 0$		$x = 2$
$\sqrt{x^2-1}$		$\sqrt{x^2-1}$		$\sqrt{x^2-1}$
Si		No		Si

$\text{dom } r: (-\infty, -1] \cup [1, +\infty)$

e) $g(x) = \dfrac{2}{\sqrt{|x-2|-1}}$

$|x-2|-1 > 0$

$|x-2| > 1$

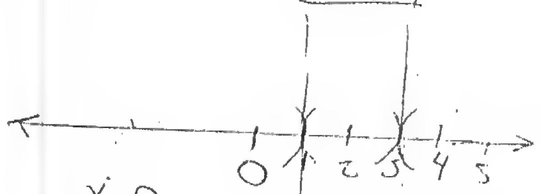
$x-2 > 1 \quad \vee \quad -x+2 > 1$

$\boxed{x > 3}$

$-x > 1-2$

$-x > -1$

$\boxed{x < 1}$



$x=0$	$x=2$	$x=4$
$\sqrt{\lvert x-2\rvert-1}$	$\sqrt{\lvert x-2\rvert-1}$	$\sqrt{\lvert x-2\rvert-1}$
sí	no	sí

dom : $(-\infty, 1) \cup (3, +\infty)$

f) $f(x) = \dfrac{x^2-1}{x^2+1}$

Punto crítico

$x^2+1=0$

$x^2 \neq -1$ NO HAY PUNTO

$x \neq \pm 1$ CRÍTICO

dom f : $x \in \mathbb{R}$

g) $f(x) = \dfrac{1}{x-1} + \dfrac{1}{x-2}$

$\Rightarrow \dfrac{x-2+x-1}{(x-1)(x-2)} \Rightarrow \dfrac{2x-3}{(x-1)(x-2)}$

Puntos críticos

$x-1=0 \qquad x-2=0$

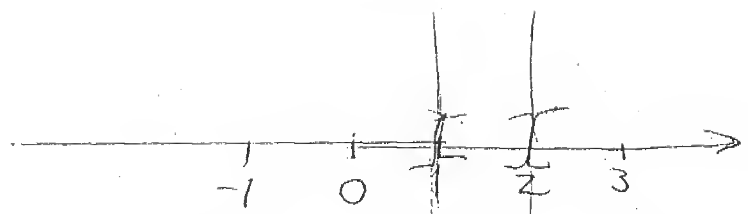
$\boxed{x=1} \qquad \boxed{x=2}$

dom f : $\mathbb{R} - \{1, 2\}$

h) $h(x) = \sqrt{x-1} + \sqrt{x-2}$

$x-1 \geq 0 \qquad x-2 \geq 0$

$\boxed{x \geq 1} \qquad \boxed{x \geq 2}$



$x = 0$ | $x = 1{,}5$ | $x = 3$

$\sqrt{x-1}$	$\sqrt{x-2}$	$\sqrt{x-2}$	$\sqrt{x-1}$	$\sqrt{x-2}$
NO	NO	NO	si	si

dom h: $[2, +\infty)$

6.– $f(x) = x^2 - x$

Vértice

$x = -\frac{b}{2a} = \frac{-(-1)}{2(1)} = \frac{1}{2}$

$y = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)$

$y = -1/4$

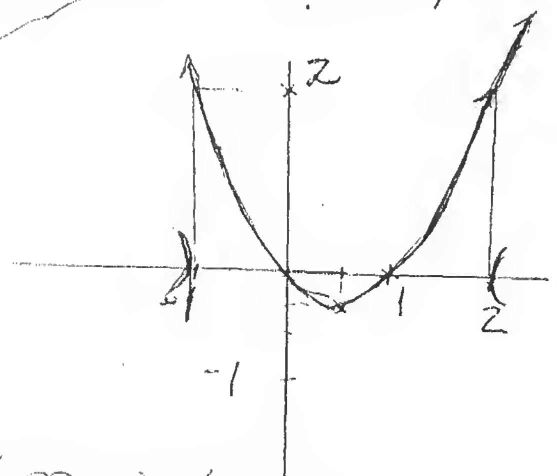
$V\left(\frac{1}{2}, -\frac{1}{4}\right)$

raices

$x^2 - x = 0$

$x(x-1) = 0$

$\{x = 0\}$ $\{x = 1\}$



$x^2 - x > 2$

$x^2 - x - 2 > 0$

$(x-2)(x+1) > 0$

$\{x > 2\}$ $\{x > -1\}$

$\therefore (-\infty, -1) \cup (2, +\infty)$

7.– $f(x) = \frac{\sqrt{4 - x^2}}{x^2 + 6x - 7}$

$4 - x^2 \geq 0$

$-x^2 \geq -4$

$x^2 \leq 4$

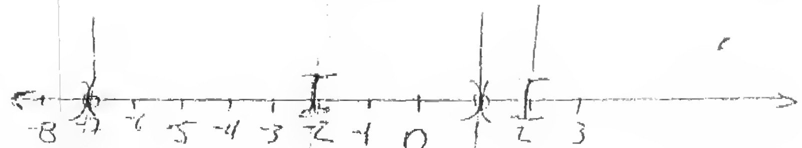
$\{x \leq \pm 2\}$

Pontos críticos

$x^2 + 6x - 7 = 0$

$(x+7)(x-1) = 0$

$\{x = -7\}$ $\{x = 1\}$



$x=-8$	$x=-3$	$x=0$	$x=1{,}5$	$x=3$
$\sqrt{4-x^2}$	$\sqrt{4-x^2}$	$\sqrt{4-x^2}$	$\sqrt{4-x^2}$	$\sqrt{4-x^2}$
NO	NO	SI	SI	NO

dom: $[-2,1)\cup(1,2]$. c) correcto

8.- $h(x)=\sqrt{x-4+|3x-5|}$

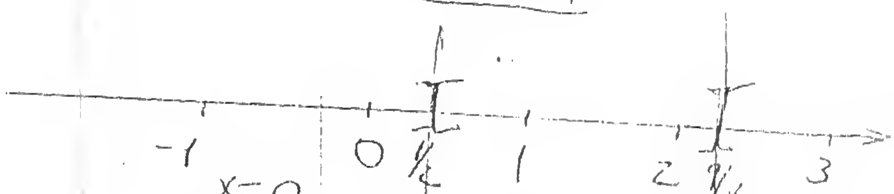
$x-4+|3x-5|\geq 0$

$x-4+3x-5\geq 0 \quad \vee \quad x-4-3x+5\geq 0$

$4x\geq 9 \qquad\qquad -2x\geq -1$

$\boxed{x\geq 9/4} \qquad\qquad 2x\leq 1$

$\boxed{x\leq 1/2}$



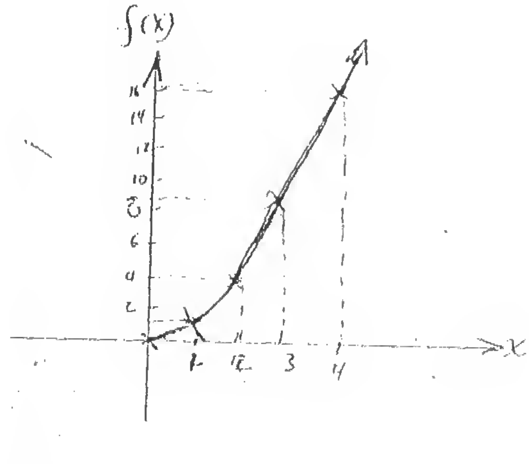
$x=0$	$x=1$	$x=3$
$\sqrt{0-4+\lvert -5\rvert}$	$\sqrt{1-4+\lvert 3-5\rvert}$	$\sqrt{3-4+\lvert 3(3)-5\rvert}$
Si	NO	Si

dom: $(-\infty,1/2]\cup[9/4,+\infty)$

dom $(1/2, 9/4)^c$ c) correcto

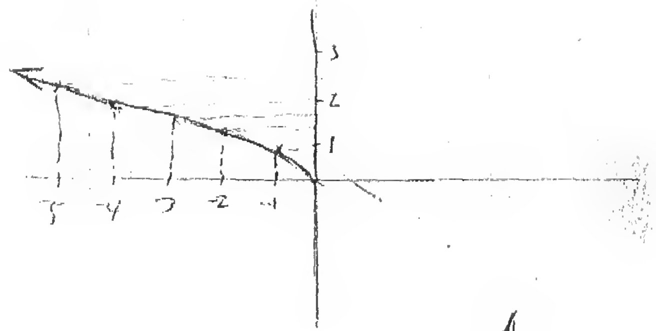
9.- a) $f(x)=x^2$; $x\geq 0$

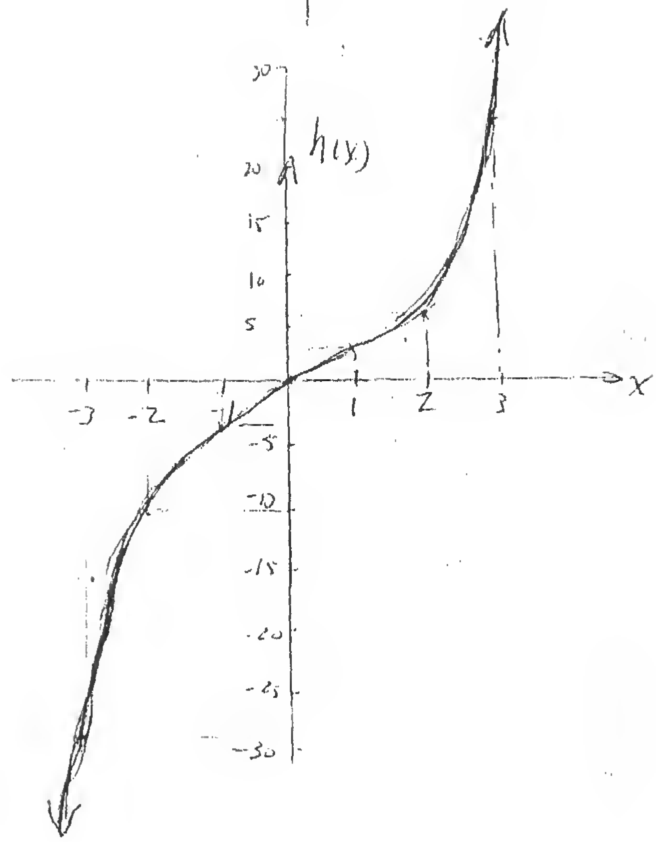
x	$f(x)$
0	0
1	1
2	4
3	9
4	16
⋮	⋮



b) $g(x) = \sqrt{-x}$; $x \leq 0$

x	$g(x)$
0	0
−1	1
−2	$\sqrt{2}$
−3	$\sqrt{3}$
−4	2
−5	$\sqrt{5}$



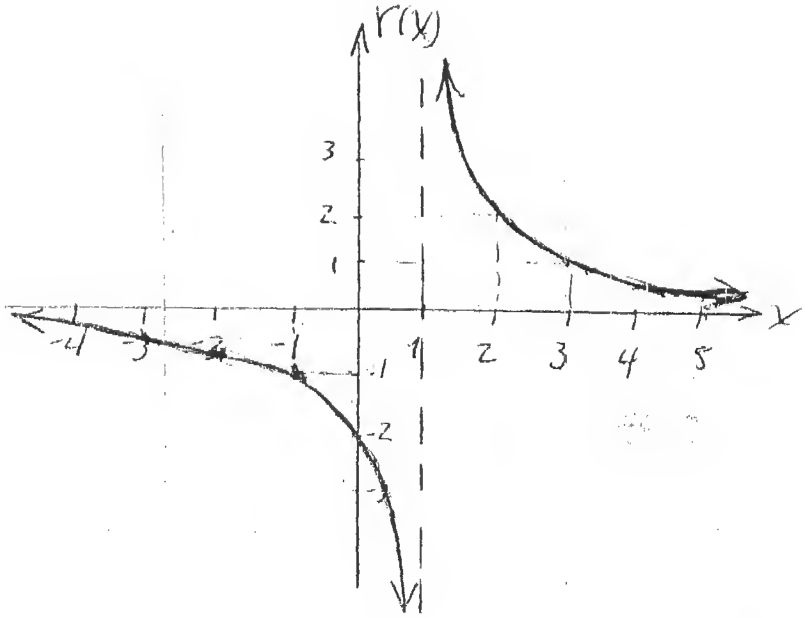


c) $h(x) = x^3 - 2$; $x \in \mathbb{R}$

x	$h(x)$
−3	−29
−2	−10
−1	−3
0	0
1	−1
2	6
3	25

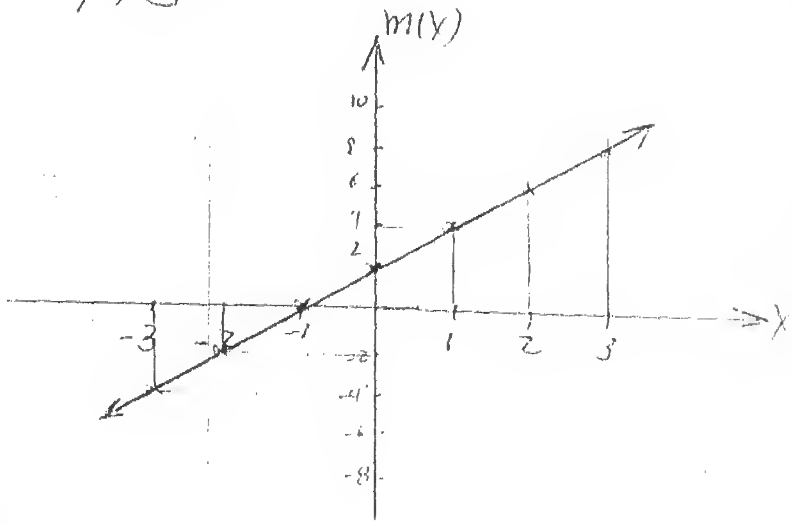
d) $r(x) = \dfrac{2}{x-1}$; $x \in \mathbb{R}, x \neq 1$

x	$r(x)$
−3	$-1/2$
−2	$-2/3$
−1	−1
0	−2
2	2
3	1
4	$2/3$
5	$1/2$



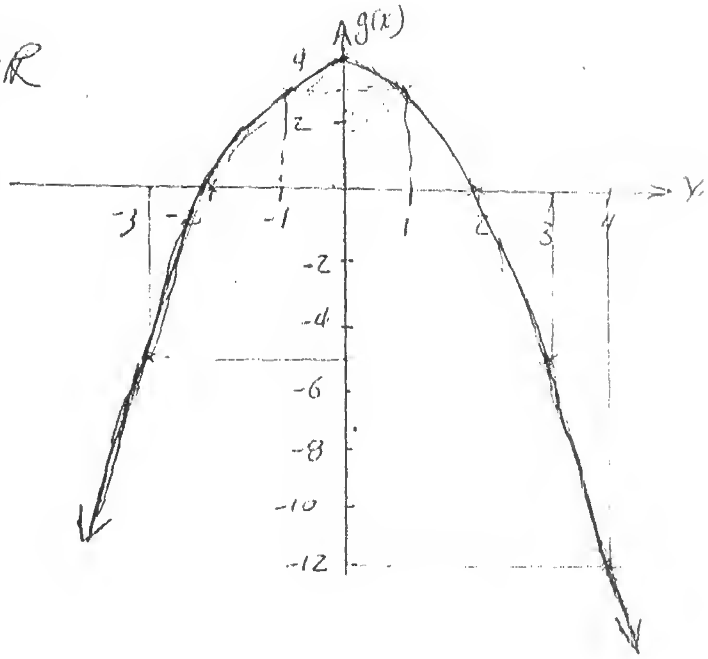
e) $m(x) = 2x+2 \quad ; x \in \mathbb{R}$

x	$m(x)$
-3	-4
-2	-2
-1	0
0	2
1	4
2	6
3	8



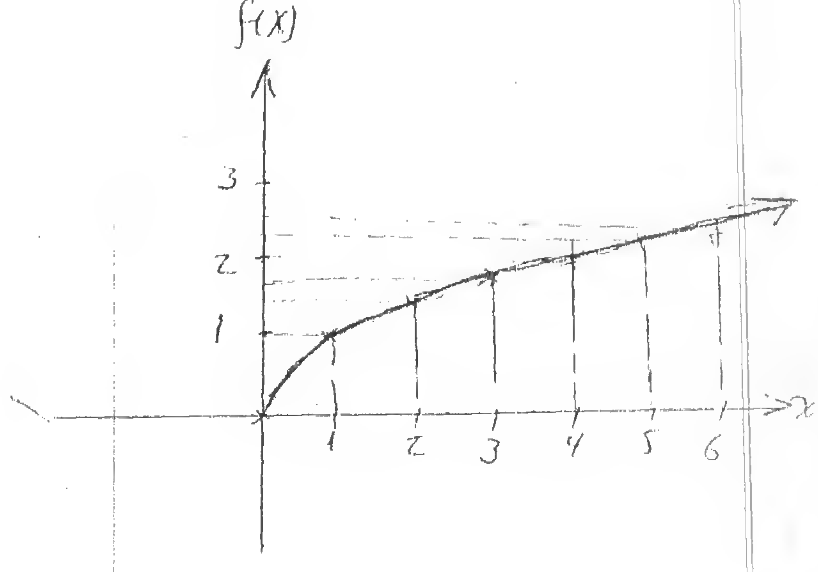
f) $g(x) = 4 - x^2 \quad ; x \in \mathbb{R}$

x	$g(x)$
-3	-5
-2	0
-1	3
0	4
1	3
2	0
3	-5
4	-12



g) $f(x) = \sqrt{x} \quad ; x \geq 0$

x	$f(x)$
0	0
1	1
2	$\sqrt{2}$
3	$\sqrt{3}$
4	2
5	$\sqrt{5}$
6	$\sqrt{6}$



10.- Si al trazar una recta vertical, intercepta más de 1 punto, la relación NO ES FUNCION.

a) Si es función

b) Si es función

c) Si es función

d) Si es función

e) Si es función

f) NO es función

g) NO es función.

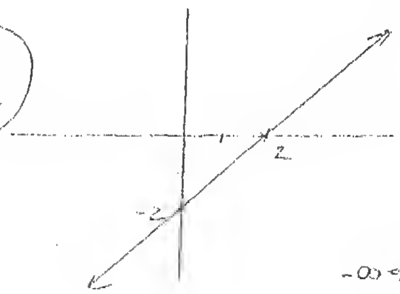
11.- b) Falso, si eso ocurre deja de ser función.

12.- $f(x) = |x-2|+1$

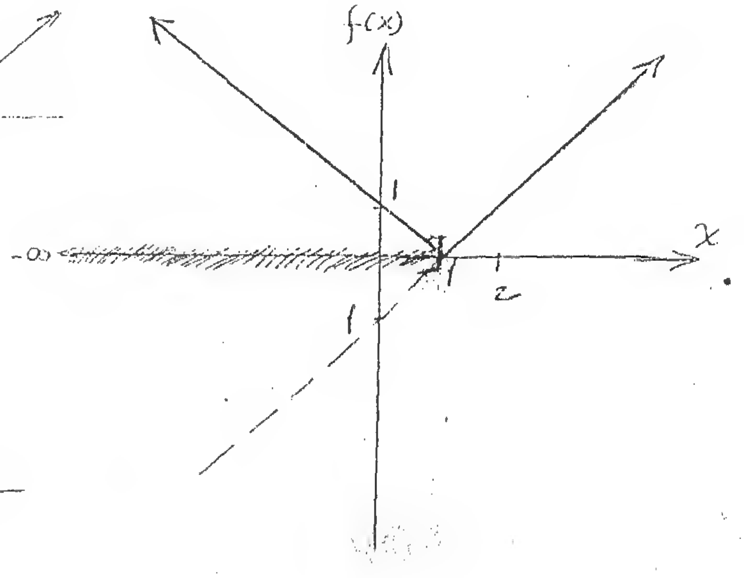
$y = x-2$

$x=0$
$y=-2$

$y=0$
$x=2$

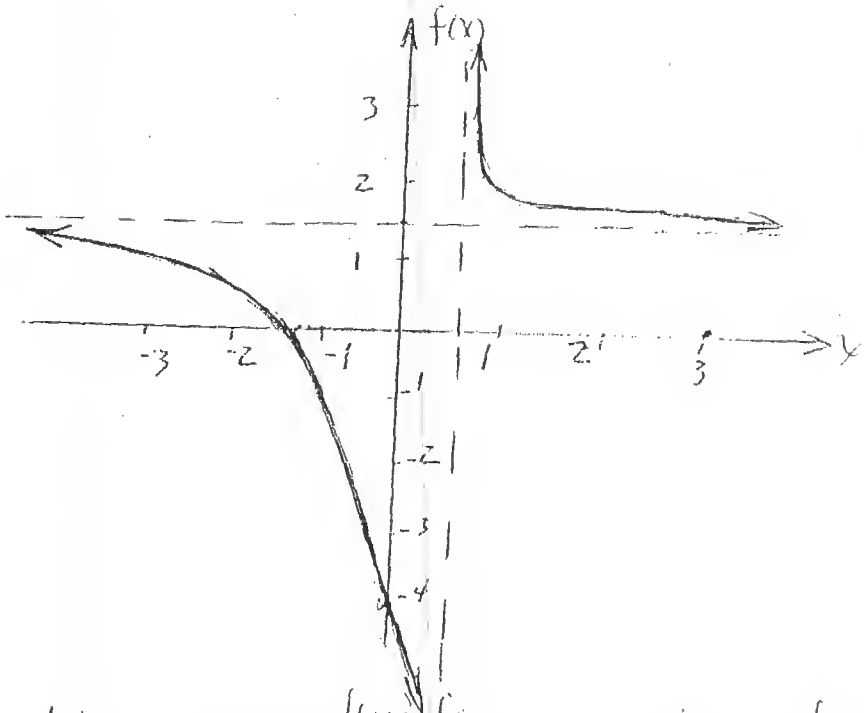


$f(x) = |x-2|+1$



a) correcto

13.- $f(x) = \dfrac{3x+4}{2x-1}$



asintota vertical

$2x-1=0$

$2x=1$

$\boxed{x=1/2}$

asintota horiz.

$\dfrac{3\cancel{x}}{2\cancel{x}}$

$\boxed{y=3/2}$

raíces

$3x+4=0$

$3x=-4$

$\boxed{x=-4/3}$

Intersecciones con y

$x=0$

$\boxed{f(0)=-4}$

14.- $g(x) = \dfrac{f(x)+f(-x)}{2}$; $h(x) = \dfrac{f(x)-f(-x)}{2}$

a) $h(-x) = \dfrac{f(-x)-f(x)}{2} \neq g(x)$; FALSO

b) Si es impar $h(x) = -h(-x)$

$\Rightarrow -\dfrac{f(-x)-f(x)}{2}$

$\Rightarrow \dfrac{f(x)-f(-x)}{2}$; CORRECTO

c) $f(a) = g(-a) - h(-a)$

$\Rightarrow \dfrac{f(-a)+f(a)}{2} - \dfrac{f(-a)-f(a)}{2}$

$\Rightarrow \dfrac{\cancel{f(-a)}+f(a)-\cancel{f(-a)}+f(a)}{2} \Rightarrow \dfrac{\cancel{2}f(a)}{\cancel{2}}$; CORRECTO

$\Rightarrow f(a)$.

d) Si $g(x)$ es par, entonces $g(-x)=g(x)$

$$g(-x)=\frac{f(-x)+f(x)}{2}$$

$$\Rightarrow \frac{f(x)+f(-x)}{2}$$; CORRECTO

e) $-g$ es par $\Rightarrow -g(x)=-g(-x)$

$$g(x)=g(-x)$$; CORRECTO

15.— a) $f(x)=\operatorname{sen} x+x^3$; $\operatorname{sen}(x)$ no es par.

¿impar?

$$f(x)=-f(-x)$$

$$\Rightarrow -\left(\operatorname{sen}(-x)+(-x)^3\right)\Rightarrow -(-\operatorname{sen} x-x^3)$$

$$\Rightarrow \operatorname{sen} x+x^3$$; $f(x)$ es impar.

b) $g(x)=|x|+1$

¿par?

$$|x|+1=|-x|+1$$

$$\Rightarrow |x|+1$$; $g(x)$ es par

c) $h(x)=|-x|-x^2$

$$\Rightarrow |x|-x^2$$

¿par?

$$|x|-x^2=|-x|-(-x)^2$$

$$\Rightarrow |x|-x^2$$; $h(x)$ es par.

d) $f(x) = |2-x| - |x+2|$

¿par?

$|+2-x| - |x+2| = |-2+x| - |-x-2|$

$= |x-2| - |x+2|$; $f(x)$ es par

e) $f(1-x) = x+2$

NO ES PAR; porque es lineal

¿impar?

$f(1-x) = -f(x-1)$

$= -(x+2)$

$\Rightarrow x-2$; NO ES IMPAR

f) $h(x) = x^2 - |x|$

¿par?

$x^2 - |x| = (-x)^2 - |-x|$

$\Rightarrow x^2 - |x|$; $h(x)$ es par

16.— $g(x) = x^2 - x + 1$

Vértice

$x = \frac{+1}{2(1)} = \boxed{1/2}$

$y = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) + 1$

$y = \frac{1}{4} - \frac{1}{2} + 1$

$y = 3/4$

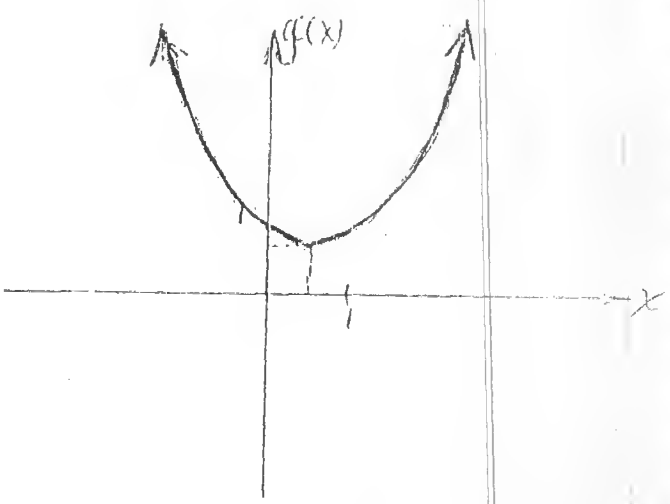
$V\left(\frac{1}{2}, 3/4\right)$

Raíces

$x^2 - x + 1 = 0$

$x = \frac{1 \pm \sqrt{1-4(1)(1)}}{2(1)}$

NO HAY raíces



17.- $K \to$ pendiente o tangente del ángulo

$K>0$; $\tan\theta$ (entre 0 y 90°) ; debe ser creciente

$K<0$; $\tan\theta$ (entre 90 y 180) ; debe ser decreciente

18.- f es función impar y estrictamente decreciente $f(x) = g(x)$.

$$\frac{2g(4) + 3f(4)}{-f(-4) + 4g(-4)} = \frac{5f(4)}{3f(-4)} = -\frac{5}{3}$$

$f(x) = -f(-x)$ $\qquad f(4)/f(-4) = -1$

a) 5/3 $\qquad$ b) -5/2 $\qquad$ ~~c)~~ -5/3 $\qquad$ d) 1 $\qquad$ e) -1

19.- $f(x) = \dfrac{3x^3+2x^2+5}{x^2-4}$

Asíntotas Verticales

$x^2-4=0$

$(x+2)(x-2)=0$

$x=-2$ / $x=2$

a) Correcto

20.-

$g(x) = \dfrac{12x-3}{9x^2-4}$

Asíntotas Verticales

$9x^2-4=0$

$(3x-2)(3x+2)=0$

$3x-2=0 \qquad 3x+2=0$

$3x=2 \qquad 3x=-2$

$x=2/3 \qquad x=-2/3$

Asíntotas Horizontales

$\dfrac{12x}{9x^2} \Rightarrow \dfrac{4}{3x}$

$\Rightarrow y=0$

b) FALSO

21.- $f(x)=\dfrac{4x^2-x}{x^2-1}$

Asíntotas V.

$x^2-1=0$

$(x+1)(x-1)=0$

$x=-1$ $x=1$

Asíntota H.

$\dfrac{4x^2}{x^2} \Rightarrow y=4$

raices

$4x^2-x=0$

$x(4x-1)=0$

$x=0$ $4x-1=0$

$4x=1$

$x=1/4$

a) correcto

b) Falso

c) correcto

d) correcto

e) correcto

22.- $h(x)=\dfrac{2x}{x^2+x-2}$

Asíntotas V.

$x^2+x-2=0$

$(x+2)(x-1)=0$

$x=-2$ $x=1$

Asíntotas H.

$\dfrac{2\cancel{x}}{x^{\cancel{2}}} \Rightarrow \dfrac{2}{x}$

$y=0$

raices

$2x=0$

$x=1$

a) correcto

23.- $g(x)=\dfrac{1}{x^2+1}$

Asíntotas V.

$x^2+1=0$

$x^2=-1$

NO TIENE ASINTOTAS

Asíntotas H.

$\dfrac{1}{x^2}$

$\Rightarrow \boxed{y=0}$

raices

NO HAY

C) FALSO

24.- a) $f(x) = \dfrac{x^2-1}{x^2+7x-8}$

Asíntotas V

$x^2+7x-8=0$

$(x+8)(x-1)=0$

$\boxed{x=-8}$ $\boxed{x=1}$

Asíntotas H

$\dfrac{x^2}{x^2} \Rightarrow y=1$

raíces

$x^2-1=0$

$x^2=1$

$\boxed{x=\pm 1}$

Intersección con y

$x=0$

$\boxed{f(x)=\frac{1}{8}}$

b) $g(x) = \dfrac{2}{x^2+1}$

Asíntotas V

$x^2+1=0$

$\boxed{x^2=-1}$

NO EXISTEN ASÍNTOTAS

Asíntota H

$\boxed{y=0}$

raíces

NO EXISTEN

Intersección con y

$g(x)=2$

c) $h(x) = \dfrac{x^2-3x+2}{x^2+1}$

Asíntotas V

$x^2+1=0$

$\boxed{x^2=-1}$

NO EXISTEN

Asíntota H

$\dfrac{x^2}{x^2} \Rightarrow \boxed{y=1}$

raíces

$x^2-3x+2=0$

$(x-2)(x-1)=0$

$\boxed{x=2}$ $\boxed{x=1}$

Intersección con y

$\boxed{h(x)=2}$

d) $r(x) = \dfrac{2x^2}{-x^2+9}$

Asíntotas V

$-x^2+9=0$

$x^2-9=0$

$(x-3)(x+3)=0$

$x=3$ $x=-3$

Asíntotas H

$\dfrac{2x^2}{-x^2}$

$y=-2$

raíces

$2x^2=0$

$x=0$

Intersección con y

$r(x)=0$

e) $f(x) = \dfrac{x}{\sqrt{1-x^2}}$

Asíntotas V

$\sqrt{1-x^2}=0$

$1-x^2=0$

$-x^2=-1$

$x^2=1$

$x=\pm 1$

Asíntotas H

No Tiene

raíces

$x=0$

Intersección con y

$r(x)=0$

f) $g(x) = \dfrac{x^3}{x^2-4}$

Asíntotas V

$x^2-4=0$

$(x-2)(x+2)=0$

$x=2$ $x=-2$

Asíntotas Oblicua

$y=x$

raíces

$x=0$

25.- $\dfrac{a}{x^2+bx+c}$

asíntotas verticales

$x=-3 \wedge x=1$

$x+3=0 \quad x-1=0$

$(x+3)(x-1)$

$\Rightarrow x^2-x+3x-3$

$\Rightarrow x^2+2x-3$

$y(2)$

$\Rightarrow \dfrac{2}{x^2+2x-3}$

26.- I) $f(x)=\dfrac{1}{2(2x-1)(2x+1)}$

Asíntotas V

$2x-1=0 \quad 2x+1=0$

$2x=1 \quad 2x=-1$

$x=1/2 \quad x=-1/2$

Asíntotas H

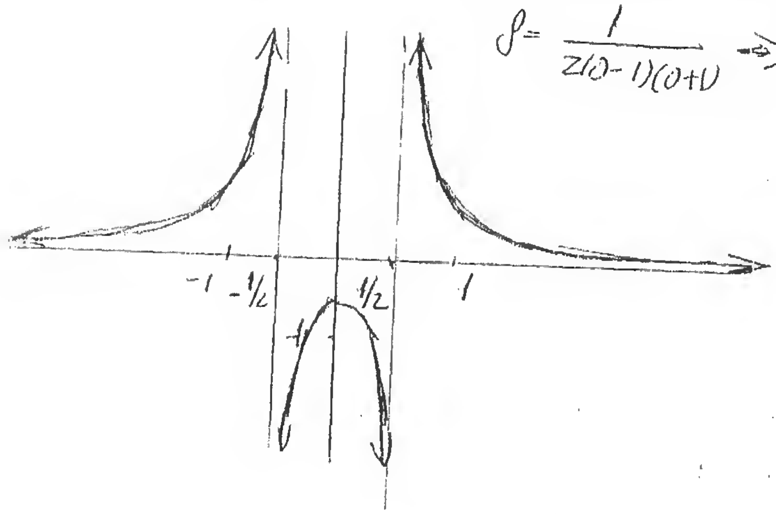
$y=0$

raíces

NO EXISTE

Intersecciones con y

$f=\dfrac{1}{2(0-1)(0+1)} \Rightarrow -\dfrac{1}{2}$



x	$f(x)$
-0,4	-
-0,3	-
-0,2	-
-0,1	-
0	-
0,1	-
0,2	-
0,3	-

II) $g(x) = \dfrac{7x^2}{4(2x-1)(2x+1)}$

Asintotas V.

$2x-1=0 \quad 2x+1=0$

$2x=1 \quad 2x=-1$

$\boxed{x=1/2} \quad \boxed{x=-1/2}$

Asintotas H

$y = \dfrac{7x^2}{16x^2} \approx 0{,}5$

$\boxed{y \approx 0{,}5}$

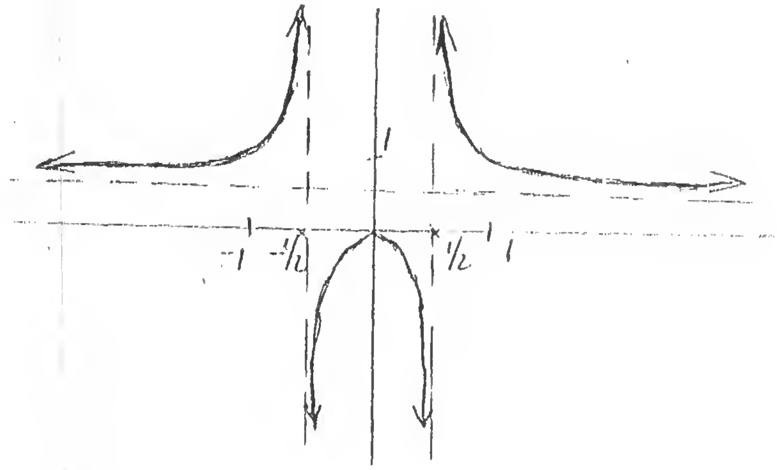
raices

$7x^2=0$

$\boxed{x=0}$

Intersecciones

$\boxed{y=0}$



x	f(x)
-0,4	-
-0,2	-
-0,1	-
0	+
0,1	-
0,2	-

III) $h(x) = \dfrac{7x^3}{4(2x+1)(2x-1)}$

Asintotas V

$2x+1=0 \quad 2x-1=0$

$2x=-1 \quad 2x=1$

$x=-1/2 \quad x=1/2$

Asintota oblicua

$\dfrac{7x^3}{16x^2-4} = \dfrac{7}{16}x + \dfrac{7x}{4(16x^2-4)}$

$\boxed{y = \frac{7}{16}x}$

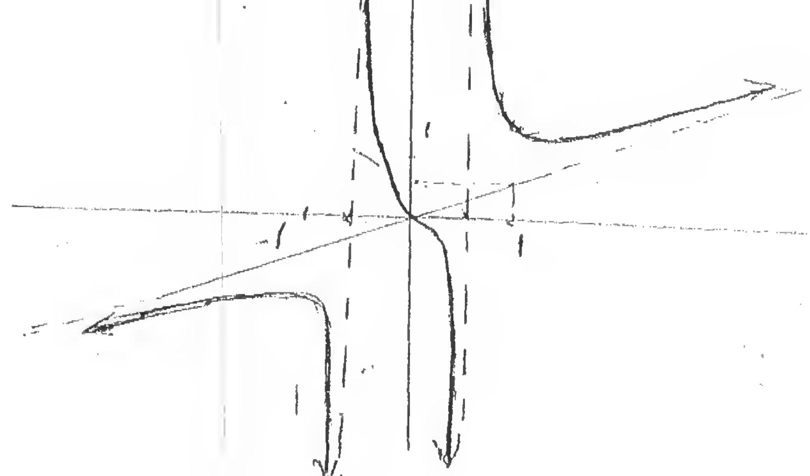
raices

$7x^3=0$

$\boxed{x=0}$

Intercecciones con eje y

$y=0$



x	f(x)
-0,4	+
-0,2	+
-0,1	+
0	+
0,1	-
0,2	-
0,4	-

IV) $r(x) = \dfrac{7x^4}{4(2x+1)(2x-1)}$

Asíntota V.

$2x+1=0$ $\quad$ $2x-1=0$

$2x=-1$ $\quad$ $2x=1$

$x=-1/2$ $\quad$ $x=1/2$

Asíntota Oblícua

$y = \frac{7}{16}x^2$

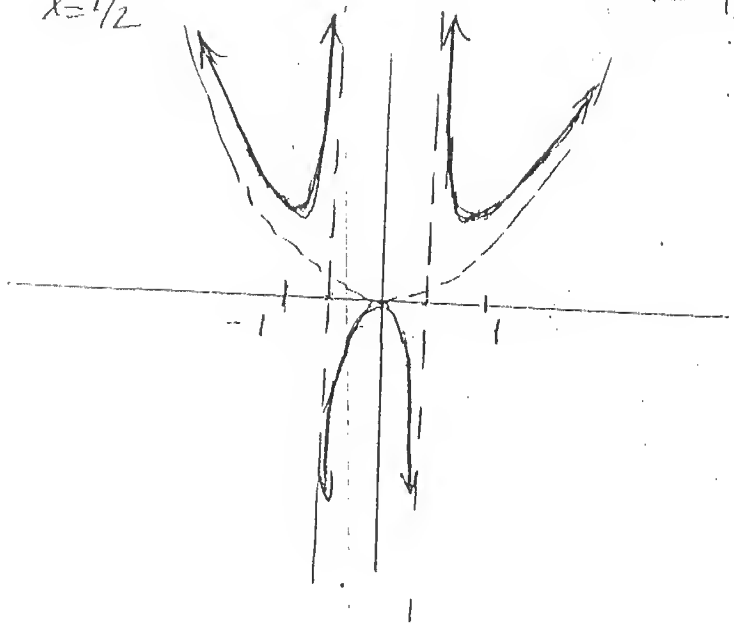
raíces

$7x^4=0$

$(x=0)$

Interacciones con Y

$\boxed{y=0}$



V) $m(x) = \dfrac{7x}{(2x-1)(2x+1)}$

Asíntotas V.

$2x-1=0$ $\quad$ $2x+1=0$

$2x=1$ $\quad$ $2x=-1$

$(x=1/2)$ $\quad$ $(x=-1/2)$

Asíntota H

$y=0$

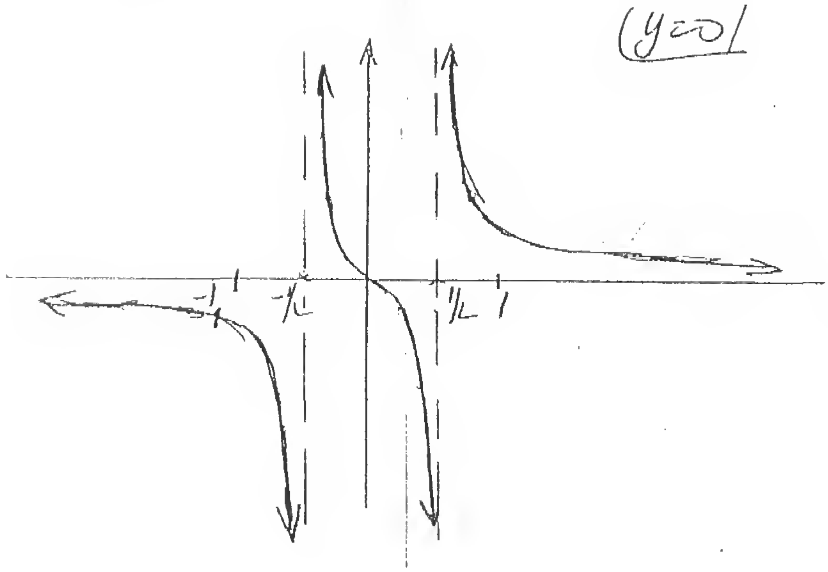
raíces

$7x=0$

$[x=0]$

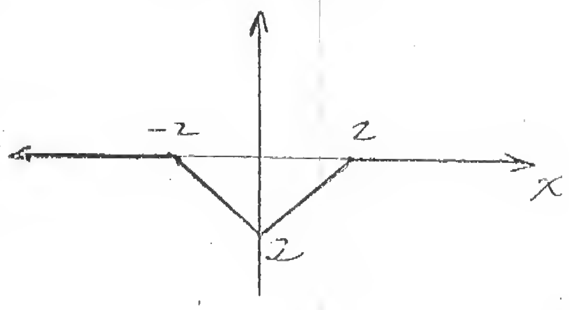
Intersección con eje Y

$(y=0)$

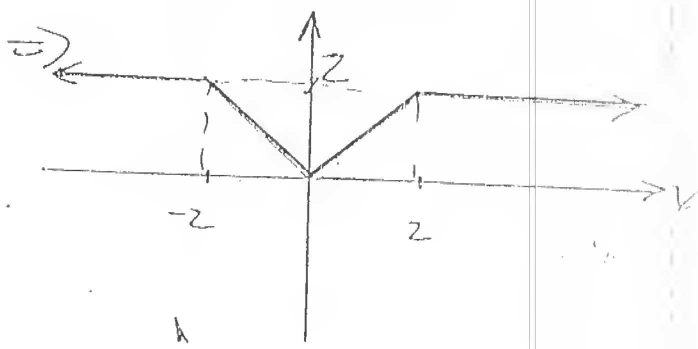


x	$m(x)$
-0,4	+
-0,2	+
-0,1	+
0	•
0,1	−
0,2	−
0,4	−

27.- $y = -f(x)$



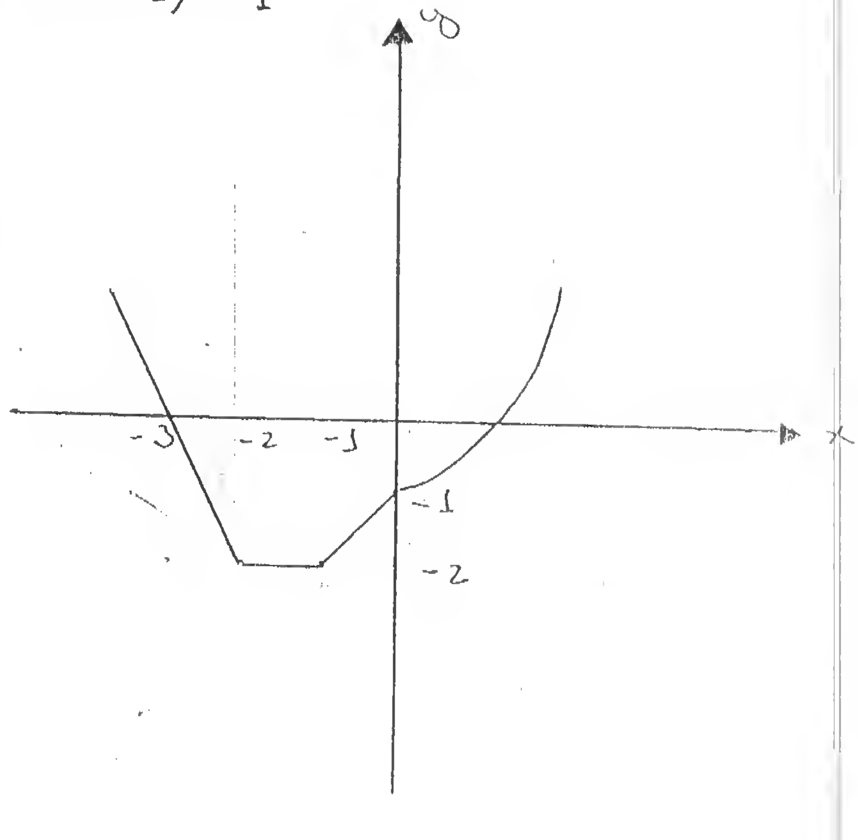
$y = -f(x)+2$



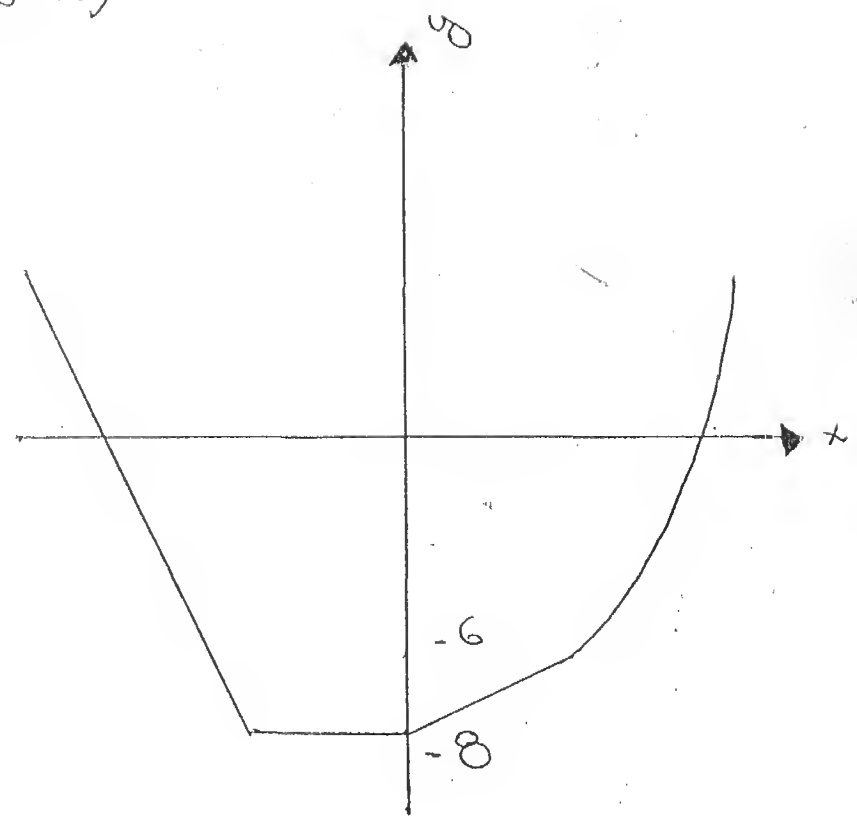
28.- $g(x) = f(|x|)$

a) correcto

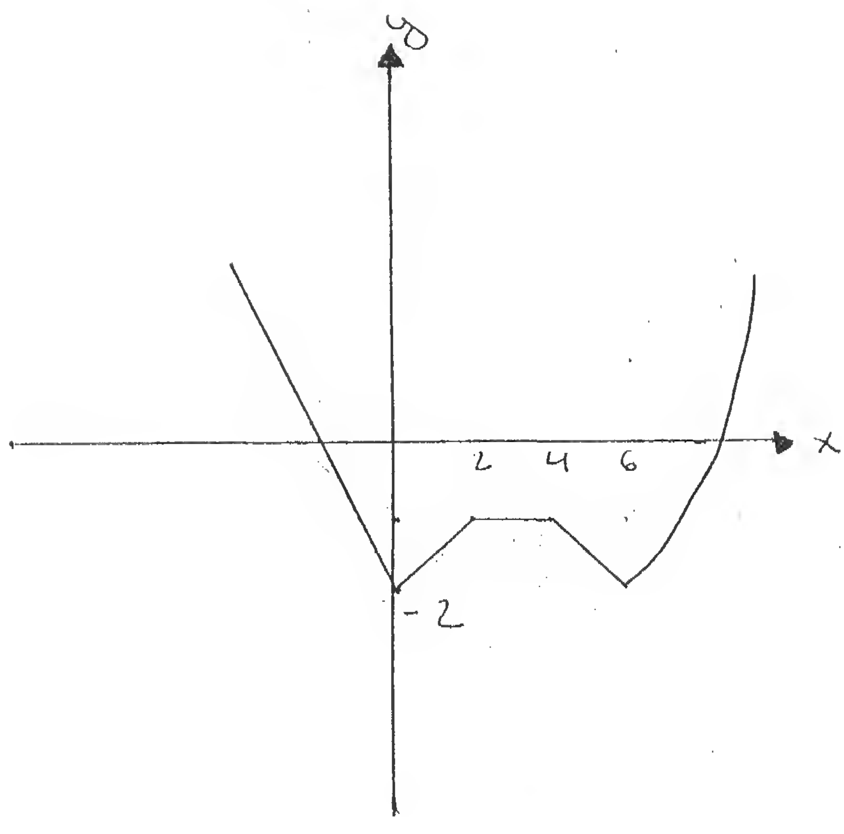
29.- a) $y = f(x+1) - 1$



) $y = -2f(3-x)$



) $y = |f(2x-4)| - 2$



30.-

$$h(x)=\begin{cases} 4+2x & ; -2\leq x\leq 0 \\ 4-2x & ; 0<x\leq 2 \\ 0 & ; |x|>2 \end{cases}$$

$h(-3)=0$

$h(1)=2$

$h(0)=4$

$h(-1)=2$

$h(5)=0$

$h(\pi)=0$

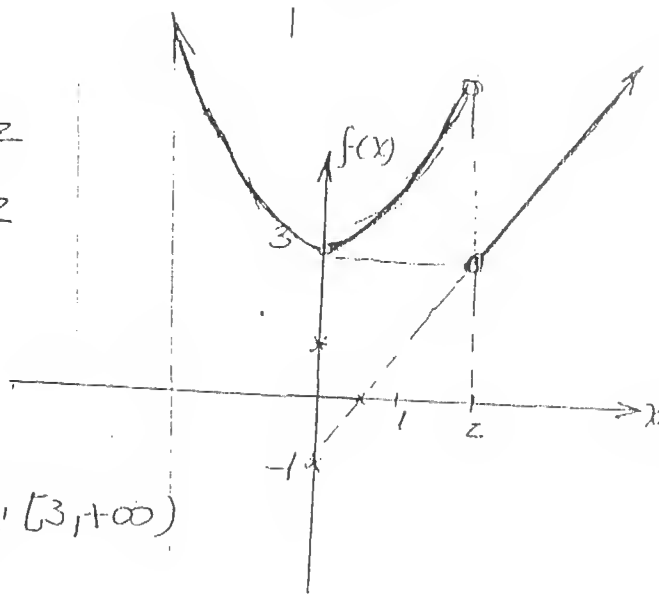
$h\left(-\frac{5\pi}{2}\right)=0$

$h(-e)=0$

$$\Rightarrow \frac{0+4-0+0}{2+2-0+0} \Rightarrow \frac{4}{4} = \underline{\underline{1}}$$ correcto

31.-

$$f(x)=\begin{cases} 2x-1 & ; x\geq 2 \\ x^2+3 & ; x<2 \end{cases}$$



$y=2x-1$

$x=0$
$y=-1$

$y=0$
$x=1/2$

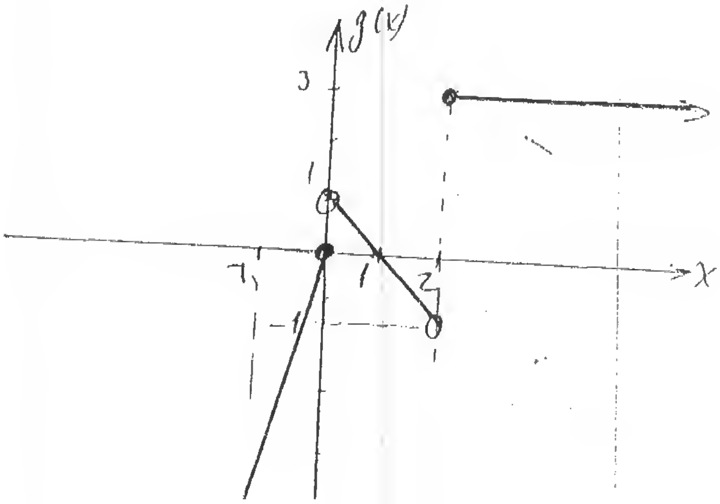
rango: $[3,+\infty)$

$$g(x)=\begin{cases} 3 & ; x\geq 2 \\ 1-x & ; 0<x<2 \\ 4x & ; x\leq 0 \end{cases}$$

$y=1-x$

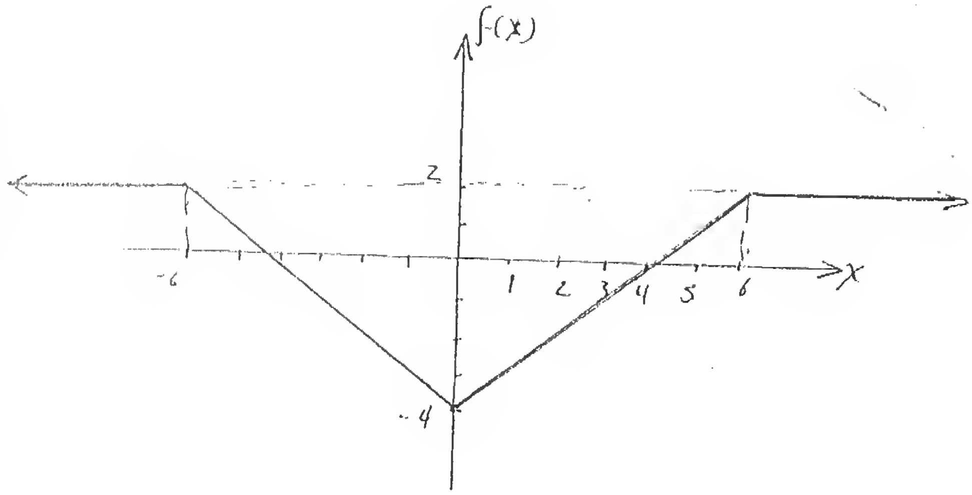
$x=0$
$y=1$

$y=0$
$x=1$



rango: $(-\infty,1)\cup\{3\}$

32.- $f(x) = \begin{cases} |x| - 4 & ; |x| \le 6 \\ 2 & ; |x| > 6 \end{cases}$



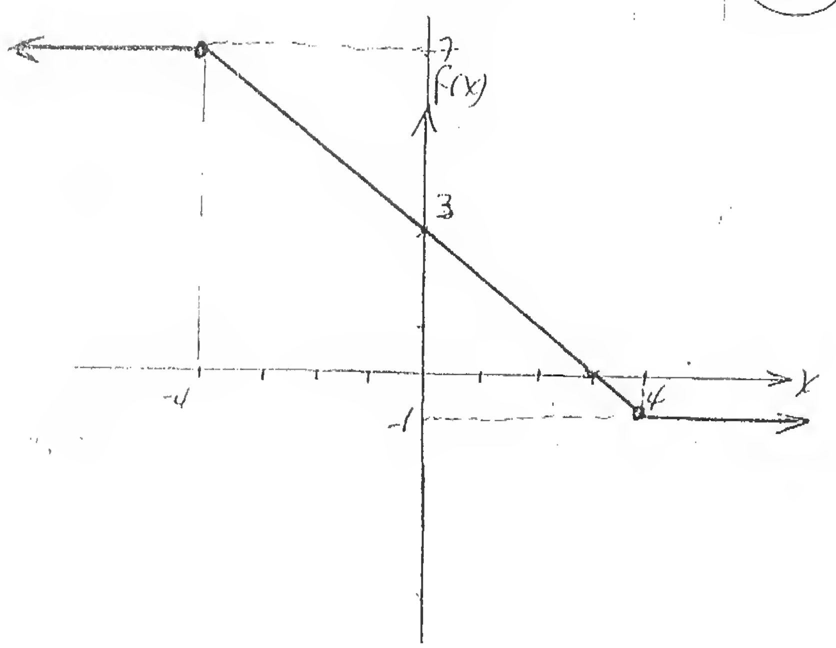
a) correcto
b) correcto
c) Falso
d) correcto
e) correcto

33.- $f(x) = \begin{cases} 7 & ; x < -4 \\ 3 - x & ; -4 \le x \le 4 \\ -1 & ; x > 4 \end{cases}$

$y = 3 - x$

$x = 0$
$y = 3$

$y = 0$
$x = 3$



a) FALSO
b) FALSO
c) FALSO
d) FALSO
e) CORRECTO

34.-

1° $f(x) = -x$

2° $f(x) = 2 - x$ (sube 2 unidades)

3° $f(x) = 2 - (x+2)$ (se mueve a la izq.)

4° $f(x) = -2 + (x+2)$ (se voltea)

$\Rightarrow f(x) = x$

$f(0) = 0$ 6) FALSO

35.-

$C(r) = nr + s$

① $\boxed{12r + s = 2925}$ ② $\boxed{20r + s = 4525}$

① (-1) + ②

$$
\begin{array}{r}
-12r - s = -2925 \\
20r + s = 4525 \\
\hline
8r = 1600
\end{array}
$$

$\boxed{r = \$200}$

$12r + s = 2925$

$s = 2925 - 12r$

$\boxed{s = \$525}$

36.-

$D(n) = 1420 + 100n$

$n = 10$

$D(10) = 1420 + 100(10)$

$\boxed{D_{10} = 2420 \text{ médicos}}$

$1420 + 100n \geq 2000$

$100n \geq 2000 - 1420$

$n \geq \frac{580}{100}$

$n \geq 5{,}8$ años $\rightarrow$ 5 años y 9 meses

$1994 + 5 + 9$ mes. $\Rightarrow$ $\boxed{1999 \text{ y } 9 \text{ meses}}$

37.- $C.F = \$\,2500$

$CT = \$3300$

$x = 200$

$P.V.P = \$5{,}25$

$C(x) = CF + CV \cdot x$

$33\cancel{00} = 25\cancel{00} + CV \cdot 2\cancel{00}$

$33 - 25 = 2CV$

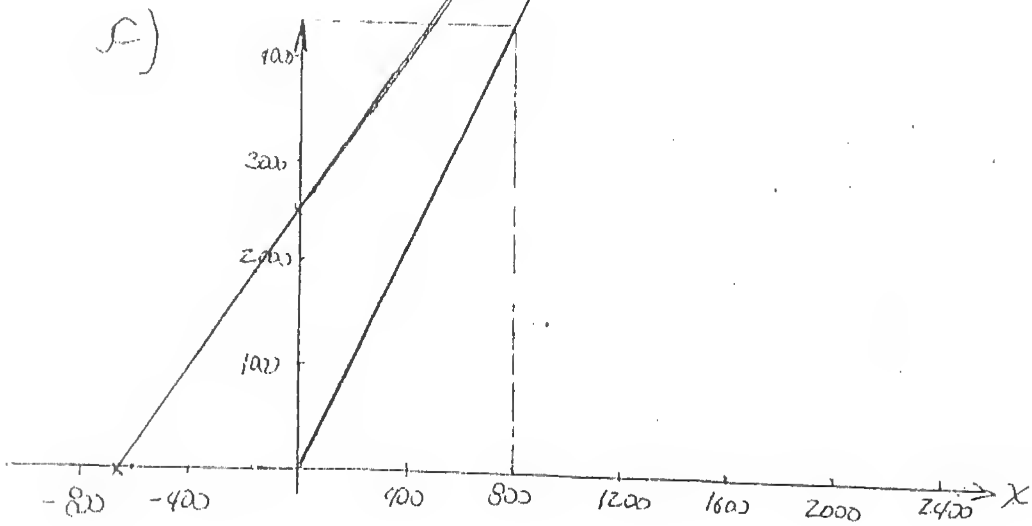
$8 = 2CV$

$CV = 4$

a) $C(x) = 2500 + 4x$

b) $I(x) = 5{,}25x$

c)



$y = 2500 + 4x$

$x = 0$
$y = 2500$

$y = 0$
$x = -625$

$y = 5{,}25x$

$x = 800$
$y = 4200$

d) $200 = 5{,}25x - (2500 + 4x)$

$200 = 5{,}25x - 2500 - 4x$

$2700 = 1{,}25x$

$x = 2160$

38.-

$y_c = 2{,}8x + 600$

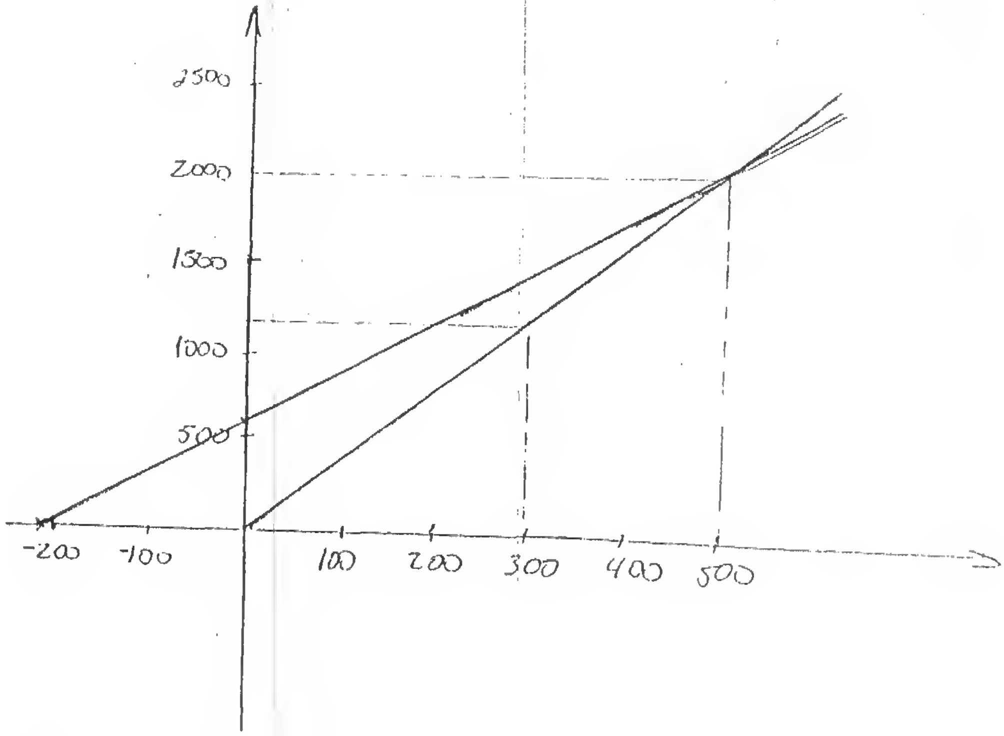
$I(x) = 4x$

a) $4x = 2{,}8x + 600$

$1{,}2x = 600$

$x = 500$ unidades

b)



$y = 2{,}8x + 600$

$x = 0$, $y = 600$

$y = 0$, $x = -214{,}2$

$y = 4x$

$x = 300$, $y = 1200$

c) Sin pérdida

$$I(x) = y_c$$

$$P \cdot x = 2{,}8x + 600$$

$$45\cancel{0}P = 2{,}8(45\cancel{0}) + 60\cancel{0}$$

$$45P = 126 + 60$$

$$P = \frac{186}{45}$$

$$P = \$4{,}13$$

39.- $\rho = \frac{m}{V} \Rightarrow V = \frac{m}{\rho}$

$$V = \frac{5000}{0{,}859}$$

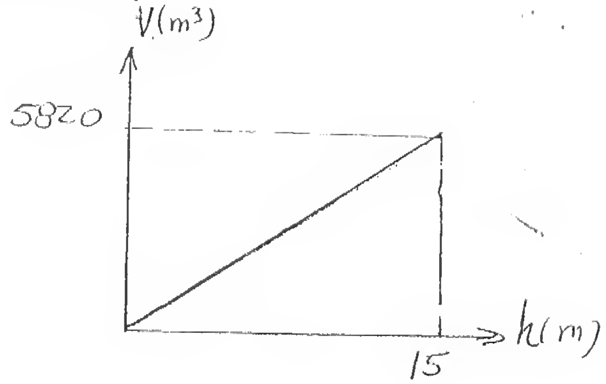
$$V = 5820 \, m^3$$

$$V = A\,h$$

$$h = \frac{V}{A} \Rightarrow \frac{5820{,}72}{388}$$

$$h = 15 \, m$$

b) $V = 388h$

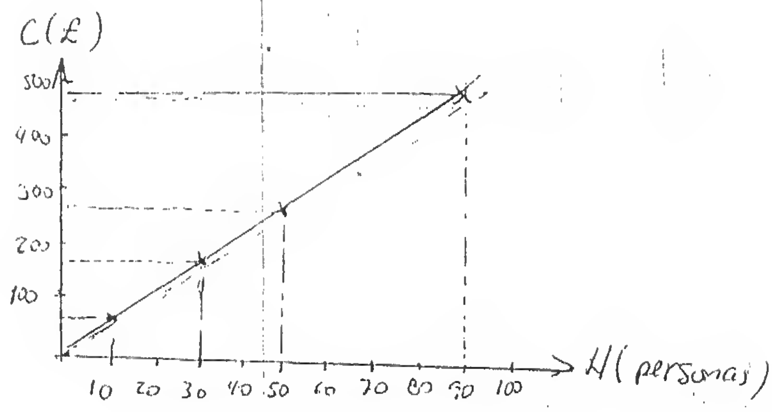


dom f : (0,15) m.

40.- a.1)

N	10	30	50	90
C	70	170	270	470

a.2)

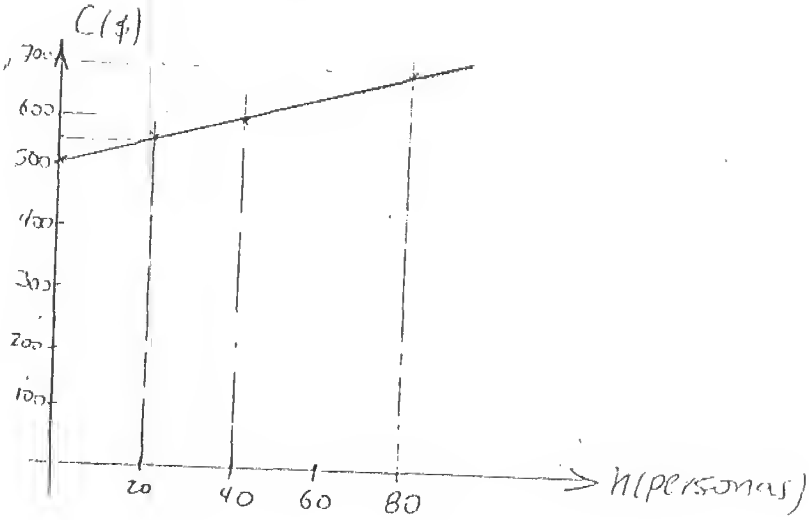


a.3) $C(n) = 5n + 20$

b.1)

N	0	20	40	80
C	500	550	600	700

b.2)



c.1) es una extrapolación; se usará ecuación:

$$C = \frac{5N}{2} + 500$$

$$C = \frac{5}{2}(208) + 500$$

$$C = \$\ 1020$$

c.2)

$$C = 2{,}5n + 500$$

$$1060 = 2{,}5n + 500$$

$$2.5n = 1060 - 500$$

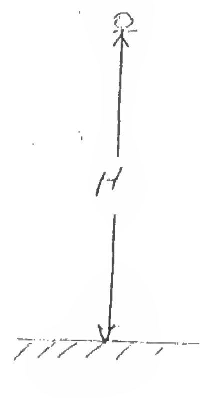
$$n = \frac{560}{2{,}5} \Rightarrow n = 224$$

41.-

$$H = 25 - 16t^2$$

$$H = 25 - 16(1{,}25)^2$$

$$H = 0$$



a) correcto

12.-

Producto de raíces: $\frac{c}{a}$

Suma de raíces: $-\frac{b}{a}$

Producto r. = Suma r.

$$\frac{c}{\cancel{a}} = -\frac{b}{\cancel{a}}$$

$b = -c$ b) correcto

13.- $g(x) = x^2 - bx + 1$

a) Dom g: $x \in \mathbb{R}$; verdadero

b) $\Delta = b^2 - 4(1)(1)$

$\Delta = b^2 - 4 \geq 0$

$b^2 \geq 4$; para que haya solución real.

$b^2 < 4$; no hay soluciones reales (no hay raíces) $g(x) \neq 0$

Verdadero

c) rango g: $\left[1 - \frac{b^2}{4}, +\infty\right)$

$$y = \frac{-\Delta}{4a} \Rightarrow \frac{-b^2 + 4ac}{4a} \Rightarrow \frac{-b^2 + 4}{4} \Rightarrow -\frac{b^2}{4} + 1 = \boxed{1 - \frac{b^2}{4}}$$

FALSO

d) Verdadero

e) Verdadero; $b \geq 2 \vee b \leq -2$

44.- $f(x) = |2x^2 - 3x + 1| - 2$

$f_1(x) = 2x^2 - 3x + 1$

Vértice

$x = -\frac{b}{2a} \Rightarrow -\frac{-3}{2(2)}$

$x = \frac{3}{4}$

$y = 2\left(\frac{3}{4}\right)^2 - 3\left(\frac{3}{4}\right) + 1$

$\Rightarrow \frac{\cancel{18}\,9}{\cancel{16}\,8} - \frac{9}{4} + 1$

$\Rightarrow \frac{9}{8} - \frac{9}{4} + 1 \Rightarrow \frac{9 - 18 + 8}{8}$

$\Rightarrow -\frac{1}{8}$

$V\left(\frac{3}{4}, -\frac{1}{8}\right)$

raices

$2x^2 - 3x + 1 = 0$

$(2x - 2)(2x - 1) = 0$

$2x - 2 = 0 \qquad 2x - 1 = 0$

$x = 1 \qquad x = 1/2$

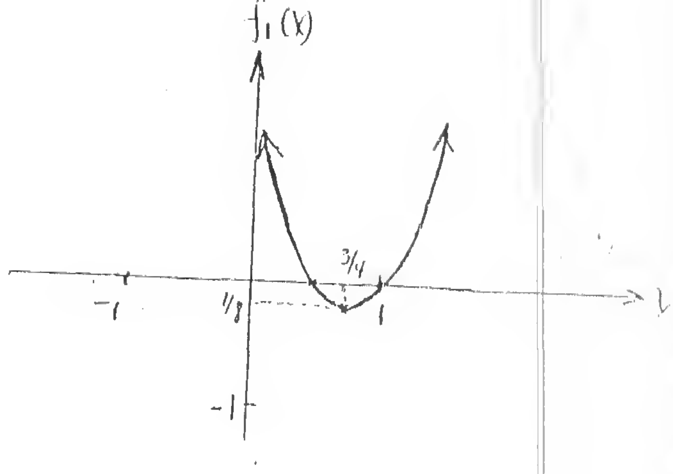
a) FALSO

b) VERDADERO

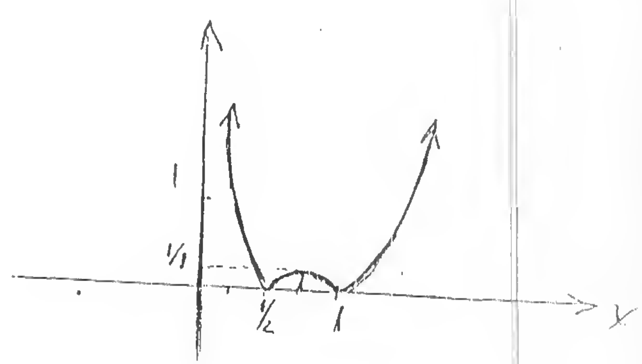
c) FALSO

d) FALSO

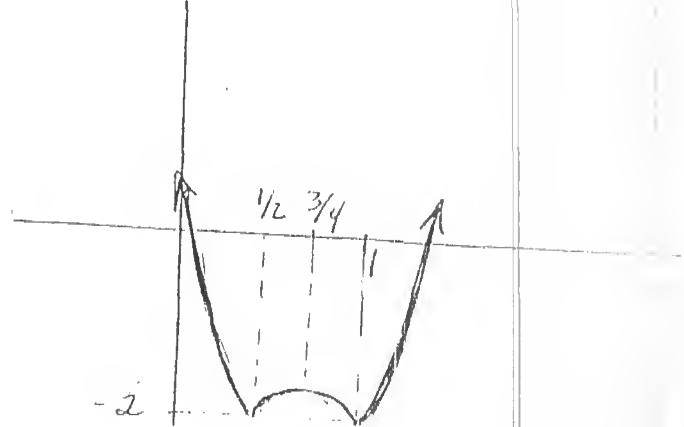
e) FALSO



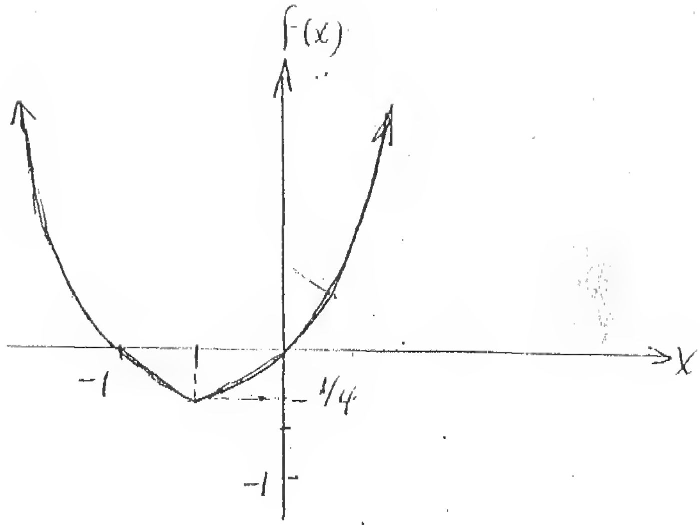
$f_2(x) = |2x^2 - 3x + 1|$



$f_3(x) = |2x^2 - 3x + 1| - 2$



45.- $f(x) = x^2 + x$



Vértice

$x = -\frac{1}{2(1)} \Rightarrow -\frac{1}{2}$

$y = \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)$

$y = \frac{1}{4} - \frac{1}{2} \Rightarrow -\frac{1}{4}$

$V\left(-\frac{1}{2}, -\frac{1}{4}\right)$

raíces

$x^2 + x = 0$

$x(x+1) = 0$

$x = 0 \; ; x = -1$

a) FALSO

b) FALSO

c) FALSO; $[-1/4, +\infty)$

d) CORRECTO

e) FALSO

46.- $V(-1, 2)$; $A(1, 0)$

$x = -\frac{b}{2a} \Rightarrow -1 = -\frac{b}{2a}$

$\Rightarrow \boxed{b = +2a}$

$y = -\frac{b^2 - 4ac}{4a} \Rightarrow 2 = -\frac{(+2a)^2 - 4ac}{4a}$

$\Rightarrow -2 = \frac{\cancel{4}a^{\cancel{2}} - \cancel{4a}c}{\cancel{4a}} \Rightarrow -2 = a - c$

$\Rightarrow \boxed{c = a + 2}$

$y = ax^2 + bx + c$

$(1, 0) \quad ax^2 + (2a)x + a + 2 = 0$

$a + 2a + a + 2 = 0$

$4a = -2 \Rightarrow \boxed{a = -1/2}$

$a = -1/2$

$b = -1$

$c = 3/2$

$y = -\frac{1}{2}x^2 - x + \frac{3}{2}$

$y = -\frac{1}{2}(x^2 + 2x - 3)$

$y = -\frac{1}{2}(x^2 + 2x + 4 - 4 - 3)$

$y = -\frac{1}{2}(x + 2)^2 + 7/2$

$y = \frac{-1}{2}(x + 2)^2 + 7/2$

$h = -2$

$k = +7/2$

$a = -\frac{1}{2}$

47.- $V(1,8)$ $A(3,0)$

$y=ax^2+bx+c$

$x=-\frac{b}{2a} \Rightarrow 1=-\frac{b}{2a}$

$\boxed{b=-2a}$

$y=-\frac{b^2-4ac}{4a} \Rightarrow 8=-\frac{(-2a)^2-4ac}{4a}$

$-8=\frac{4a^2-4ac}{4a} \Rightarrow -8=a-c$

$\boxed{c=a+8}$

$y=ax^2-2ax+a+8$

$0=a(3)^2-2a(3)+a+8$

$0=9a-6a+a+8$

$-4a=8$

$\boxed{a=-2}$ $\boxed{b=4}$ $\boxed{c=6}$

$\boxed{y=-2x^2+4x+6}$

$y=-2(x^2-2x+3)$

$y=-2(x-2)(x-1)$

48.- $V(3,1)$ $P(5,9)$

$y=ax^2+bx+c$

$x=\frac{-b}{2a} \Rightarrow 3=\frac{-b}{2a}$

$b=-6a$

$y=-\frac{b^2-4ac}{4a}$

$-1=\frac{(-6a)^2-4ac}{4a}$

$-1=\frac{36a^2-4ac}{4a}$

$-1=9a-c \Rightarrow \boxed{c=9a+1}$

$\Longrightarrow y=ax^2-6ax+9a+1$

$P(5,9)$

$9=a(5)^2-6a(5)+9a+1$

$25a-30a+9a=8$

$4a=8$

$\boxed{a=2}$ $\boxed{b=-12}$ $\boxed{c=19}$

$y=2x^2-12x+19$

$y=(2x^2-12x+18)+1$

$y=2(x^2-6x+9)+1$

$$y = 2(x-3)^2+1$$

$a = 2$

$h = 3$

$k = 1$

49.- $f(x) = 30x - 5x^2$

a) raices: $30x - 5x^2 = 0$

$5x(6 - x) = 0$

$\boxed{x = 0}$ $\quad x - 6 = 0$

$\boxed{x = 6}$

b) c = vertice

$$x = \frac{-30}{2(-5)} = 3$$

$y = 30(3) - 5(3)^2$ $\quad C(3, 45)$

$y = 90 - 45$

$y = 45$

c) $x = 3 \Rightarrow x - 3 = 0$

50.- $V(2,3)$ $\quad A(0,5)$

$$x = -\frac{b}{2a}$$

$$2 = -\frac{b}{2a} \Rightarrow \boxed{b = -4a}$$

$$y = -\frac{b^2 - 4ac}{4a}$$

$$3 = -\frac{(-4a)^2 - 4ac}{4a}$$

$$-3=\frac{\cancel{16}^{4}a^{\cancel{2}}-\cancel{4a}c}{\cancel{4a}} \Rightarrow -3=4a-c$$

$$\boxed{c=4a+3}$$

$$y=ax^2-4ax+4a+3$$

$A(0,5)$ $5=0-0+4a+3$

$$4a=2$$

$$\boxed{a=1/2} \quad \boxed{b=-2} \quad \boxed{c=5}$$

$$y=\frac{1}{2}x^2-2x+5$$

$$\Rightarrow y=\frac{1}{2}x^2-2x+2+3$$

$$\Rightarrow y=\frac{1}{2}(x^2-4x+4)+3$$

$$\Rightarrow y=\frac{1}{2}(x-2)^2+3$$

$h=2$
$a=1/2$

51.- $f(x)=x^2+bx+c$ $A(2,0)$ $B(3,0)$

$$0=2^2+b(2)+c \qquad 0=3^2+b(3)+c$$

$$(1)\ \boxed{2b+c=-4} \qquad (2)\ \boxed{3b+c=-9}$$

$(1)*(-1)+(2)$

$$-2b-c=4$$
$$3b+c=-9$$
$$\boxed{b=-5}$$

$$c=-9-3b$$
$$c=-9+15$$
$$\boxed{c=6}$$

52.- $f(x) = 2x^2 + x + k$

$f(x) \Rightarrow$ no se intercepta con el eje x; entonces no hay raíces; por lo tanto $b^2 - 4ac < 0$

$1^2 - 4(2)k < 0$

$-8k < -1$

$8k > 1$

$\boxed{k > 1/8}$ c) correcto

53.- Pventa $= \$100$

$y_c = x^2 + 20x + 700$

a) $I(x) = 100x$

b) $U(x) = 100x - x^2 - 20x - 700$

c) $U(x) = -x^2 + 80x - 700$

vértice

$x = \dfrac{-80}{2(-1)} =$ 40 unidades

$U(x) = -(40)^2 + 80(40) - 700$

$U(x) = \$900$

54.- **Demanda**

$P^2+x^2=169$

$P_1=\sqrt{169-x^2}$

Oferta

$P_2=x+7$

$$P_1=P_2$$

$$\left(\sqrt{169-x^2}\right)^2=(x+7)^2$$

$$169-x^2=x^2+14x+49$$

$$x^2+x^2+14x+49-169=0$$

$$2x^2+14x-120=0$$

$$x^2+7x-60=0$$

$$(x+12)(x-7)=0$$

$$\boxed{x=7}$$

$$P=x+7$$

$$P=\$14$$

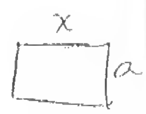
55.- a)

Longitud(m)	Ancho(m)	Área(m²)
1	11	11
2	10	20
3	9	27
4	8	32

b) $x \rightarrow$ largo

$24-x \rightarrow$ ancho

$$A(x)= x(12-x)$$



$2x+2a=24$

$x+a=12$

$a=12-x$

c) $A(x) = x(12 - x)$

$\Rightarrow 12x - x^2$

$x = \frac{-12}{2(-1)} = 6\,m$

56.- a) $h(t) = -5t^2 + 30t$

$t = \frac{-b}{2a} = \frac{-30}{2(-5)}$

$t = 3s$

b) $h = -5(3)^2 + 30(3)$

$h = 45m$

57.- CV = \$15.

número : 100.000 − 4000x

P.V.P = x

a) $U = x \cdot (100.000 - 4000x) - 15(100.000 - 4000x)$

b) $U(x) = 100.000x - 4000x^2 - 1'500.000 + 60.000x$

$U(x) = -4000x^2 + 160.000x - 1'500.000$

$x = \frac{-160.000}{2(-4000)}$

$x = 20$

c) $100.000 - 4000(20) \Rightarrow$ 20.000 libros

58.-

$f(x)$: x^2+3 for $x<2$; $2x-1$ for $x \ge 2$

$g(x)$: $4x$ for $x<0$; $1-x$ for $0 \le x < 2$; 3 for $x \ge 2$

$f-g$: x^2-4x+3 ; x^2+x+2 ; $2x-4$

$\frac{f}{g}$: $\frac{x^2+3}{4x}$; $\frac{x^2+3}{1-x}$; $\frac{2x-1}{3}$

$f+g$: x^2+4x+3 ; x^2-x+4 ; $2x+2$

$f\,g$: $4x(x^2+3)$; $(x^2+3)(1-x)$; $3(2x-1)$

$$\begin{array}{r} x^2+3 \\ 1-x \\ \hline x^2+3-x^3-3x \end{array}$$

$$f \cdot g(x) = \begin{cases} 4x^3+12x & ; x \le 0 \\ -x^3+x^2-3x+3 & ; 0<x<2 \\ 6x-3 & ; x \ge 2 \end{cases}$$

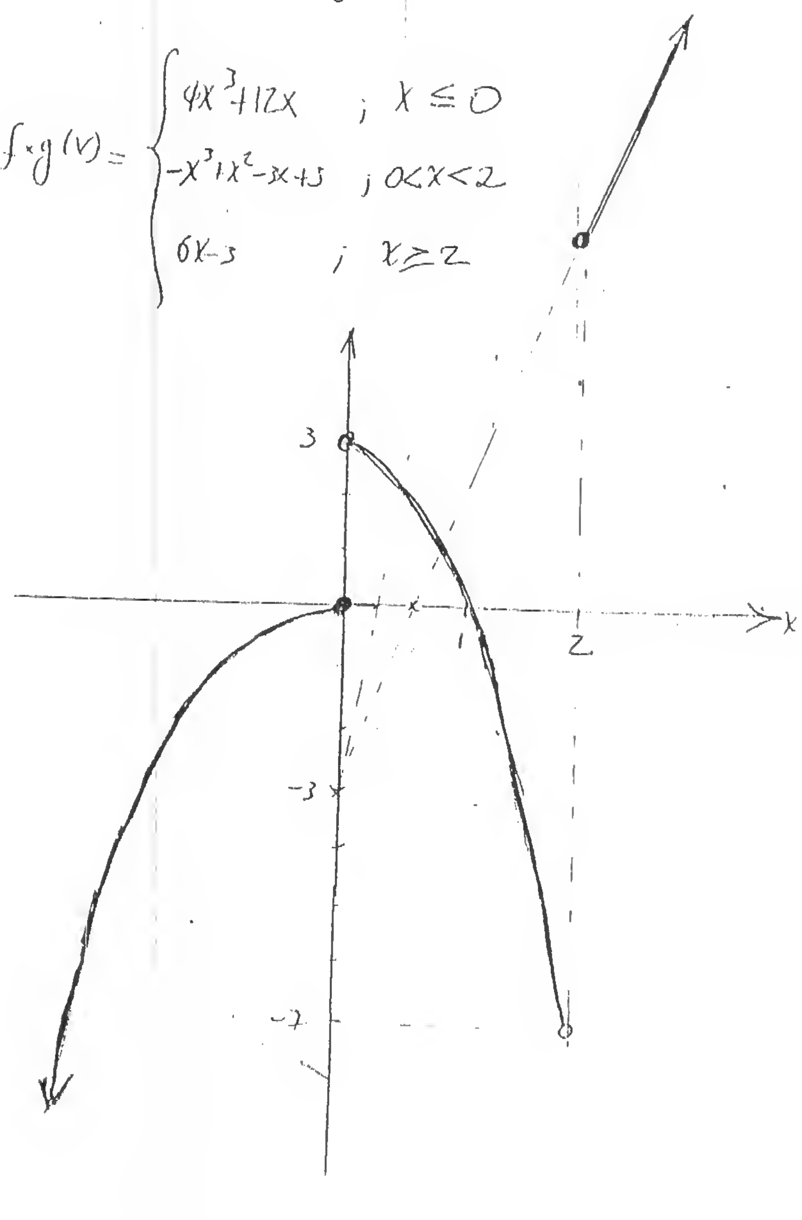
$y = 6x-3$

$x=0$, $y=-3$; $y=0$, $x=\frac{1}{2}$

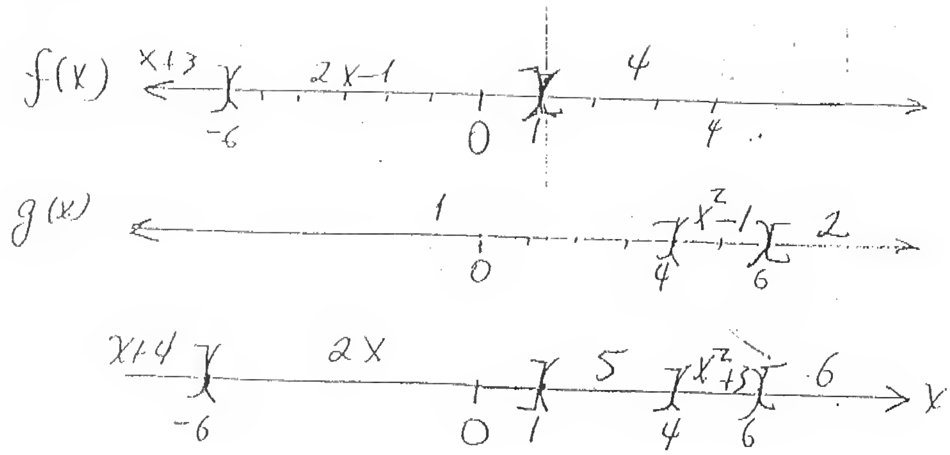
$4x^3+12x=0$

$4x(x^2+3)=0$

$\boxed{x=0}$



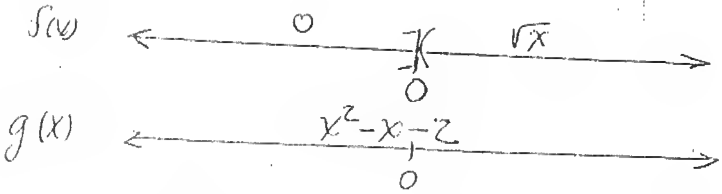
59.-



$$(f+g)(x)\begin{cases} x+4 & ; x\leq -6 \\ 2x & ; -6<x\leq 1 \\ 5 & ; 1<x\leq 4 \\ x^2+3 & ; 4<x<6 \\ 6 & ; x\geq 6. \end{cases}$$

C) correcto

60.-



a) $\begin{cases} x^2-x-2 & ; x\leq 0 \\ x^2-x+\sqrt{x}-2 & ; x>0 \end{cases}$

b) $\begin{cases} x^2-x-2 & ; x\leq 0 \\ x^2-x-\sqrt{x}-2 & ; x>0 \end{cases}$

d) $\begin{cases} 0 & ; x\leq 0 \\ \dfrac{\sqrt{x}}{x^2-x-2} & ; x>0 \end{cases}$

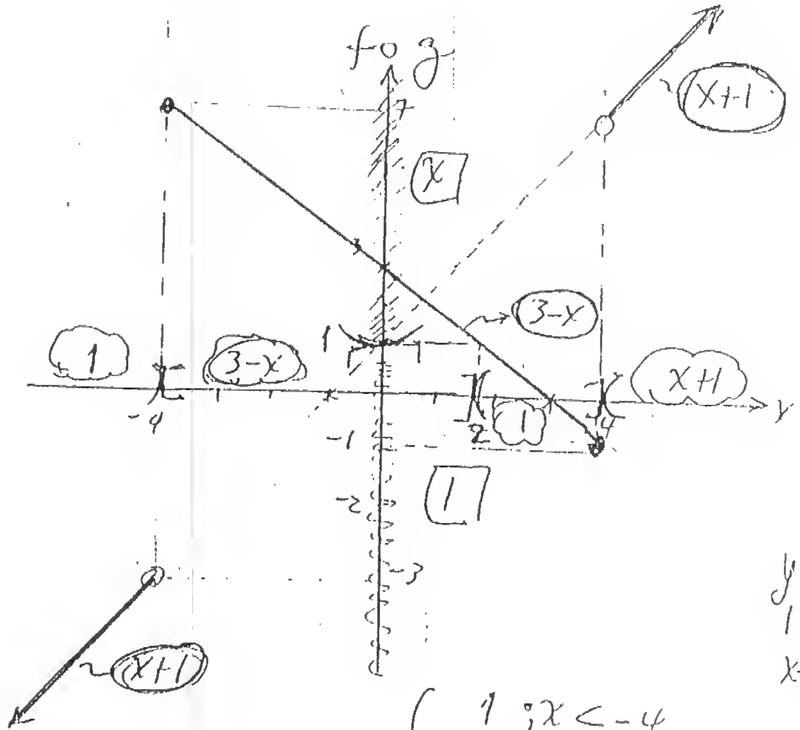
c) $\begin{cases} 0 & ; x\leq 0 \\ \dfrac{\sqrt{x}}{x^2-x-2} & ; x>0 \end{cases}$

e) $\begin{cases} \text{NO DEFINIDO} & ; x\leq 0 \\ \dfrac{x^2-x-2}{\sqrt{x}} & ; x>0 \end{cases}$

f) $\begin{cases} 0 & ; x\leq 0 \\ \sqrt{x}(x^2-x-2) & ; x>0 \end{cases}$

61.- $f(x) = \begin{cases} x & ; x > 1 \\ 1 & ; x \leq 1 \end{cases}$

$g(x) = \begin{cases} 3-x & ; |x| \leq 4 \\ x+1 & ; |x| > 4 \end{cases}$



$y = 3-x$

$x=0$, $y=3$ $\quad$ $y=0$, $x=3$.

$y = x+1$

$x=0$, $y=1$ $\quad$ $y=0$, $x=-1$

$y = 3-x$

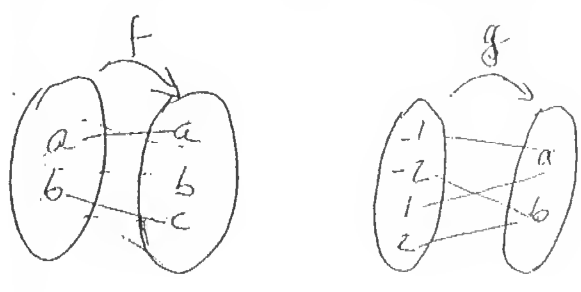
$1 = 3-x$

$x = 2$.

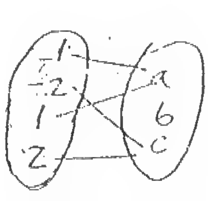
$f \circ g \begin{cases} 1 & ; x < -4 \\ 3-x & ; -4 \leq x \leq 2 \\ 1 & ; 2 \leq x \leq 4 \\ x+1 & ; x > 4 \end{cases}$

e) correcto

62.- FALSO, NO NECESARIAMENTE.

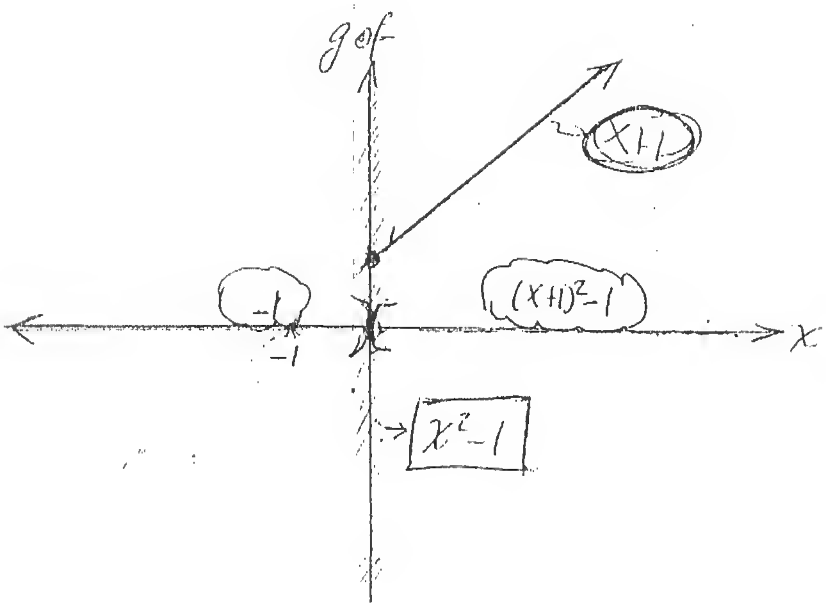


$f \circ g$



63.-

$$f(x) = \begin{cases} 0 & ; x < 0 \\ x+1 & ; x \geq 0 \end{cases} \qquad g(x) = x^2 - 1 \ ; x \in \mathbb{R}$$



$y = x+1$

$x = 0$
$y = 1$

$y = 0$
$x = -1$

$$gof = \begin{cases} -1 & ; x < 0 \\ x^2+2x & ; x \geq 0 \end{cases}$$

64.-

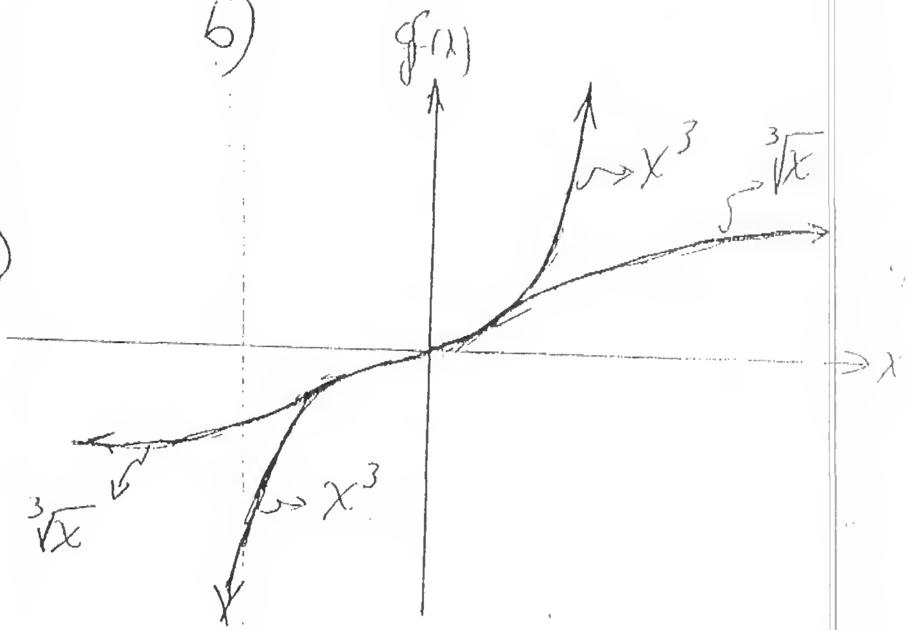
$g(x) = x^3$

a) $\sqrt[3]{g(x)} = \sqrt[3]{x^3}$

$x = \sqrt[3]{g(x)}$

$g(x)^{-1} = \sqrt[3]{x}$

b)



c) $g(x)$: $x^3 = \sqrt[3]{x}$

$(x^3)^3 = (\sqrt[3]{x})^3$

$\Rightarrow x^9 = x$

$\Rightarrow x^9 - x = 0$

$\Rightarrow x(x^8 - 1) = 0 \Rightarrow x(x^4+1)(x^4-1) = 0 \Rightarrow x(x^4+1)(x^2+1)(x^2-1) = 0$

$\Rightarrow x(x^4+1)(x^2+1)(x+1)(x-1) = 0$

$\boxed{x=0}$ $x-1=0$ $x+1=0$

$\boxed{x=1}$ $\boxed{x=-1}$

$\Lambda g(x): \{-1, 0, 1\}$

65.- $f(x) = \sqrt{x}$ $g(x) = x^2$ $h(x) = x-1$

$f \circ g \circ h$

$g \circ h = (x-1)^2 \Rightarrow f \circ (g \circ h) = \sqrt{(x-1)^2}$

$f \circ (g \circ f) = x-1$

56.- $\int og(x) = x^2+2x+6$

$f(0)=9$

a)

$f(0)= ax^2+6x+9$

$f(x)= x^2+6x+9$

$f(x)= x^2+4x+9$

$x-k$

$fog = x^2+4x+9$

$fog = (x-k)^2+4(x-k)+9$

$fog = x^2-2kx+k^2+4x-4k+9$

$fog = x^2+(-2k+4)x+(k^2-4k+9) = x^2+2x+6$

$\cancel{x}(-2k+4) = 2\cancel{x}$

$-2k+4=2$

$2k=-2$

$k=1$ $\quad k\in N$

b) $f(x) = x^2+4x+9$ $\quad k=1 ;\ k\in Z$

57.- $f(x) = \begin{cases} 2(x-3) ; & x\leq 3 \\ (x-2)^2 ; & x>3 \end{cases}$

$g(x) = 1-2x \quad ; x\leq 0$

eterminar la regla de correspondencia de $f\circ g$

$(f\circ g)(x) = f(g(x))$

$f(g(x)) = \begin{cases} 2(g(x)-3) ; & g(x)\leq 3 \\ (g(x)-2)^2 ; & g(x)>3\ * \end{cases}$

No se puede efectuar ya que $g(x) ; x\leq 0$

$f(g(x)) = 2(1-2x-3) \quad ; x\leq 0$

$f(g(x)) = 2(-2-2x) \quad ; x\leq 0$

$f(g(x)) = -4-4x \quad ; x\leq 0$

68.- $f(x) = -[\![x]\!]$

a) verdadero

69.- $f(x) = x^3$

$$y = x^3$$

$$\sqrt[3]{y} = \sqrt[3]{x^3}$$

$$x = \sqrt[3]{y}$$

$$f^{-1}(x) = \sqrt[3]{x}$$

$$f(x) = f^{-1}(x)$$

$$x^3 = \sqrt[3]{x}$$

$$(x^3)^3 = (\sqrt[3]{x})^3$$

$$x^9 = x$$

$$x^9 - x = 0 \Rightarrow x(x^8-1) = 0 \Rightarrow x(x^4+1)(x^4-1) = 0 \Rightarrow x(x^4+1)(x^2+1)(x^2-1)$$

$$\Rightarrow x(x^4+1)(x^2+1)(x+1)(x-1) = 0 \quad (x=0) \; (x=-1) \; (x=1)$$

$$Ap(x): \{-1, 0, 1\}$$

Suma: $-1+0+1 \Rightarrow \underline{\underline{0}}$ b) correcto

70.-

Opción A: $A(x) = 6+3x$

a). Costo fijo: \$6.

b) $A(x) = 6+3(2,4)$

$A(x) = \$/. 13,2$

c)

$$B(x)\begin{cases} \$4 & ; 0{,}5 \le x < 1 \\ \$6 & ; 1 \le x < 1{,}5 \\ \$8 & ; 1{,}5 \le x < 2 \\ \$10 & ; 2 \le x \le 2{,}5 \\ \$12 & ; 2{,}5 \le x < 3 \\ \$14 & ; 3 \le x < 3{,}5 \end{cases}$$

d) $B(x) = 2*([\![2x]\!]+1)$

e) $B(1{,}6) = 2*([\![2(1{,}6)]\!]+1)$

$\Rightarrow 2*([\![3{,}2]\!]+1)$

$\Rightarrow 2*4$

$\Rightarrow \$8$

f) $A(x) = 6+3x$

$22{,}5 = 6+3x$

$\dfrac{22{,}5-6}{3} = x \Rightarrow x = 5{,}5\ kg$

g) $B(5{,}5) = 2*([\![2(5{,}5)]\!]+1)$

$\Rightarrow 2*([\![11]\!]+1)$

$\Rightarrow \$24$

h) $6+3x = 2([\![2x]\!]+1)$

$6+3x = 2[\![2x]\!]+2$

$2[\![2x]\!]-3x = 4$

$x=1$ NO

$x=2$ NO

$x=3$ NO

$x=4$ SI $\quad x=4$

$A(x) = 6+3x$

$A(x) = \$18$

71.- $h(x) = |x-2| - |x| + 2$

$|x-2| - |x| + 2$

$x-2-|x|+2=0 \quad \wedge \quad -x+2-|x|+2=0$

$\cancel{x-2-x+2=0} \wedge x-2+x+2=0 \wedge -x+2-x+2=0 \wedge -x+2+x+2=0$

$0 \wedge 2x=0 \wedge$ $\quad -2x+4=0 \quad$ $4=0$ NO.

$\boxed{x=0}$ $\quad -2x=-4$

$\boxed{x=2}$

$|x-2|$: $-x+2$ para $x<2$; $x-2$ para $x>2$ (puntos 0, 2)

$-|x|$: $+x$ para $x<0$; $-x$ para $x>0$ (punto 0)

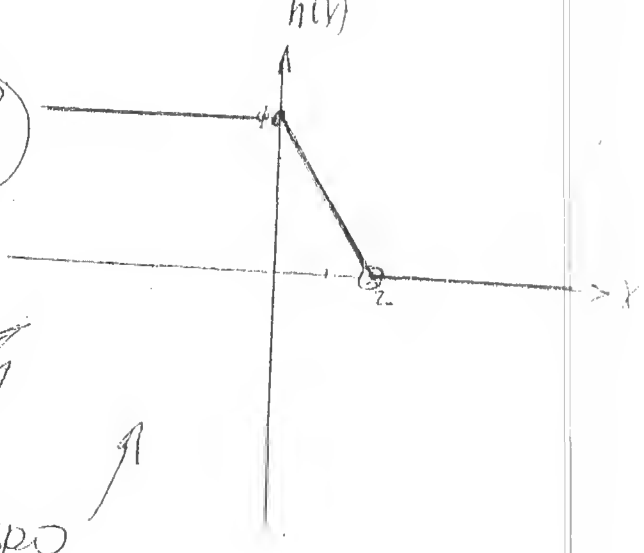
2: 2; 2 (punto 0)

$h(x)$: 4 para $x<0$; $-2x+4$ entre 0 y 2; 0 para $x>2$

$y=-2x+4$

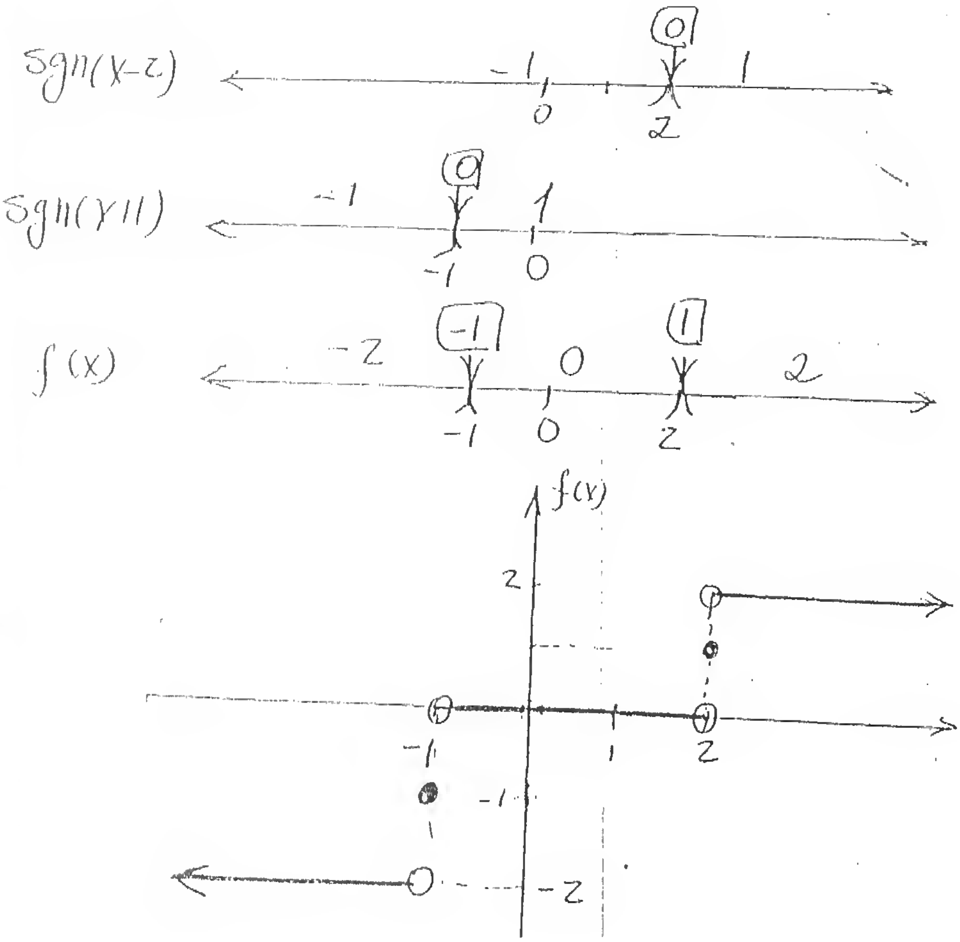
$x=0$
$y=4$

$y=0$
$x=2$



a) FALSO
b) FALSO
c) VERDADERO
d) FALSO
e) FALSO

72.- $f(x) = \operatorname{sgn}(x-2) + \operatorname{sgn}(x+1)$



a) Verdadero

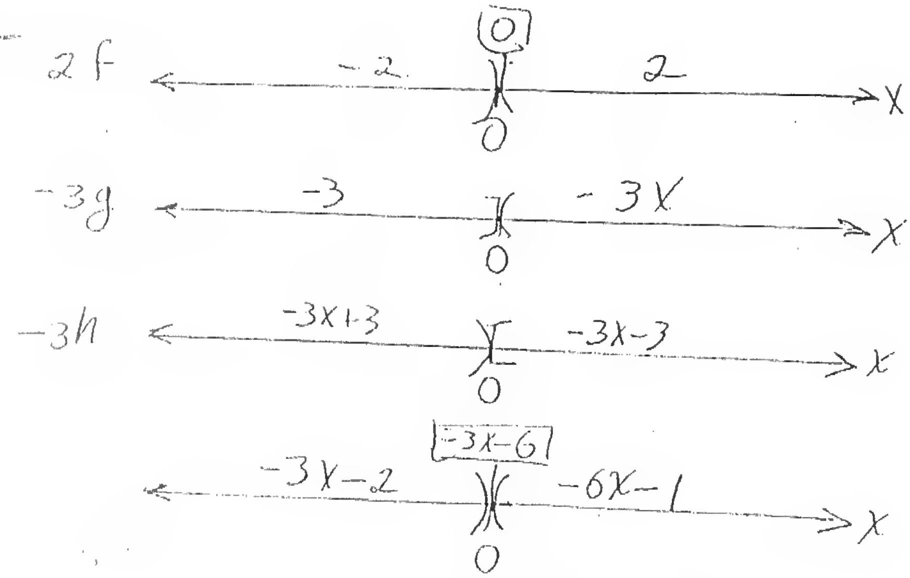
b) Verdadero

c) Verdadero

d) Falso

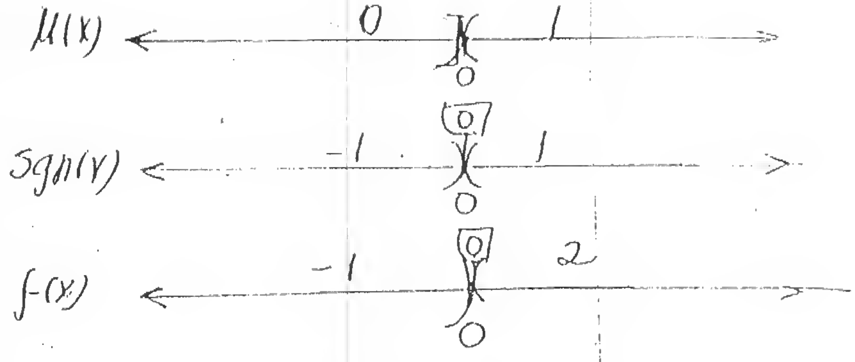
e) Verdadero

73.-



$$(2f-3g-3h)(x)\begin{cases} -3x-2 & ; x<0 \\ -3x-6 & ; x=0 \\ -6x-1 & ; x>0 \end{cases}$$

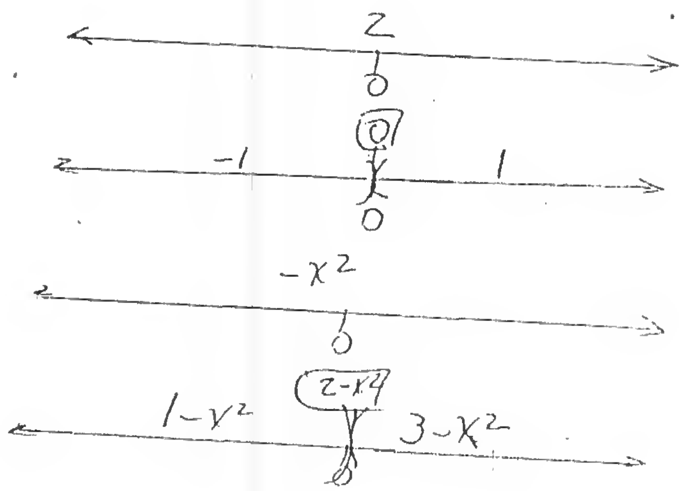
74.- $\mu(x)$

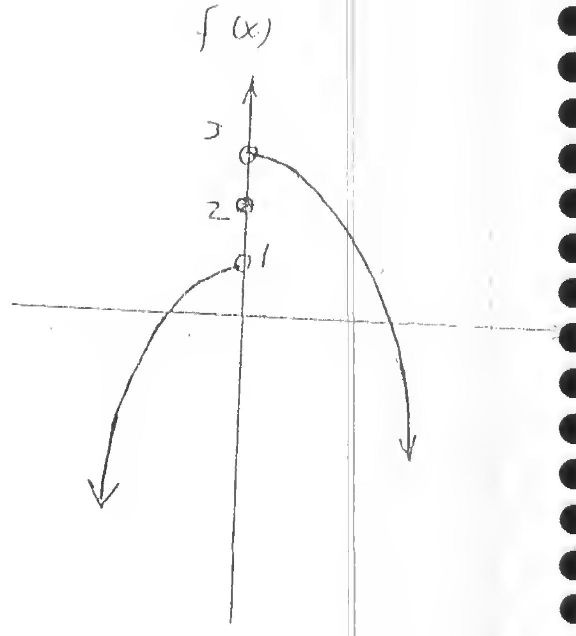


$$f(x)=\begin{cases} 2 & ; x>0 \\ 0 & ; x=0 \\ -1 & ; x<0 \end{cases}$$

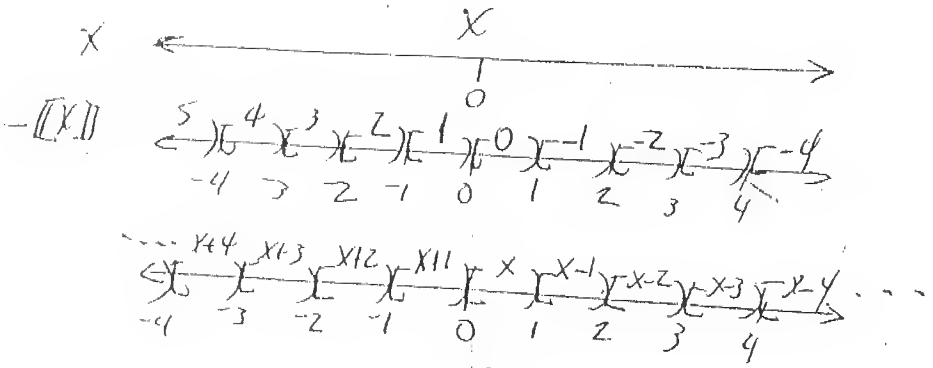
d) correcto

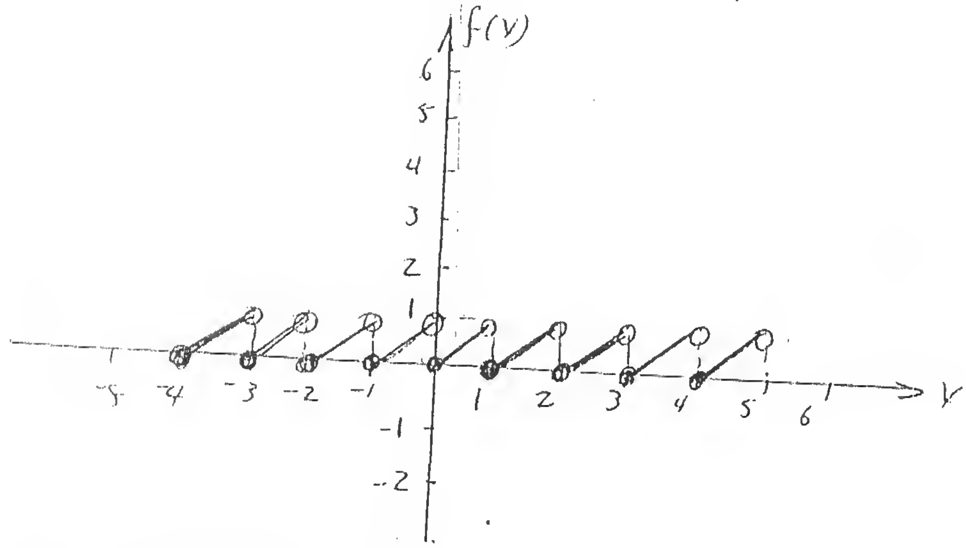
75.- a)



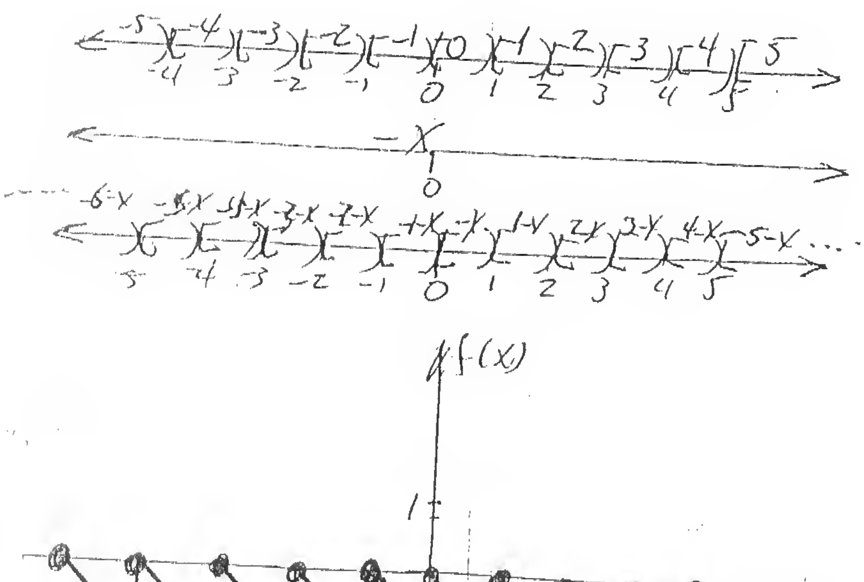


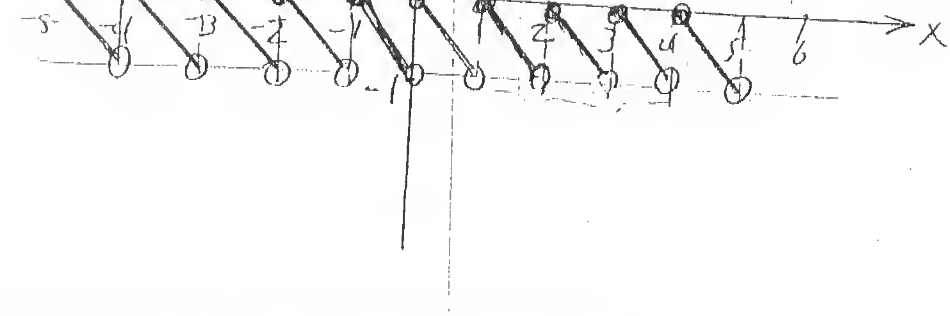
b) $f(x) = x - [\![x]\!]$





c) $f(x) = \begin{cases} x - [\![x]\!] \; ; \; x \geq 0 \\ [\![x]\!] - x \; ; \; x < 0 \end{cases}$



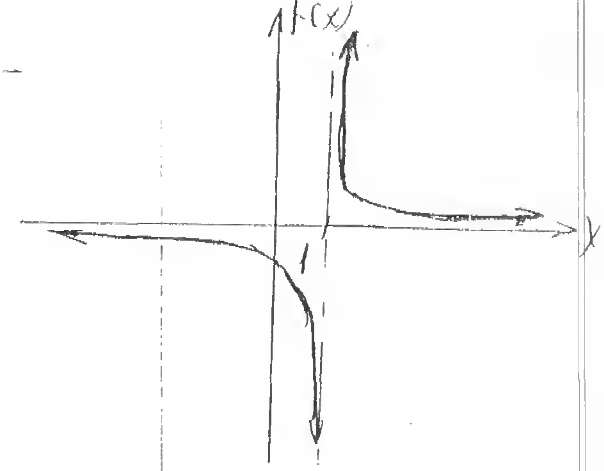


d) $f(x) = \frac{1}{x-1}$

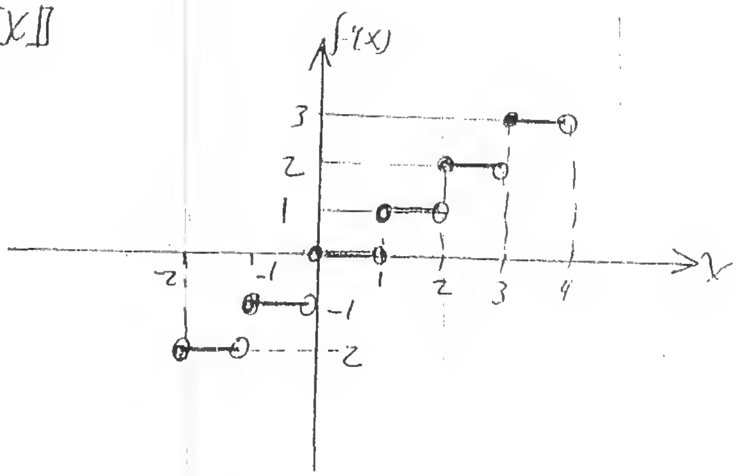
asintota

$x - 1 = 0$

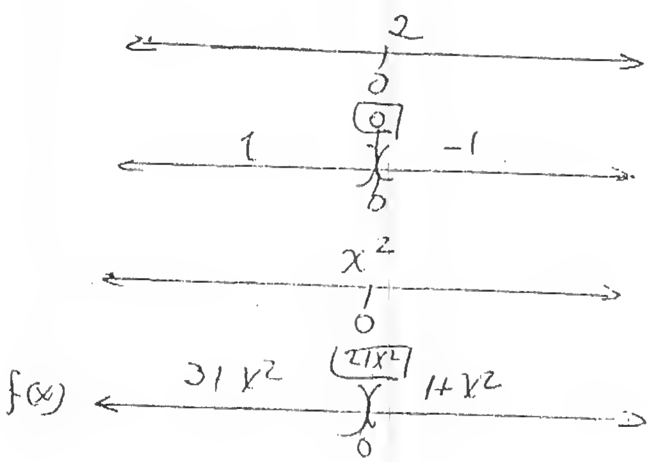
$x = 1$

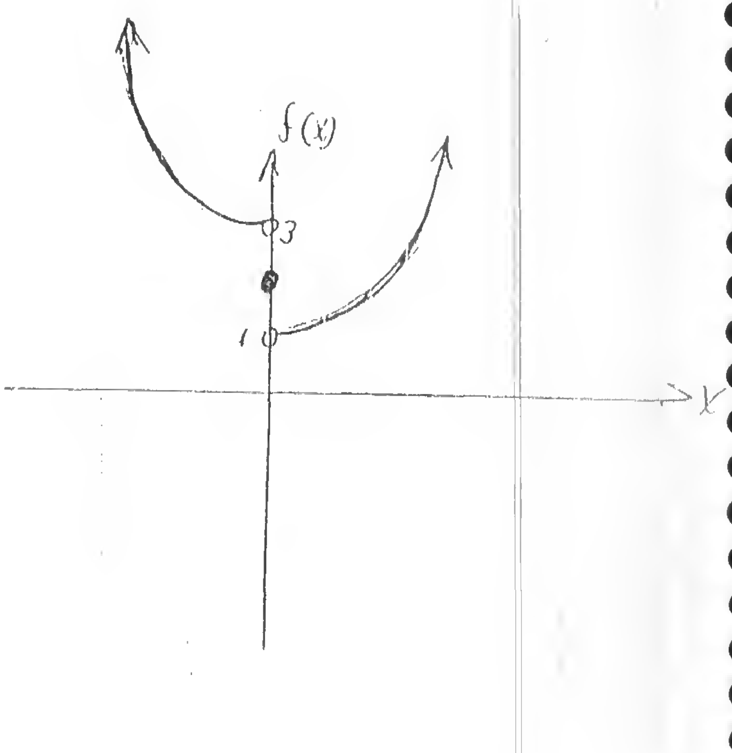


e) $f(x)$ $[\![x]\!]$



f) $f(x) = [2 - \operatorname{sgn}(x)] + x^2$





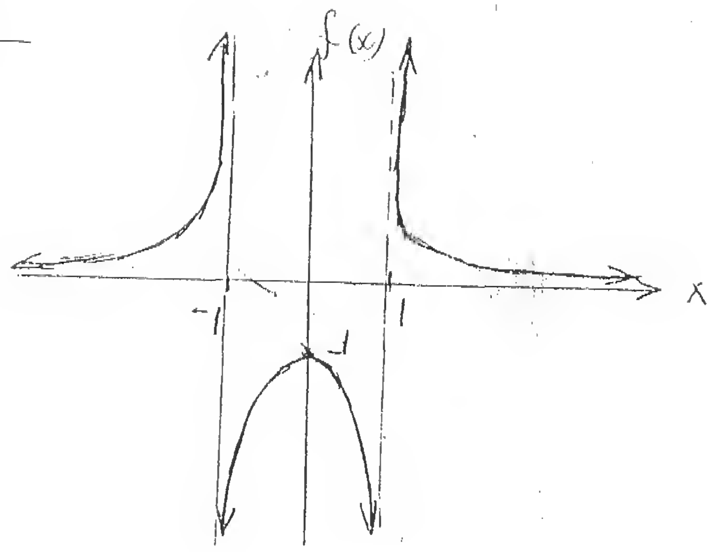
g) $f(x)=\dfrac{1}{x^2-1}$

asíntota

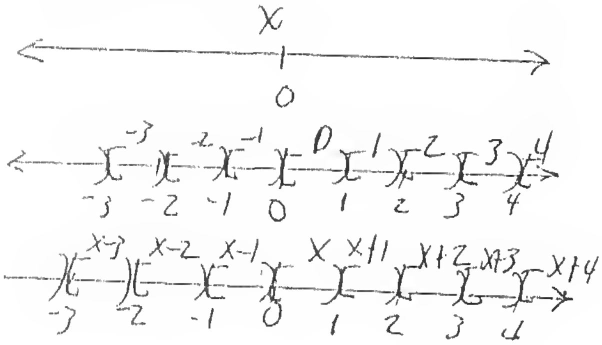
$x^2-1=0$

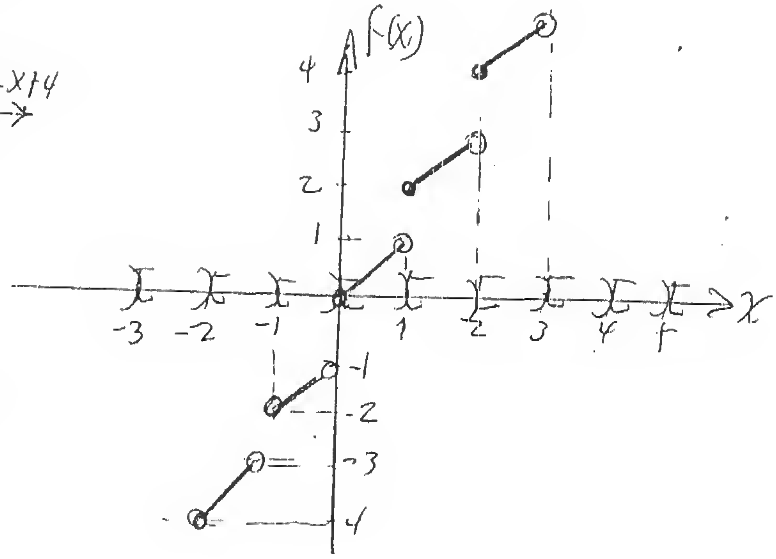
$x^2=1$

$x=\pm 1$



h) $f(x)=x+[\![x]\!]$





a) verdadero

b) Falso; tomando en cuenta el ejercicio anterior, la inversa también deberá estar en el 1er cuadrante

78.— $f^{-1}(a) = 2$

$f(f^{-1}(a)) = f(2)$

$a = f(2)$ a) verdadero

79.— $x^2 - 4x + 3$ $x \in (-\infty, 2)$

rango: $(-7, +\infty)$

$y = x^2 - 4x - 3$

$y = x^2 - 4x + 4 - 4 - 3$

$y = (x-2)^2 - 7$

$\sqrt{y-7} = \sqrt{(x-2)^2}$

$x - 2 = \sqrt{y-7}$

$x = \sqrt{y-7} + 2$ $\Rightarrow$ $f^{-1}(x) = \sqrt{x-7} + 2$; $x \geq -7$ c) correcto

80.— a) $f(x) = \dfrac{8}{x}$

$y = \dfrac{8}{x}$

$x = \dfrac{8}{y}$ $\Rightarrow$ $f^{-1}(x) = \dfrac{8}{x}$, NO es ni par ni impar

b) $g(x) = x^2$ $f^{-1}(x) = \dfrac{8}{x}$

$f^{-1} \circ g(x) = \dfrac{8}{x^2}$ $\Rightarrow$ es función par.

c) $(f^{-1}(x) \circ g) = x$

$$\frac{8}{x^2} = x$$

$$\sqrt[3]{x^3} = \sqrt[3]{8}$$

$$\boxed{x = 2}$$

81.- a) $g(x) = \dfrac{6-x}{2}$

$$y = \frac{6-x}{2}$$

$$2y = 6 - x$$

$$x = 6 - 2y$$

$$f^{-1}(x) = 6 - 2x$$

b) $f(x) = 4(x-1)$ $\qquad g(x) = \dfrac{6-x}{2}$

$$y = 4x - 4$$

$$y - 4 = 4x$$

$$x = \frac{y-4}{4}$$

$f^{-1} \circ g$

$$f^{-1}(x) = \frac{x-4}{4} \Rightarrow \frac{\left(\frac{6-x}{2}\right) - 4}{4}$$

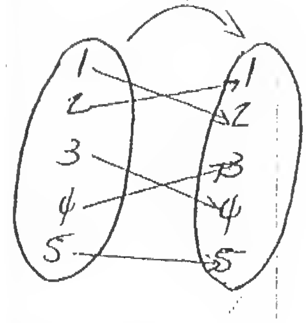
$$\Rightarrow \frac{\frac{6-x-8}{2}}{4} \Rightarrow \frac{-x-2}{8}$$

$$\frac{-x-2}{8} = 4 \Rightarrow -x - 2 = 32$$

$$-x = 34 \Rightarrow x = -34$$

82.—

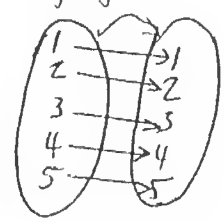
f



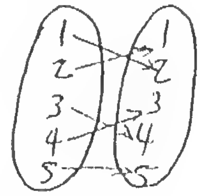
a) correcto

b)

$f \circ f$



$(f \circ f) \circ f$



correcto

c) correcto

d) correcto

e) correcto

83.—

a) Son funciones lineales, tienen que ser inyectivas,

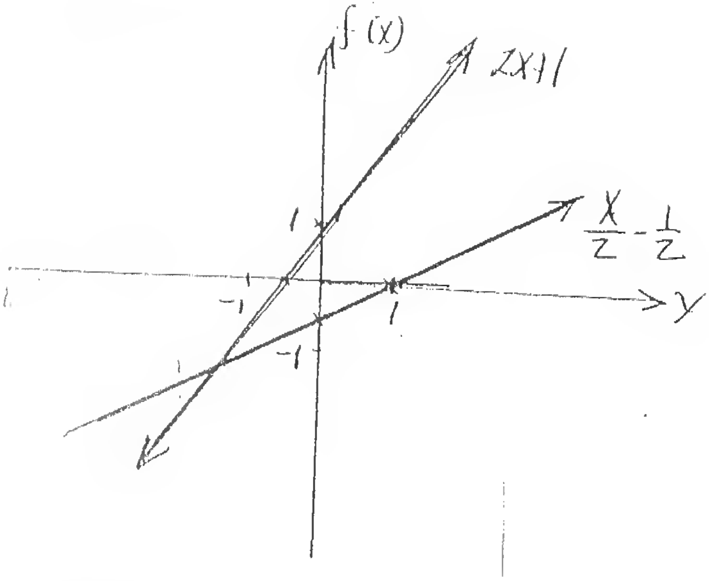
$(X_1 = X_2) \Rightarrow f(x_1) = f(x_2)$

$f(x) = 2x+1$	$g(x) = 3x-4$
$y = 2x+1$	$y = 3x-4$
$y-1 = 2x$	$y+4 = 3x$
$x = \frac{y-1}{2}$	$x = \frac{y+4}{3}$

Solo a un valor de y, corresponde X

b) $f(x)^{-1} = \frac{x-1}{2}$

c)



$y = 2x+1$

$x=0$, $y=1$ $\quad$ $y=0$, $x=-\frac{1}{2}$

$y = \frac{x}{2} - \frac{1}{2}$

$x=0$, $y=-\frac{1}{2}$ $\quad$ $y=0$, $x=1$

d) $(g \circ f)(-2)$

$f(-2) = -3$

$g(-3) = -13$ //

e) $(f \circ g)(x) = 2(3x-4)+1$

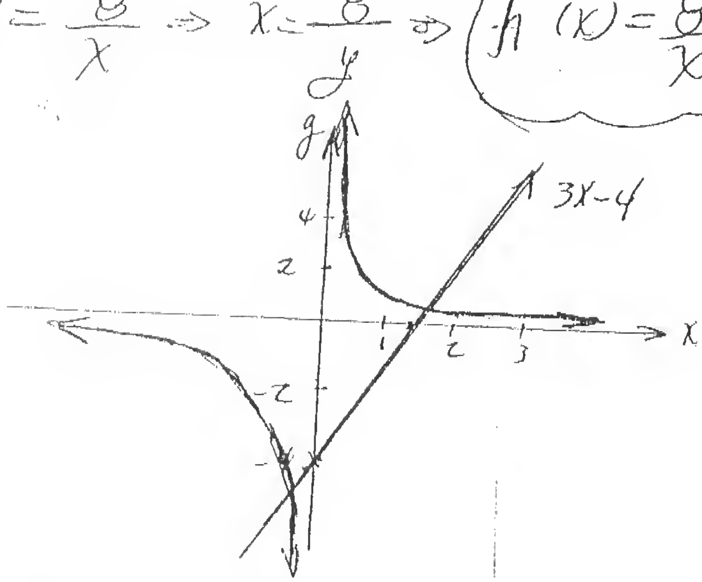
$\Rightarrow 6x-8+1$

$\Rightarrow 6x-7$

f) $h(x) = \frac{8}{x}$

$y = \frac{8}{x} \Rightarrow x = \frac{8}{y} \Rightarrow h^{-1}(x) = \frac{8}{x}$

g)



$g = 3x-4$

$x=0$, $g=-4$ $\quad$ $g=0$, $x=\frac{4}{3}$

h) $h^{-1}_{(x)} = \frac{8}{x}$; $r(x) = x^2$

$$(h^{-1} o\, r)(x) = \frac{8}{x^2}$$

i) $(h^{-1} o\, r)(x) = \frac{1}{2}$

$$\frac{8}{x^2} = \frac{1}{2}$$

$$16 = x^2$$

$$\boxed{x = \pm 4}$$

84.- $f(x) = \sqrt{x-5} - 5$ dom: $[5, +\infty)$

rango: $[-5 + \infty)$

$f^{-1}(x) \Rightarrow$ dom: $[-5, +\infty)$

85.- $f(x) = \frac{1}{ax^2 - 1}$ $A(2, \frac{1}{3})$

a) $\frac{1}{3} = \frac{1}{a(2)^2 - 1} \Rightarrow 4a - 1 = 3 \Rightarrow 4a = 4$

$a = 1$

b) $y = \frac{1}{x^2 - 1} \Rightarrow x^2 - 1 = \frac{1}{y} \Rightarrow x^2 = \frac{1}{y} + 1$

$x = \sqrt{\frac{1+y}{y}} \Rightarrow f^{-1}(x) = \sqrt{\frac{1+x}{x}}$

c) $g(x) = \frac{1}{f(x)}$ $x^2 - 1$

d) $f \circ g(x) \Rightarrow \frac{1}{(x^2-1)^2 - 1} \Rightarrow \frac{1}{x^4 - 2x^2 + 1 - 1} \Rightarrow \frac{1}{x^4 - 2x^2}$

- Una raiz (en una ecuación) corresponde a un factor en un polinomio

a) correcto

- b) FALSO ; No hay relación alguna entre a, b y el residuo.

- $\frac{P(x)}{x-z} = D(x) + \frac{k}{x-z}$

$\frac{P(x)}{\cancel{x-z}} = \frac{D(x)(x-z) + k}{\cancel{x-z}}$

$P(x) = \underbrace{Dx(x-z)}_{\downarrow} + k$

grado 4

también debe ser grado 4.

a) Verdadero

89.-

$$P(x) = x^3 + x^2 - (k+7)x + \frac{21}{8}$$

$(x - \frac{1}{2}) \Rightarrow$ factor $\Rightarrow x = \frac{1}{2}$ $\quad P(x) = 0$

$$\Rightarrow \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - (k+7)\left(\frac{1}{2}\right) + \frac{21}{8} = 0$$

$$8 \times \left(\frac{1}{8} + \frac{1}{4} - \frac{k+7}{2} + \frac{21}{8}\right) = 0 \times 8$$

$$1 + 2 - 4(k+7) + 21 = 0$$

$$-4(k+7) = -24$$

$$k+7 = 6$$

$k = -1$ $\quad$ a) correcto

90.- $x^3 + ax^2 + b \quad | \quad x^2 - x - 2$

$-x^3 + x^2 + 2x \qquad x + (a+1)$

$// \ (a+1)x^2 + 2x + b$

$-(a+1)x^2 + (a+1)x + 2(a+1)$

$// \ (a+3)x + (2a+b+2)$

Para que sea divisible $P(x)/q(x)$; residuo $\underline{\underline{0}}$

$a+3=0$

$a=-3$

$2a+b+2=0$

$b=-2a-2$

$b=-2(-3)-2$

$b=4$

$a+b=1$

a) correcto.

91 $q(x) = k^2x^3 - 4kx + 4 \quad ; x=1$

$k^2(1)^3 - 4k(1) + 4 = 1$

$k^2 - 4k + 3 = 0$

$(k-3)(k-1) = 0$

$k=3 \quad k=1$

$k_1 + k_2 = 4$

a) correcto

92.- $P(x): x^3+mx^2-x-2$; $x=2$; residuo: 4

$(2)^3+(2)^2-(2)-2=4$

$8+4m-4=4$

$4m=8-8$

$m=0$ b) correcto

93.- $P(x)=x^4-ax^2-5x+b$; $x=2$; $P(1)+10=0$

$(2)^4-(2)^2a-5(2)+b=0$

$16-4a-10+b=0$

$\boxed{-4a+b=-6}$

$(1)^4-a(1)^2-5(1)+b+10=0$

$1-a-5+b+10=0$

$\boxed{-a+b=-6}$

Es un sistema de ecuaciones inconsistente, ESTA MAL PLANTEADA LAS CONDICIONES

94.- $P(x): (a+1)x^5+(b-2)x^4-31x^3-39x^2+76x-20$

$x=1$ residuo: 0

$(a+1)+(b-2)-31-39+76-20=0$

$\boxed{a+b=+15}$

$(a+1)(-3)^5+(b-2)(-3)^4-31(-3)^3-39(-3)^2+76(-3)-20=400$

$-243(a+1)+81(b-2)+837-351-228-20=400$

$-243(a+1)+81(b-2)=162 \Rightarrow 3(a+1)+b-2=2$

$3a+3+b-2=2$

$\boxed{3a+b=1}$ $\quad$ $\boxed{a+b=15}\,(-1)$

$-a-b=-15$

$3a+b=1$

$2a=-14$

$\boxed{a=-7}$

$b=15-a$

$b=22$

$a+b=15$ e) correcto

95.- $q(x)=x^2+ax+b$; $x=1$; residuo: -3

; $x=2$; residuo: -7

$(1)^2+a(1)+b=-3$

$\boxed{a+b=-4}\,(-1)$

$(2)^2+a(2)+b=-7$

$\boxed{2a+b=-11}$

$-a-b=4$

$2a+b=-11$

$\boxed{a=-7}$

$b=-11-2a$

$b=-11-2(-7)$

$\boxed{b=3}$

$a \cdot b=-21$ c) correcto

96.- $P(x): x^3-x^2-14x+24$

$\Rightarrow (x^2+x-12)(x-2)$

$\Rightarrow (x+4)(x-3)(x-2)$

$x_1=-4$ $\quad$ $x_2=3$ $\quad$ $x_3=2$

97.- $P(x): ax^4 + bx^3 + cx^2 + dx + e$

$P(x): x^4 + bx^3 + cx^2 + dx + e$

$P(1): 1 + b + c + d + e = 0$

① $\{ \boxed{b+c+d+e=-1}$

$P(0) = x^4 + bx^3 + cx^2 + dx + e = -2$

$0 + 0 + 0 + 0 + e = -2$

$e = -2$

$$\begin{cases} b+c+d = 1 \\ 2b-2c+d = 0 \\ 4b-2c = 6 \end{cases}$$

$(-2)(1)$ $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & -2 & 1 & 0 \\ 2 & -1 & 0 & 3 \end{array}\right)$

$\Rightarrow \begin{array}{c} \\ (-3) \\ (4) \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -4 & -1 & -2 \\ 0 & -3 & -2 & 1 \end{array}\right)$

$\Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -4 & -1 & -2 \\ 0 & 0 & -5 & 10 \end{array}\right)$

$-5d = 10$

$d = -2$

$$\begin{array}{l|l} x^4 + bx^3 + cx^2 + dx + e & x^2 + 2x + 2 \\ \hline -x^4 - 2x^3 - 2x^2 & x^2 + (b-2)x + (-2b+c+2) \\ \hline // \ (b-2)x^3 + (c-2)x^2 + dx & \\ -(b-2)x^3 - 2(b-2)x^2 - 2(b-2)x & \\ \hline // \ (c-2-2b+4)x^2 + (-2b+4+d)x & \\ (c-2b+2)x^2 + (-2b+d+4)x + e & \\ -(-2b+c+2)x^2 - 2(-2b+c+2)x - 2(-2b+c+2) & \\ \hline // \ (-\cancel{2b}+d+4+4b-2c-4)x + (4b-2c \ldots & \\ (2b-2c+d)x + (4b-2c+e-4) & \end{array}$$

residuo: 0

② $\{ \boxed{2b-2c+d = 0}$

③ $\boxed{4b-2c+e = 4}$

$-3c - 2d = 1$

$-3c = 1 + 2d$

$-3c = 1 + (-4)$

$-3c = -3$

$c = 1$

$b + c + d = 1$

$b = 1 - c - d$

$b = \cancel{1} - \cancel{1} + 2$

$b = 2$

$P(x): x^4 + 2x^3 + x^2 - 2x - 2$

98.- $g(x) = x^3 - 8x^2 + 9x + K$

$\left.\begin{array}{l} x_1 = a \\ x_2 = 2a \end{array}\right\}$ Son raíces

$(a)^3 - 8(a)^2 + 9(a) + K = 0$

$a^3 - 8a^2 + 9a + K = 0$

$(2a)^3 + 9(2a) - 8(2a)^2 + K = 0$

$8a^3 - 32a^2 + 18a + K = 0$

$a^3 - 8a^2 + 9a = -K$

$8a^3 - 32a^2 + 18a = -K$

$a^3 - 8a^2 + 9a = 8a^3 - 32a^2 + 18a$

$7a^3 - 24a^2 + 9a = 0$

$a(7a^2 - 24a + 9) = 0$

$a = 0 \wedge a = \dfrac{24 \pm \sqrt{24^2 - 4(7)(9)}}{2(7)}$

$a = \dfrac{24 \pm 18}{14} = \begin{cases} 3 \\ \dfrac{3}{7} \end{cases}$

$K = -a^3 + 8a^2 - 9a$

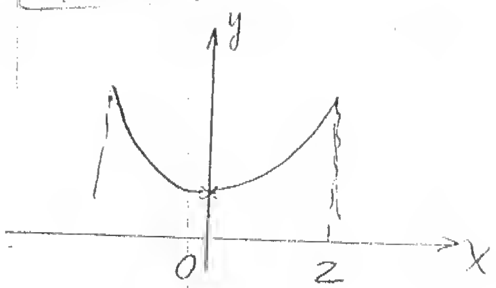
$K = -(3)^3 + 8(3)^2 - 9(3) \wedge K = (-\frac{3}{7})^3 + 8(\frac{3}{7})^2 - 9(\frac{3}{7})$

$K = 18$

$K = -\dfrac{846}{343}$

Exponencial

99.- $y = 2^x + 2^{-x}$



$x = 0$

$y = 2^0 + 2^0$

$y = 2$ $P(0,2)$ b) correcto

100.- $y = 2^2 + 2^{-2}$

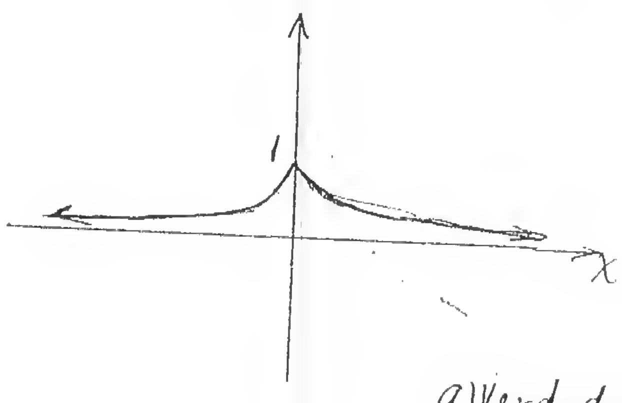
$y = 4 + \frac{1}{4} \rightarrow \frac{16+1}{4} = \frac{17}{4}$

rango: $\left[2, \frac{17}{4}\right]$ d) correcto

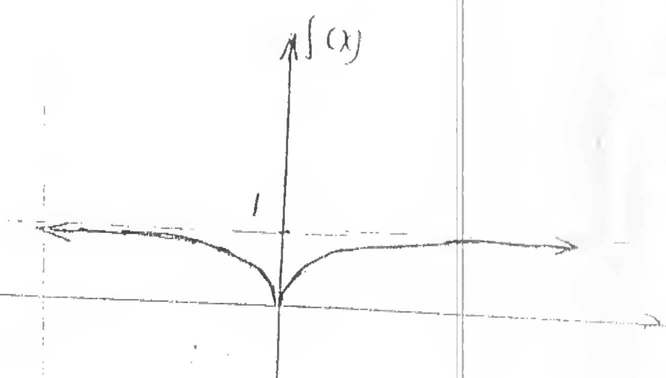
101.- $f(x) = -\frac{1}{2^{|x|}} + 1$

$\Rightarrow f(x) = -\left[\frac{1}{2}\right]^{|x|} + 1$

$f(x) = \left(\frac{1}{2}\right)^{|x|}$



$f(x) = -\left(\frac{1}{2}\right)^{|x|} + 1$



a) Verdadero
b) Verdadero
c) Verdadero
d) Verdadero
f) Falso

102.- $4 \times 4^{1/3} \times 4^{1/9} \times 4^{1/27} \ldots$

$\Rightarrow 4^{1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots}$

$S = 1+\frac{1}{3}+\frac{1}{9}+\ldots$

$\boxed{r=\frac{1}{3}}$

$S = \frac{a_1}{1-r} = \frac{1}{1-\frac{1}{3}}$

$S = \frac{1}{\frac{2}{3}} \Rightarrow \frac{3}{2}$

$\Rightarrow 4^{3/2} \Rightarrow (\sqrt{4})^3 \Rightarrow 8$ c) correcto

103.-

a) $P(x) = 3^{x+1} + 3^x + 3^{x-1} = 39$

$\Rightarrow 3 \cdot 3^x + 3^x + \frac{3^x}{3} = 39$

$\Rightarrow 3^x(\underbrace{3+1}_{4}+\frac{1}{3}) = 39 \Rightarrow 3^x\left(\frac{12+1}{3}\right) = 39$

$\Rightarrow 3^x = \frac{\overset{3}{\cancel{39}} \times 3}{\underset{1}{\cancel{13}}} \Rightarrow \cancel{3}^x = \cancel{3}^2$

$x = 2$

b) $q(x): 2^{x+1} + 4^x = 80$

$\Rightarrow 2 \cdot 2^x + (2^x)^2 - 80 = 0$

$a = 2^x$

$2a + a^2 - 80 = 0 \Rightarrow a^2 + 2a - 80 = 0$

$(a+10)(a-8) = 0$

$\cancel{a = -10}$ $a = 8$

$2^x = 8 \Rightarrow \cancel{2}^x = \cancel{2}^3$

$x = 3$

c) $r(x): 6(3^{2x}) - 13(6^x) + 6(2^{2x}) = 0$

$$\Rightarrow 6(3^x)^2 - 13(2\times 3)^x + 6(2^x)^2 = 0$$

$$a = 3^x \quad ; b = 2^x$$

$$6a^2 - 13ab + 6b^2 = 0$$

$$(6a - 9b)(6a - 4b) = 0$$

$6a - 9b = 0$ $\qquad$ $6a - 4b = 0$

$6a = 9b$ $\qquad$ $6a = 4b$

$\boxed{a = \frac{3}{2}b}$ $\qquad$ $\boxed{a = \frac{2}{3}b}$

$\frac{a}{b} = \frac{3}{2}$ $\qquad$ $\frac{a}{b} = \frac{2}{3}$

$\Rightarrow \frac{3^x}{2^x} = \frac{3}{2}$ $\qquad$ $\Rightarrow \frac{3^x}{2^x} = \frac{2}{3}$

$\Rightarrow \cancel{\left(\frac{3}{2}\right)}^x = \cancel{\frac{3}{2}}$ $\qquad$ $\Rightarrow \cancel{\left(\frac{3}{2}\right)}^x = \cancel{\left(\frac{3}{2}\right)}^{-1}$

$x = 1$ $\qquad$ $x = -1$

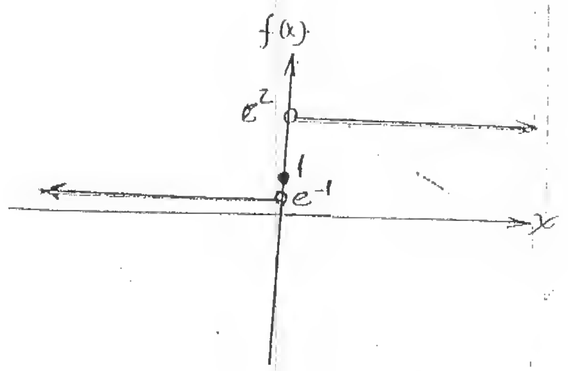
104.- $f(x) = e^{\operatorname{sgn}(x) + u(x)}$

$$f(x) = e^{\operatorname{sgn}(x)} \cdot e^{u(x)}$$

$e^{\operatorname{sgn}(x)}$: e^{-1} | $\boxed{1}$ at 0 | e

$e^{u(x)}$: 1 | 0 | e

$f(x)$: e^{-1} | $\boxed{1}$ at 0 | e^2



a) FALSO; es creciente

b) FALSO

c) FALSO

d) VERDADERO

e) FALSO; rango $f: \{\frac{1}{e}, e^2\}$

105.-

a) $P(x): 4^x + 2^{x+1} = 8$

$(2^x)^2 + 2^x . 2 - 8 = 0$

$a = 2^x$

$a^2 + 2a - 8 = 0$

$(a+4)(a-2) = 0$

$a+4=0 \qquad a-2=0$

$\cancel{a=-4} \qquad a=2 \qquad 2^x = 2$

$\log_2 2^x = \log_2 2$

$x . \cancel{\log_2 2} = \cancel{\log_2} 2$

$x = \underline{\underline{1}}$

b) $h(x): 2^x + (0{,}5)^{2x-3} - 5(0{,}5)^{x-1} = -1$

$\Rightarrow 2^x + \left(\frac{1}{2}\right)^{2x}\left(\frac{1}{2}\right)^{-3} - 5\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{-1} = -1$

$\Rightarrow 2^x + \left[\left(\frac{1}{2}\right)^x\right]^2 \times 2^3 - 5\left(\frac{1}{2}\right)^x \times 2 = -1$

$\Rightarrow 2^x + \left(\frac{1}{2^x}\right)^2 \times 8 - 10 \times \frac{1}{2^x} + 1 = 0$

$\Rightarrow \qquad a = 2^x$

$\Rightarrow a + \frac{8}{a^2} - \frac{10}{a} + 1 = 0 \Rightarrow \frac{a^3 + 8 - 10a + a^2}{a^2} = 0$

$\Rightarrow a^3 + a^2 - 10a + 8 = 0 \Rightarrow (a^2 + 2a - 8)(a-1) = 0$

$\Rightarrow (a+4)(a-2)(a-1) = 0$

$a = 4 \; ; \; a = 2 \; ; \; a = 1$

$\cancel{2^x = 4}$ $\quad$ $\cancel{2^x = 2^1}$ $\quad$ $2^x = 1$

$x = 1$ $\quad$ $\cancel{2^x = 2^0}$

$x = 0$

c) $g(x): 16^x - 6(4^x) = -8$

$\Rightarrow (4^x)^2 - 6(4^x) + 8 = 0$

$a = 4^x$

$\Rightarrow a^2 - 6a + 8 = 0$

$\Rightarrow (a-4)(a-2) = 0$

$a = 4 \; ; \; a = 2$

$4^x = 2^1$ $\quad$ $\cancel{4^x = 4^1}$

$\Rightarrow \cancel{2}^{2x} = \cancel{2}^1$ $\quad$ $x = 1$

$2x = 1$

$x = 1/2$

d) $r(x) = 9^x + 3^{x+1} - 4 = 0$

$(3^x)^2 + 3^x \cdot 3 - 4 = 0$

$a = 3^x$ $\quad \Rightarrow \cancel{3^x = -4} \; ; \; 3^x = 1$

$a^2 + 3a - 4 = 0$ $\quad \Rightarrow \cancel{3^x = 3^0}$

$(a+4)(a-1) = 0$ $\quad x = 0$

$a = -4 \; ; \; a = 1$

$g(x):\ 3^x + 9^x = 6642$

$\Rightarrow 3^x + (3^x)^2 - 6642 = 0$

$a = 3^x$

$a + a^2 - 6642 = 0$

$a^2 + a - 6642 = 0$

$(a+82)(a-81) = 0$

$a = -82 \quad a = 81$

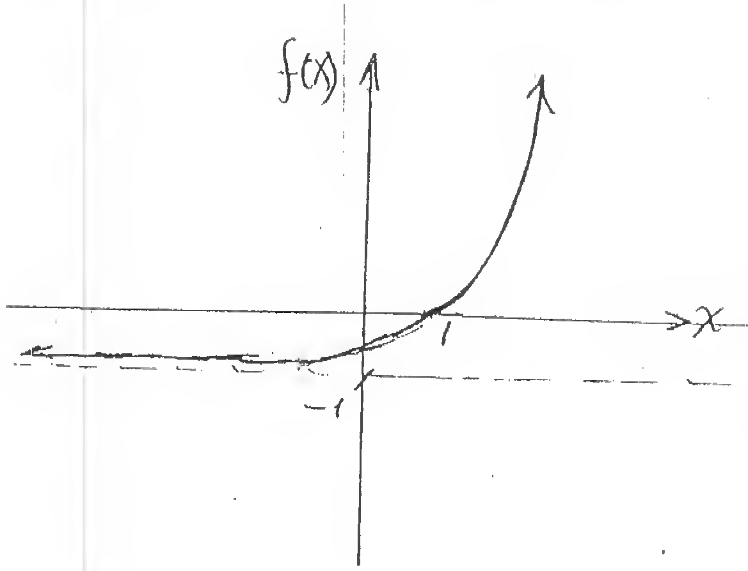
$\cancel{3^x = 82} \qquad 3^x = 81$

$\cancel{3}^x = \cancel{3}^4 \qquad x = 4$

6642	2
3321	3
1107	3
369	3
123	3
41	41
0	

106.-

$F(x) = e^{x-1} - 1$



a) FALSO

b) FALSO

c) VERDADERO

d) FALSO

e) FALSO

107.-

a) $\sqrt[3]{2^{\frac{3x-1}{x-1}}} < 8^{\frac{x-3}{3x-7}}$

$\cancel{2}^{\frac{1}{3}\cdot\left(\frac{3x-1}{x-1}\right)} < \cancel{2}^{3\cdot\frac{x-3}{3x-7}}$

$\frac{3x-1}{3x-3} < \frac{3x-9}{3x-7}$

$\frac{3x-1}{3x-3} - \frac{3x-9}{3x-7} < 0$

$\frac{(3x-1)(3x-7)-(3x-9)(3x-3)}{(3x-3)(3x-7)} < 0$

$9x^2-21x-3x+7-(9x^2-9x-27x+27)<0$

$\cancel{9x^2}-24x+7-\cancel{9x^2}+36x-27<0$

$12x-20<0$

$\boxed{x<\frac{5}{3}}$

Pto críticos

$3x-3=0$

$x=1$

$3x-7=0$

$3x=7$

$x=7/3$

0 1 2 3, $5/3$, $7/3$

$x=0$

$\frac{1}{3}<\frac{9}{7}$

Sí

$x=1,5$

$\frac{3,5}{0,5}<\frac{-4,5}{-2,5}$

NO

$x=2$

$\frac{5}{3}<\ldots$

Sí

Sol: $(-\infty,1)\cup\left(\frac{5}{3},7/3\right)$

b) $(0{,}04)^{5x-x^2} < 625$

$\Rightarrow \left(\frac{\cancel{4}\,1}{\cancel{100}\,25}\right)^{5x-x^2} < 5^4$

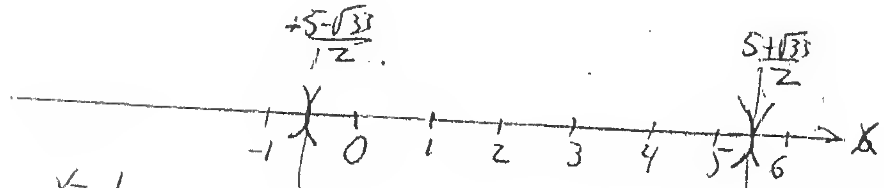
$\Rightarrow \left(\frac{1}{25}\right)^{5x-x^2} < 5^4 \Rightarrow \left(5^{-2}\right)^{5x-x^2} < 5^4$

$\Rightarrow \cancel{5}^{-10x+2x^2} < \cancel{5}^4 \Rightarrow 2x^2-10x < 4$

$\Rightarrow 2(x^2-5x-2) < 0$ $\qquad x = \frac{5 \pm \sqrt{25-4(1)(-2)}}{2(1)}$

$\Rightarrow x^2-5x-2 < 0$

$x = \frac{5 \pm \sqrt{33}}{2}$



$x=-1$	$x=0$	$x=6$
$(0{,}04)^{-6} < 625$	$(0{,}04)^0 < 625$	$(0{,}04)^{-6} < 625$
$\Rightarrow \left(\frac{1}{25}\right)^{-6} < 625$	$1 < 625$	$\left(\frac{1}{25}\right)^{-6} < 25^2$
$\Rightarrow 25^6 < 25^2$	si	$25^6 < 25^2$
NO		NO

Sol: $\left(\frac{5-\sqrt{33}}{2}, \frac{5+\sqrt{33}}{2}\right)$

c) $2^{x+2} - 2^{x+3} - 2^{x+4} \geq \cancel{5^{x+1}} - \cancel{5^{x+1}}$

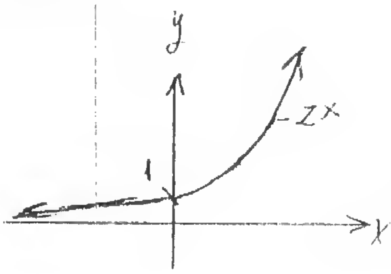
$$\Rightarrow 2^x \cdot 2^2 - 2^x \cdot 2^3 - 2^x \cdot 2^4 \geq 0$$

$$\Rightarrow 2^2(2^x - 2 \cdot 2^x - 2^2 \cdot 2^x) \geq 0$$

$$2^x - 2 \cdot 2^x - 4 \cdot 2^x \geq 0$$

$$5 \cdot 2^x \geq 0$$

$$2^x \geq 0$$



Sol: $x \in \mathbb{R}$

d.) $\dfrac{1}{(0{,}5)^x - 1} - \dfrac{1}{1 - (0{,}5)^{x+1}} \geq 0$

$$\Rightarrow \frac{1}{\left(\frac{1}{2}\right)^x - 1} - \frac{1}{1 - \left(\frac{1}{2}\right)^{x+1}} \geq 0 \Rightarrow \frac{1}{\frac{1}{2^x} - 1} - \frac{1}{1 - \frac{1}{2^{x+1}}} \geq 0$$

$$\Rightarrow \frac{1}{\frac{1-2^x}{2^x}} - \frac{1}{\frac{2^{x+1} - 1}{2^{x+1}}} \geq 0 \Rightarrow \frac{2^x}{1-2^x} - \frac{2^{x+1}}{2^{x+1} - 1} \geq 0$$

$$\Rightarrow \frac{2^x(2^{x+1} - 1) - 2^{x+1}(1 - 2^x)}{(1-2^x)(2^{x+1} - 1)} \geq 0 \Rightarrow 2^x(2^x \cdot 2 - 1) - 2^x \cdot 2(1 - 2^x) \geq 0$$

$$\Rightarrow 2^x[(2 \cdot 2^x - 1) - 2(1 - 2^x)] \geq 0 \quad \Rightarrow 2^x[2 \cdot 2^x - 1 - 2 + 2 \cdot 2^x] \geq 0$$

$$\Rightarrow 2^x[4 \cdot 2^x - 3] \geq 0 \Rightarrow 2^x \geq 0$$

$x \in \mathbb{R}$

$$4 \cdot 2^x - 3 \geq 0$$

$$4 \cdot 2^x \geq 3$$

$$2^x \geq \frac{3}{4}$$

$$\log_2 2^x \geq \log_2(3/4)$$

$$x \geq \log_2(3/4)$$

l) $0 < 8^x + 18^x - 2(27)^x$

$\Rightarrow 2^{3x} + (9\times 2)^x - 2(3^3)^x > 0$

$\Rightarrow 2^{3x} + 3^{2x}\times 2^x - 2\times 3^{3x} > 0$

$a = 2^x \quad ; \quad b = 3^x$

$\Rightarrow a^3 + ab^2 - 2b^3 > 0$

$\Rightarrow (a^3 - ab^2) + (2ab^2 - 2b^3) > 0 \Rightarrow a(a^2 - b^2) + 2b^2(a-b) > 0$

$\Rightarrow a(a+b)(a-b) + 2b^2(a-b) > 0 \Rightarrow (a-b)(a(a+b) + 2b^2) > 0$

$a - b > 0 \qquad a(a+b) + 2b^2 > 0$

$a > b$

$2^x > 3^x$

$\frac{2^x}{3^x} > 1 \Rightarrow \left(\frac{2}{3}\right)^x > 1 \Rightarrow \cancel{\left(\frac{2}{3}\right)}^x > \cancel{\left(\frac{2}{3}\right)}^0$

$x > 0$

108.

a) $f \circ g = 2^{\frac{x}{x-2}}$

b) $g(x) = \frac{x}{x-2}$

$\Rightarrow y = \frac{x}{x-2}$

$\Rightarrow y(x-2) = \cancel{x}$

$\Rightarrow yx - 2y = x$

$xy - x = 2y$

$x(y-1) = 2y$

$x = \frac{2y}{y-1}$

$f^{-1}(x) = \frac{2x}{x-1}$

dom ; $x \in \mathbb{R} - \{1\}$

c) $g^{-1} \circ g$ $\quad g^{-1}(x) = \frac{2x}{x-1} \quad g(x) = \frac{x}{x-2}$

$$g^{-1} \circ g = \frac{2\left(\frac{x}{x-2}\right)}{\frac{x}{x-2} - 1} \Rightarrow \frac{\frac{2x}{\cancel{x-2}}}{\frac{x-(x-2)}{\cancel{x-2}}} \Rightarrow \frac{\cancel{2}x}{\cancel{x-x+2}}$$

$g^{-1} \circ g = x$ dom $x \in \mathbb{R}$

109.- $\log(x) \cdot \log(x) \cdot \log(x) = 3\log(x)$

$\Rightarrow (\log(x))^3 \neq 3\log(x)$ b) Falso

110.- $p(x): \ln(\log_x 2) = -1$

$e^{-1} = \log_x 2 \Rightarrow \log_x 2 = e^{-1}$

$\Rightarrow x^{e^{-1}} = 2 \Rightarrow x^{\frac{1}{e}} = 2 \Rightarrow \left(x^{\frac{1}{e}}\right)^e = (2)^e$

$x = 2^e$ a) Verdadero

111 $\quad a < b \quad c \in \mathbb{R}^+ - \{1\}$

$\log_c a < \log_c b \qquad a = 1 \quad b = c = 2$

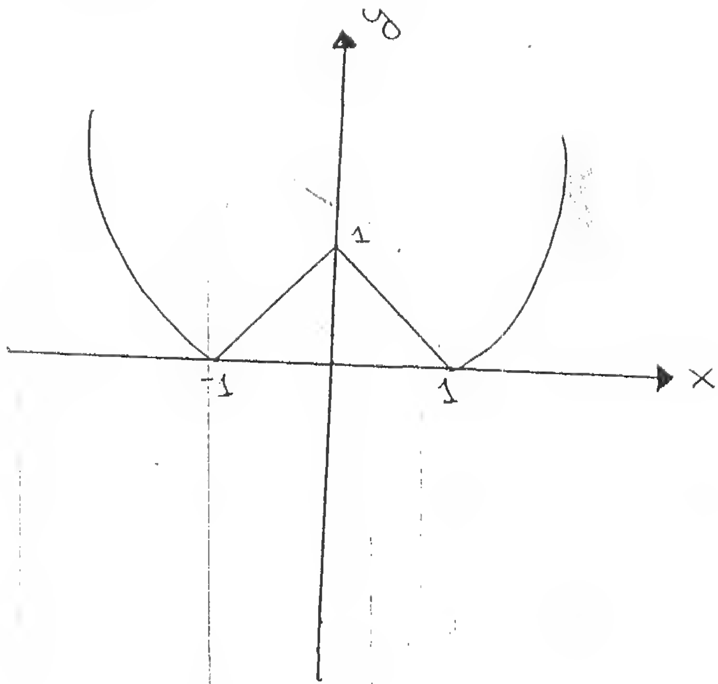
$\log_2 1 < \log_2 2$

$0 < 1$

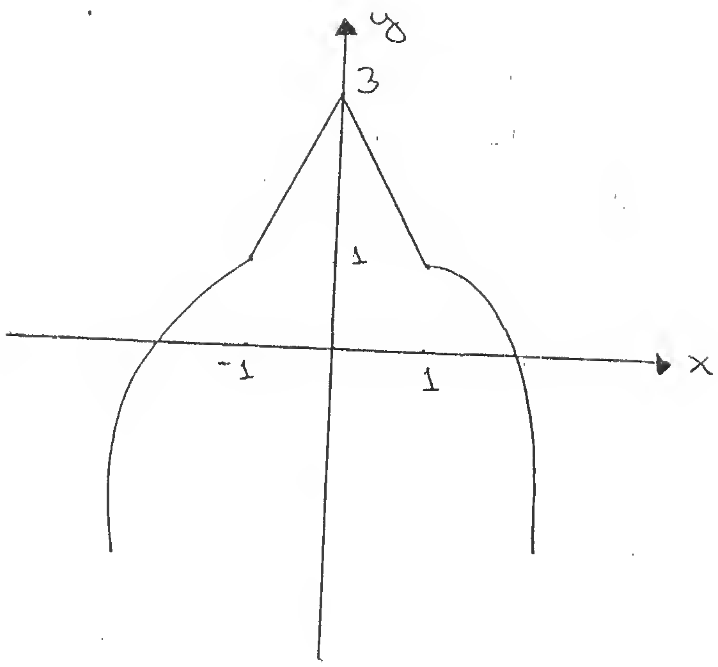
~~a)~~ Verdadero

b) Falso

d) $y = |f(-|x|)|$



e) $y = 1 - 2f(|x|)$



$-2 > -5$ b) Falso

112.- $\longrightarrow$ $(1,0)$ $(a,1)$

$\log_a 1 = 0$ $\log_a a = 1$

a) verdadero

113.- $\log 2 = a$ $\log 3 = b$

$\log(75) = \log\left(\frac{300}{4}\right)$

$\Rightarrow \log 300 - \log 4$

$\Rightarrow \log(3 \times 100) - \log 2^2$

$\Rightarrow \log 3 + \log 100 - 2 \cdot \log 2$

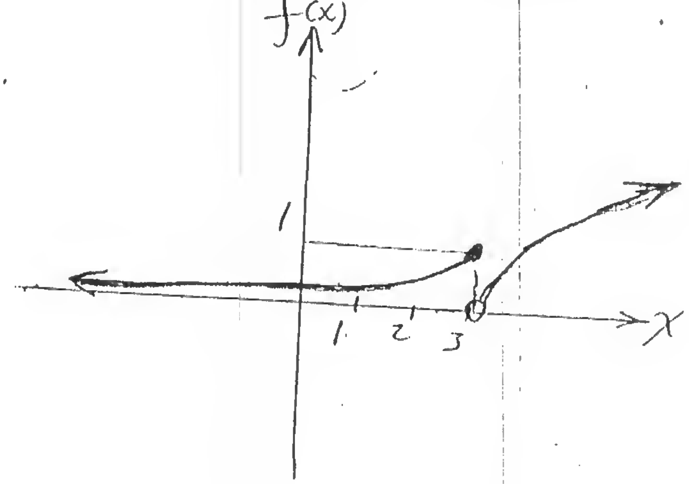
$\Rightarrow \log 3 + \log 10^2 - 2 \log 2$

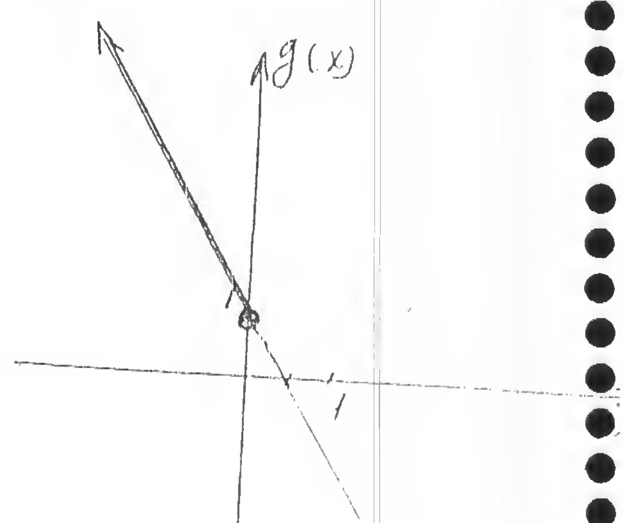
$\Rightarrow b + 2 - 2a$ e) correcto

114.- $f(x) = \begin{cases} 2^{x-3} & ; x \leq 3 \\ \log_3(x-2) & ; x \in (3, +\infty) \end{cases}$

$g(x) = 1 - 2x \; ; \; x \leq 0$

b)





$y = \log_3(x-2)$

asíntota

$x - 2 = 0$

$x = 2$

raíz

$x - 2 = 1$

$x = 3$

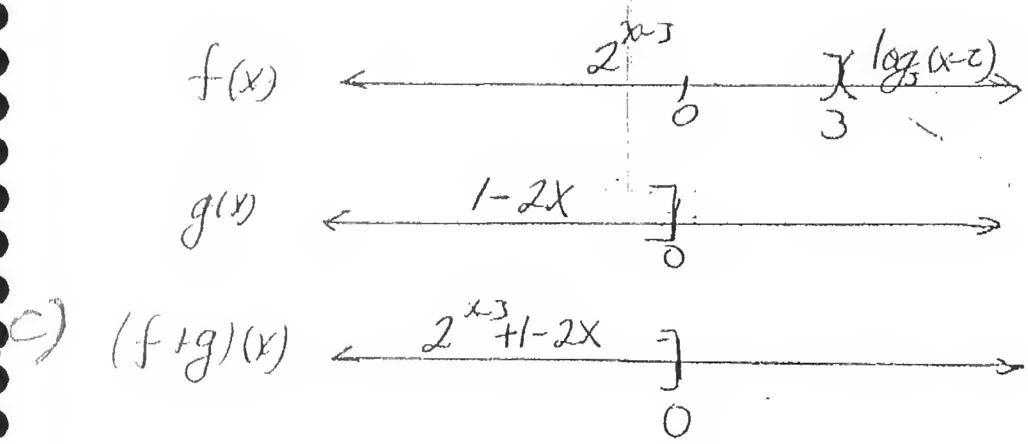
$y = 1 - 2x$

$x = 0$ $y = 1$

$y = 0$ $x = 1/2$

a) rango de f : $(0, +\infty)$

rango g : $[1, +\infty)$



$$(f+g)(x) = 2^{x-3}+1-2x \quad ; x \le 0$$

15.- a) $p(x): \log(x-1) = \log\sqrt{5+x} + \log\sqrt{5-x}$

$\log(x-1) = \log\left[\sqrt{5+x}\right]\left[\sqrt{5-x}\right]$

$\log(x-1) = \log\sqrt{25-x^2}$

$(x-1) = \sqrt{25-x^2}$

$(x-1)^2 = (\cancel{\sqrt{25-x^2}})^{\cancel{2}}$

$x^2-2x+1 = 25-x^2$

$x^2-2x+1+x^2-25=0$

$2x^2-2x-24=0$

$2(x^2-x-12)=0$

$x^2-x-12=0$

$(x-4)(x+3)=0$

$x-4=0 \qquad x+3=0$

$x=4 \qquad x=-3$ *

Pero $x=-3$ no satisface a $\log(x-1)$

$x=4$ es solución ya que:

$\log(4-1)=\log\sqrt{5+4}+\log\sqrt{5-4}$

$\log 3=\log\sqrt{9}+\log\sqrt{1}$

$\log 3=\log 3+\log 1$

$\log 3=\log 3$ $\qquad \log 1\equiv 0$

$A\,p(x)=\{4\}$

b) $q(x)$: $\log(x^2-4)-\log(x+2)=3\log(x-2)$

$$\log\frac{x^2-4}{x+2}=\log(x-2)^3$$

$$\frac{x^2-4}{x+2}=(x-2)^3 \qquad \frac{\cancel{(x+2)}(x-2)}{\cancel{x+2}}=(x-2)^3$$

$$\frac{(x-2)^{\cancel{3}2}}{\cancel{x-2}}=1 \qquad (x-2)^2=1$$

$x^2-4x+4=1$ $\qquad x^2-4x+4-1=0$

$x^2-4x+3=0$ $\qquad (x-3)(x-1)=0$

$x-3=0$ $\qquad x-1=0$

$x=3$ $\qquad x=1$ *

Pero $x=1$ no satisface a $\log(x-2)$ ni a $\log(x^2-4)$

$x=3$ es solución ya que:

$\log(3^2-4)-\log(3+2)=3\log(3-2)$

$\log(9-4)-\log(3+2)=3\log(3-2)$

$\log 5-\log 5=3\log 1$ $\qquad \log 1=0$

$0=3(0)$ $\qquad 0=0$

$A\,q(x)=\{3\}$

c) $p(x): e^{x} - e^{-x} = 1$

$e^{x}(e^{x} - e^{-x}) = e^{x}(1)$

$e^{2x} - 1 = e^{x}$ $\quad\quad e^{2x} - e^{x} - 1 = 0$

* Hagamos $e^{x} = y$

$y^2 - y - 1 = 0 \quad\quad a = 1 \quad\quad b = -1 \quad\quad c = -1$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$y = \frac{1 \pm \sqrt{1+4}}{2} \quad\quad y = \frac{1 \pm \sqrt{5}}{2}$$

$$y_1 = \frac{1+\sqrt{5}}{2} \quad\quad y_2 = \frac{1-\sqrt{5}}{2}$$

* Luego reemplazamos $y = e^{x}$

$e^{x} = \frac{1+\sqrt{5}}{2}$ $\quad\quad \ln(e)^{x} = \ln\left(\frac{1+\sqrt{5}}{2}\right)$

$x \ln e = \ln\left(\frac{1+\sqrt{5}}{2}\right)$ $\quad\quad x_1 = \ln\left(\frac{1+\sqrt{5}}{2}\right)$

$e^{x} = \frac{1-\sqrt{5}}{2}$ $\quad\quad \ln(e)^{x} = \ln\left(\frac{1-\sqrt{5}}{2}\right)$

$x \ln e = \ln\left(\frac{1-\sqrt{5}}{2}\right)$ $\quad\quad x_2 = \ln\left(\frac{1-\sqrt{5}}{2}\right)$

x_2 no satisface ya que $\frac{1-\sqrt{5}}{2} < 0$.

d) $r(x): 5^{x} - 5^{-x} = 2$

$A\,p(x) = \left\{ \ln\left(\frac{1+\sqrt{5}}{2}\right) \right\}$

$5^{x}(5^{x} - 5^{-x}) = 2(5^{x})$

$5^{2x} - 1 = 2(5^{x})$

$(5)^{2x} - 2(5)^{x} - 1 = 0$

* Hagamos $y = 5^{x}$

$y^{2} - 2y - 1 = 0$ $\qquad a = 1 \qquad b = -2 \qquad c = -1$

$$y = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$$

$$y = \frac{-(-2) \pm \sqrt{(-2)^{2} - 4(1)(-1)}}{2(1)}$$

$$y = \frac{2 \pm \sqrt{4 + 4}}{2} \qquad y = \frac{2 \pm \sqrt{8}}{2}$$

$$y = \frac{2 \pm 2\sqrt{2}}{2} \qquad y = \frac{\cancel{2}(1 \pm \sqrt{2})}{\cancel{2}}$$

$y = 1 \pm \sqrt{2}$

$y_1 = 1 + \sqrt{2}$ $\qquad y_2 = 1 - \sqrt{2}$

* Reemplazamos en $y = 5^{x}$

$5^{x} = 1 + \sqrt{2}$ $\qquad \log_5 (5)^{x} = \log_5 (1 + \sqrt{2})$

$x \log_5 5 = \log_5 (1 + \sqrt{2})$

$x_1 = \log_5 (1 + \sqrt{2})$

$5^x = 1 - \sqrt{2}$ $\qquad \log_5 (5)^x = \log_5 (1 - \sqrt{2})$

$x \log_5 (5) = \log_5 (1 - \sqrt{2})$

$x_2 = \log_5 (1 - \sqrt{2})$

no es solución ya que $1 - \sqrt{2} < 0$, y argumento no puede ser negativo.

$A\, r(x) = \{\log_5 (1 + \sqrt{2})\}$

$h)\ h(x):\ 5^{1+2x} + 6^{1+x} = 30 + 150^x$

$5(5)^{2x} + 6(6)^x = 30 + (5^2 \cdot 6)^x$

$5(5)^{2x} + 6(6)^x = 30 + (5)^{2x}(6)^x$

$5(5)^{2x} - (5)^{2x}(6)^x = 30 - 6(6)^x$

$(5)^{2x}[5 - 6^x] = 6[5 - 6^x]$

$5^{2x} = 6 \qquad 25^x = 6$

$\log_{25} (25)^x = \log_{25} (6)$

$x \log_{25} (25) = \log_{25} (6)$

$x = \log_{25} (6)$

$A\, h(x) = \{\log_{25} (6)\}$

116. a) $10^{x^2+3x} = 200$

$\log(10)^{x^2+3x} = \log(200)$

$(x^2+3x)\log 10 = \log(100 \cdot 2)$

$x^2+3x = \log 100 + \log 2$

$x^2+3x = \log(10)^2 + \log 2$

$x^2+3x = 2\log 10 + \log 2$

$x^2+3x = 2 + \log 2$

$x^2+3x-2-\log 2 = 0$

$a = 1 \qquad b = 3 \qquad c = -2-\log 2$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{(3)^2-4(1)(-2-\log 2)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9+8+4\log 2}}{2}$$

$$x = \frac{-3 \pm \sqrt{9+8+\log(2)^4}}{2}$$

$$x = \frac{-3 \pm \sqrt{17+\log 16}}{2}$$

b) $\ln(x+4) = 5y + \ln C$

$\ln(x+4) = \ln(e)^{5y} + \ln C$

$\ln(x+4) = \ln(e^{5y} \cdot C)$

$x+4 = e^{5y} \cdot C$

$x = Ce^{5y} - 4$

117.- a) $\log_2 4 \times \log_4 6 \times \log_6 8 = -3$

$\Rightarrow \cancel{\log_2 4} \cdot \dfrac{\log_2 6}{\cancel{\log_2 4}} \cdot \dfrac{\log_2 8}{\cancel{\log_2 6}}$

$\Rightarrow \log_2 8 \Rightarrow \log_2 2^3 \Rightarrow 3\cancel{\log_2 2}$

$\Rightarrow 3$; FALSO.

b) $(\log x)^n = n \log x$

$\Rightarrow \log x^n$; FALSO

c) $\ln(1+2+3) = \ln 1 + \ln 2 + \ln 3$

$\Rightarrow \ln(6) \neq \ln 1 + \ln 2 + \ln 3$; FALSO

d) $e^{\ln \sqrt{13}} = 13^{1/2}$

$\Rightarrow \cancel{e}^{\cancel{\log_e} \sqrt{13}} = \sqrt{13}$

$\Rightarrow \sqrt{13} = \sqrt{13}$; VERDADERO

e) $2^{\log_2 2} + \log_4 \left(\frac{1}{16}\right) = 4$

$2 + \log_4 1 - \log_4 16 \Rightarrow 2 + 0 - \log_4 4^2$

$\Rightarrow 2 - 2\cancel{\log_4 4}$

$\Rightarrow 2 - 2$

$\Rightarrow 0$

118.- a) $36^{\log_6 5} + 10^{1-\log 2} - 3^{\log_9 36}$

$\Rightarrow 6^{2\log_6 5} + 10 \times 10^{-\log 2} - 3^{2 \times \frac{1}{2}\log_9 36}$

$\Rightarrow 6^{\log_6 5^2} + 10 \times 10^{\log 2^{-1}} - 9^{\log_9 36^{1/2}}$

$\Rightarrow 5^2 + 10 \times 2^{-1} - 36^{1/2} \Rightarrow 25 + \frac{10}{2} - \sqrt{36}$

$\Rightarrow 25 + 5 - 6 \quad \Rightarrow 24$

b) $81^{\frac{1}{\log_5 3}} - 27^{\log_9 36} - 3^{\frac{4}{\log_7 9}}$

$\frac{1}{\log_5 3} \Rightarrow \frac{1}{\frac{\log_3 3}{\log_3 5}} \Rightarrow \log_3 5$

$\log_9 36 \Rightarrow \frac{\log_3 36}{\log_3 9} \Rightarrow \frac{\log_3 36}{\log_3 3^2} \Rightarrow \frac{\log_3 36}{2\cancel{\log_3 3}} \Rightarrow \frac{\log_3 36}{2}$

$\Rightarrow \frac{1}{2}\log_3 36 \Rightarrow \log_3 36^{1/2} \Rightarrow \log_3 6$

$\frac{4}{\log_7 9} \Rightarrow \frac{4}{\frac{\log_3 9}{\log_3 7}} \Rightarrow \frac{4\log_3 7}{\log_3 9} \Rightarrow \frac{4\log_3 7}{\log_3 3^2} \Rightarrow \frac{\cancel{4}^{2}\log_3 7}{\cancel{2}_{1}\cancel{\log_3 3}}$

$\Rightarrow 2\log_3 7 \Rightarrow \log_3 7^2 \Rightarrow \log_3 49$

$\Rightarrow 81^{\log_3 5} - 27^{\log_3 6} - 3^{\log_3 49} \Rightarrow 3^{4\log_3 5} - 3^{3\log_3 6} - 3^{\log_3 49}$

$\Rightarrow 3^{\log_3 5^4} - 3^{\log_3 6^3} - 49 \Rightarrow 5^4 - 6^3 - 49 \Rightarrow 625 - 216 - 49$

$\Rightarrow 360$

c) $\log_8 \log_4 (\log_2 16)$

$\log_2 16 \Rightarrow \log_2 2^4 \Rightarrow 4\cancel{\log_2 2}^1 \Rightarrow 4$

$\Rightarrow \log_8 \cancel{\log_4 4}^1$

$\Rightarrow \log_8 1 \Rightarrow \underline{0}$

d) $2 - \log_2 \log_3 \sqrt{\sqrt[4]{3}}$

$\Rightarrow 2 - \log_2 (\log_3 3^{1/8})$

$\sqrt{\sqrt[4]{3}} \Rightarrow ((3)^{1/4})^{1/2}$

$\Rightarrow 3^{1/8}$

$\log_3 3^{1/8} \Rightarrow \frac{1}{8}\cancel{\log_3 3}^1$

$\Rightarrow 1/8$

$\Rightarrow 2 - \log_2 (\frac{1}{8})$

$\Rightarrow 2 - (\log_2 1 - \log_2 8) \Rightarrow 2 - (0 - \log_2 2^3)$

$\Rightarrow 2 - 0 + 3\cancel{\log_2 2}^1$

$\Rightarrow 5$

e) $1 + \log(\log \sqrt{\sqrt[5]{10}})$

$\sqrt{\sqrt[5]{10}} \Rightarrow ((10)^{\frac{1}{5}})^{\frac{1}{2}}$

$\Rightarrow 10^{1/10}$

$\Rightarrow 1 + \log(\log 10^{\frac{1}{10}})$

$\Rightarrow \log_{10} 10^{1/10} \Rightarrow \frac{1}{10}\cancel{\log_{10} 10}^1 \Rightarrow \frac{1}{10}$

$\Rightarrow 1 + \log(\frac{1}{10}) \Rightarrow 1 - (\log 1 - \log 10) \Rightarrow 1 - (0 - 1)$

$\Rightarrow 2$

f) $\log\left(11-\log_{\frac{1}{3}}\sqrt{3}\cdot\log_{\sqrt{3}}\left(\frac{1}{\sqrt{3}}\right)\right)$

$\log_{\sqrt{3}}1-\log_{\sqrt{3}}\sqrt{3}$

$\Rightarrow 0-1$

$\Rightarrow -1$

$\log_{\frac{1}{3}}(\sqrt{3})$

$\Rightarrow \dfrac{\log_3\sqrt{3}}{\log_3\left(\frac{1}{3}\right)} \Rightarrow \dfrac{\log_3 3^{1/2}}{\log_3 1-\log_3 3}$

$\Rightarrow \dfrac{\frac{1}{2}\cancel{\log_3 3}^{1}}{0-1} \Rightarrow -\frac{1}{2}$

$\Rightarrow \log\left(11-\left(-\frac{1}{2}\right)(-1)\right) \Rightarrow \log\left(11-\frac{1}{2}\right)$

$\Rightarrow \log\frac{21}{2}$

g) $\log_3 7\cdot\log_7 5\cdot\log_5 4+1$

$\Rightarrow \cancel{\log_3 7}\cdot\dfrac{\cancel{\log_3 5}}{\cancel{\log_3 7}}\cdot\dfrac{\log_3 4}{\cancel{\log_3 5}}+1$

$\Rightarrow \log_3 4+1$

119.- $\dfrac{1}{\log_a n}+\dfrac{1}{\log_{a^2}n}+\dfrac{1}{\log_{a^3}n}+\dfrac{1}{\log_{a^4}n}+\dfrac{1}{\log_{a^5}n}=15\log_n a$

$\Rightarrow \dfrac{\log_n a}{\cancel{\log_n n}1}+\dfrac{\log_n a^2}{\cancel{\log_n n}1}+\dfrac{\log_n a^3}{\cancel{\log_n n}1}+\dfrac{\log_n a^4}{\cancel{\log_n n}1}+\dfrac{\log_n a^5}{\cancel{\log_n n}1}$

$\Rightarrow \log_n a + 2\log_n a + 3\log_n a + 4\log_n a + 5\log_n a$

$\Rightarrow 15\log_n a$

120.-

a) $\log_2 5 = a$ $\quad \log_{100} 40 \times \frac{10}{10} \Rightarrow \log_{100} \frac{400}{10}$

$\Rightarrow \log_{100} 400 - \log_{100} \sqrt{10^2}$

$\Rightarrow \log_{100}(4 \times 100) - \log_{100} \sqrt{100}$

$\Rightarrow \log_{100} 4 + \cancel{\log_{100}} 100^{1} - \frac{1}{2} \cancel{\log_{100}} 100^{1}$

$\Rightarrow \log_{100} 4 + 1 - \frac{1}{2}$

$\Rightarrow \frac{\log_2 4}{\log_2 100} + \frac{1}{2} \Rightarrow \frac{\log_2 2^2}{\log_2 (25 \times 4)} + \frac{1}{2}$

$\Rightarrow \frac{2\log_2 2}{\log_2 25 + \log_2 4} + \frac{1}{2} \Rightarrow \frac{2}{\log_2 5^2 + \log_2 2^2} + \frac{1}{2}$

$\Rightarrow \frac{2}{2\log_2 5 + 2\log_2 2} + \frac{1}{2} \Rightarrow \frac{2}{2a+2} + \frac{1}{2}$

$\Rightarrow \frac{\cancel{2}}{\cancel{2}(a+1)} + \frac{1}{2} \Rightarrow \frac{2+a+1}{2(a+1)}$

$\Rightarrow \frac{3+a}{2(a+1)}$

b) $\log_6 2 = a \quad \wedge \quad \log_6 5 = b$

$$\log_3 5 \Rightarrow \frac{\log_6 5}{\log_6 3} \Rightarrow \frac{b}{\log_6\left(\frac{6}{2}\right)} \Rightarrow \frac{b}{\cancel{\log_6 6}^{1} - \log_6 2}$$

$$\Rightarrow \frac{b}{1-a}$$

c) $\log_3 20 \quad \wedge \quad \log_3 15 = b$

$$\log_2 360 \Rightarrow \frac{\log_3(360)}{\log_3 2} \Rightarrow \frac{\log_3(40 \times 9)}{\log_3 2} \Rightarrow \frac{\log_3 40 + \log_3 9}{\log_3 2}$$

$$\Rightarrow \frac{\log_3(2 \times 20) + \log_3 3^2}{\log_3 2} \Rightarrow \frac{\log_3 2 + \overset{a}{\log_3 20} + 2\,\cancel{\log_3 3}^{1}}{\log_3 2} \Rightarrow \frac{\log_3 2 + a + 2}{\log_3 2}$$

$$\Rightarrow \frac{\frac{a-b+1}{2} + a + 2}{\frac{a-b+1}{2}} \Rightarrow \frac{\frac{a-b+1+2a+4}{\cancel{2}}}{\frac{a-b+1}{\cancel{2}}} \Rightarrow \frac{3a-b+5}{a-b+1}$$

$\log_3 225 = \log(25 \times 9)$

$\log_3 15^2 = \log_3 5^2 + \log_3 3^2$

$\cancel{2}\log_3 15 = \cancel{2}\log_3 5 + \cancel{2}\,\cancel{\log_3 3}^{1}$

$b = \log_3 5 + 1$

$(\log_3 5 = b-1)$

$a = \log_3 20$

$a = \log_3(5 \times 4)$

$a = \log_3 5 + \log_3 2^2$

$a = b - 1 + 2\log_3 2$

$\frac{a-b+1}{2} = \log_3 2$

d) $\log_6(28(6^{1-2a}))$ $\quad \log_6 2 = \frac{a}{4}$ $\quad \log_6 7 = \frac{3a}{2}$

$\Rightarrow \log_6 28 + \log_6 6^{1-2a}$

$\Rightarrow \log_6(7\times 4) + (1-2a)\cancel{\log_6 6}^{1} \Rightarrow \log_6 7 + \log_6 4 + 1 - 2a$

$\Rightarrow \frac{3a}{2} + \log_6 2^2 + 1 - 2a \Rightarrow 1 - \frac{a}{2} + 2\log_6 2$

$\Rightarrow 1 - \frac{a}{2} + 2\left(\frac{a}{4}\right) \Rightarrow 1 - \cancel{\frac{a}{2}} + \cancel{\frac{a}{2}}$

$\Rightarrow 1$

e) $\log_{m^2}(mn^3)$ $\quad \therefore \log_a m = x \quad ; \log_a n = y$

$\Rightarrow \frac{\log_a(mn^3)}{\log_a m^2} \Rightarrow \frac{\log_a m + \log_a n^3}{2\log_a m} \Rightarrow \frac{x+3y}{2x}$

21.- $3^x(4^{2x+1}) = 6^{x+2}$

$\ln(3^x(4^{2x+1})) = \ln 6^{x+2}$

$\ln 3^x + \ln 4^{2x+1} = (x+2)\cdot \ln 6$

$x\cdot \ln 3 + (2x+1)\ln 4 = (x+2)\ln 6$

$x\ln 3 + 2x\ln 4 + \ln 4 = x\ln 6 + 2\ln 6$

$x\ln 3 + x\ln 4^2 - x\ln 6 = 2\ln 6 - \ln 4$

$x(\ln 3 + \ln 16 - \ln 6) = \ln 6^2 - \ln 4$

$x\ln\left(\frac{3\times 16}{6}\right) = \ln\left(\frac{36}{4}\right)$

$x\ln 8 = \ln 9$

$x = \frac{\ln 9}{\ln 8}$

122.- $\log_{a^2}\sqrt{108}$ $\qquad \log_a 2 = \frac{5}{2}$, $\log_a 3 = \frac{1}{3}$

$$\Rightarrow \frac{\log_a \sqrt{108}}{\log_a a^2} \Rightarrow \frac{\log_a 108^{1/2}}{2\cancel{\log_a a}^{1}} \Rightarrow \frac{\frac{1}{2}\log_a(12\times 9)}{2}$$

$$\Rightarrow \frac{1}{4}\log_a(3\times 2^2\times 3^2) \Rightarrow \frac{1}{4}\log_a(3^3\times 2^2) \Rightarrow \frac{1}{4}\log_a(3^3)+\frac{1}{4}\log_a(2^2)$$

$$\Rightarrow \frac{3}{4}\log_a 3 + \frac{2}{4}\log_a 2 \Rightarrow \frac{\cancel{3}}{4}\left(\frac{1}{\cancel{3}}\right)+\frac{1}{2}\left(\frac{5}{2}\right)$$

$$\Rightarrow \frac{1}{4}+\frac{5}{4} \Rightarrow \frac{\cancel{6}^{3}}{\cancel{4}_{2}} = \frac{3}{2}$$

123) $2^x + \left(\frac{1}{2}\right)^{2x-3} - 6\left(\frac{1}{2}\right)^x = 1$

$2^x + (2)^{3-2x} - 6(2)^{-x} = 1$

$2^x + \frac{8}{2^{2x}} - \frac{6}{2^x} = 1$ $\qquad * \boxed{2^x = a} *$

$a + \frac{8}{a^2} - \frac{6}{a} = 1$ $\qquad a^3 + 8 - 6a = a^2$

$a^3 - a^2 - 6a + 8 = 0$

$$\begin{array}{l|l} a^3 - a^2 - 6a + 8 & a - 2 \\ \hline -a^3 + 2a^2 & a^2 + a - 4 \\ \quad a^2 - 6a & \\ \quad -a^2 + 2a & \\ \qquad -4a + 8 & \\ \qquad 4a - 8 & \\ \qquad // \; // & \end{array}$$

$= (a-2)(a^2+a-4) = 0$

$\boxed{a_1 = 2}$

Resolviendo $a^2 + a - 4$ $\qquad a = 1 \quad b = 1 \quad c = -4$

$a = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad a = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-4)}}{2(1)}$

$a = \frac{-1 \pm \sqrt{1+16}}{2}$ $\qquad a = \frac{-1 \pm \sqrt{17}}{2}$

$\boxed{a_2 = \frac{-1+\sqrt{17}}{2}}$ $\qquad \boxed{a_3 = \frac{-1-\sqrt{17}}{2}}$

Reemplazo en $* \boxed{2^x = a} *$

Con $a = \frac{-1+\sqrt{17}}{2}$

$2^x = a \qquad 2^x = \frac{-1+\sqrt{17}}{2} \qquad \log_2(2)^x = \log_2\left(\frac{-1+\sqrt{17}}{2}\right)$

$x \log_2(2) = \log_2\left(\frac{-1+\sqrt{17}}{2}\right) \qquad \boxed{x = \log_2\left(\frac{-1+\sqrt{17}}{2}\right)}$

Con $a = \frac{-1-\sqrt{17}}{2}$

$2^x = a \qquad 2^x = \frac{-1-\sqrt{17}}{2} \qquad \log_2(2)^x = \log_2\left(\frac{-1-\sqrt{17}}{2}\right)$ Imposible

124.- una de las alternativas no es correcta, identifíquela

a) $F(x) = \ln(x-1)$ $\qquad \operatorname{dom} F = (1, \infty)$

verdadero ya que $x-1 > 0 \longrightarrow x > 1$

b) $F(x) = \frac{1}{\log_{x+1} 2}$ $\qquad \operatorname{dom} F = (-1, 0) \cup (0, \infty)$

verdadero ya que $x+1 > 0$ y $x \neq 0$

c) $F(x) = \frac{1}{|x| - x}$ $\qquad \operatorname{dom} F = \mathbb{R} - \{0\}$

$\boxed{\text{Falso}}$ ya que $|x| - x = 0$

d) $F(x) = \frac{x^8 - x^3 + x - \sqrt{2}}{\log(x^2+1)}$; $\operatorname{dom} F = \mathbb{R} - \{0\}$

verdadero ya que $x^2+1 \neq 1$

e) $F(x) = \log(x[u(x)])$; $\operatorname{dom} F = \mathbb{R}^+$

verdadero ya que $u(x) = 1 \wedge x > 0$

125.-

a) $5 > 3$

b) $5 \ln\left(\frac{1}{2}\right) > 3 \ln(1/2)$

ERROR; En una desigualdad al multiplicar por un número negativo $(\ln 1/2)$, entonces varía el símbolo.

126.-

$$f(x) = 2e^{x-3}$$

$$y = 2e^{x-3}$$

$$\Rightarrow \ln y = \ln(2e^{x-3})$$

$$\Rightarrow \ln y = \ln 2 + \ln e^{x-3}$$

$$\Rightarrow \ln y - \ln 2 = (x-3)\cancel{\ln e}$$

$$\Rightarrow \ln\left(\frac{y}{2}\right) + 3 = x$$

$$\Rightarrow f(x)^{-1} = \ln\left(\frac{x}{2}\right) + 3$$

127.-

$$f(x) = \begin{cases} e^x & ; x > 0 \\ 1 - x^2 & ; x \leq 0 \end{cases}$$

rango: $(1, +\infty)$

$$y = e^x$$

$$\ln y = \ln(e^x)$$

$$\ln y = x \cancel{\ln e} \Rightarrow f(x)^{-1} = \ln x \; ; x > 1$$

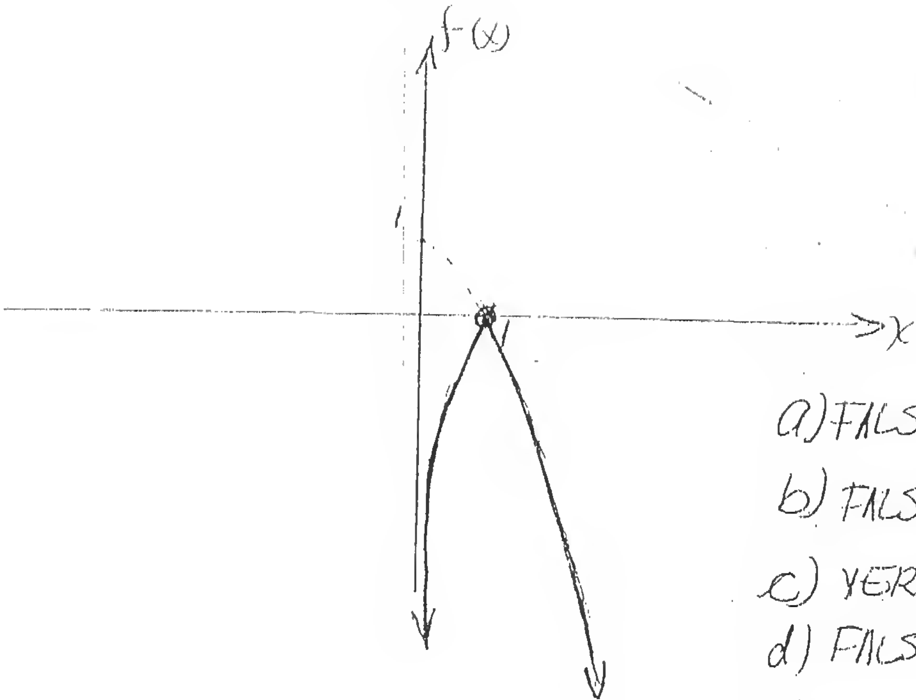
$$y = 1 - x^2$$

$$x^2 = 1 - y$$

$$x = \sqrt{1-y} \Rightarrow f(x)^{-1} = \sqrt{1-y} \; ; x \leq$$

c) correcta

128.- $f(x)=\begin{cases} 1-x^2 & ; x\geq 1 \\ \ln x & ; x<1 \end{cases}$



a) FALSO

b) FALSO

c) VERDADERO

d) FALSO

e) FALSO

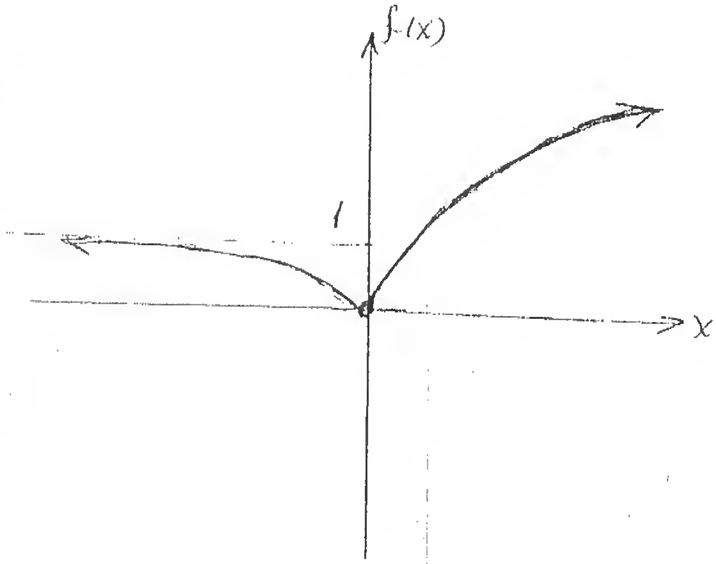
129.- $f(x)=\begin{cases} \ln(x+1) & ; x\geq 0 \\ 1-e^x & ; x<0 \end{cases}$

$y=\ln(x+1)$

asíntota

$x+1=0$

$x=-1$



130.- $\log_a(MN) = \log_a(M) + \log_a(N)$

$$x = \log_a M \quad ; \quad y = \log_a N$$

$$M = a^x \quad ; \quad N = a^y$$

$$M \times N = a^x \cdot a^y$$

$$MN = a^{x+y}$$

$$\log_a(MN) = x + y \Rightarrow \log_a(MN) = \log_a M + \log_a N$$

131.- a) $p(x)$: $\log(x+4) + \log(2x+3) = \log(1-2x)$

$$\cancel{\log}[(x+4)(2x+3)] = \cancel{\log}(1-2x)$$

$$2x^2 + 3x + 8x + 12 = 1 - 2x$$

$$2x^2 + 11x + 2x + 12 - 1 = 0$$

$$2x^2 + 13x + 11 = 0$$

$$(2x+11)(2x+2) = 0$$

$$\cancel{x = -11/2} \qquad x = -1$$

NO

b) $p(x)$: $\ln\left(\frac{x}{x-1}\right) + \ln\left(\frac{x+1}{x}\right) - \ln(x^2-1) + 2 = 0$

$$\ln\left(\frac{\frac{\cancel{x}}{x-1} \cdot \frac{x+1}{\cancel{x}}}{\frac{x^2-1}{1}}\right) = -2 \Rightarrow \ln\left(\frac{(x+1)}{(x-1)(x^2-1)}\right) = -2$$

$$\Rightarrow \ln\left(\frac{\cancel{x+1}}{(x-1)(x-1)\cancel{(x+1)}}\right) = -2 \Rightarrow e^{-2} = \frac{1}{(x-1)^2}$$

$$\Rightarrow \frac{1}{e^2} = \frac{1}{(x-1)^2} \Rightarrow \sqrt{(x-1)^2} = \sqrt{e^2}$$

$$x-1 = {}^{+}e$$

$$x = 1^{+}e$$

c) $r(x): (\log_2 x)^2 = \log_2 x^{2}$

$$a = \log_2 x$$

$$a^2 = 2a$$

$$a^2 - 2a = 0$$

$$a(a-2) = 0$$

$$\{a = 0 \wedge a = 2\}$$

$$\log_2 x = 0 \qquad \log_2 x = 2$$

$$x = 2^0 \qquad x = 2^2$$

$$x = 1 \qquad x = 4$$

d) $h(x): \cancel{\log_3}(x^2-3x-5) = \cancel{\log_3}(7-2x)$

$$x^2 - 3x - 5 = 7 - 2x$$

$$x^2 - x - 12 = 0$$

$$(x-4)(x+3) = 0$$

$$\cancel{x=4} \vee x = -3$$

e) $m(x): \log_2(9^{x-1}+7) = 2 + \log_2(3^{x-1}+1)$

$$\log_2(9^{x-1}+7) - \log_2(3^{x-1}+1) = 2$$

$$\Rightarrow \log_2\left(\frac{9^{x-1}+7}{3^{x-1}+1}\right) = 2$$

$$\frac{9^{x-1}+7}{3^{x-1}+1} = 4 \Rightarrow 9^{x-1}+7 = 4(3^{x-1}+1)$$

$$\Rightarrow 9^{x-1}+7 = 4(3^{x-1})+4$$

$$\Rightarrow 9^{x-1} - 4(3^{x-1}) + 3 = 0$$

$$\Rightarrow 9^{x} \cdot 9^{-1} - 4(3^{x-1}) + 3 = 0 \Rightarrow 3^{2x} \cdot 3^{-2} - 4(3^{x-1}) + 3 = 0$$

$$\Rightarrow 3^{2x-2} - 4(3^{x-1}) + 3 = 0 \Rightarrow 3^{2(x-1)} - 4(3^{x-1}) + 3 = 0$$

$$a = 3^{x-1}$$

$$\Rightarrow a^2 - 4a + 3 = 0 \Rightarrow (a-3)(a-1) = 0 \Rightarrow \boxed{a=3} \wedge \boxed{a=1}$$

$$\cancel{3}^{x-1} = \cancel{3}^{1}$$

$$x-1 = 1$$

$$x = 2$$

$$3^{x-1} = 1$$

$$\Rightarrow \cancel{3}^{x-1} = \cancel{3}^{0}$$

$$x-1 = 0$$

$$x = 1$$

f) $p(x): \log_5(5^{1/x}+125) = \log_5 6 + 1 + \frac{1}{2x}$

$\Rightarrow \log_5(5^{1/x}+125) - \log_5 6 = \frac{2x+1}{2x}$

$\Rightarrow \log_5\left(\frac{5^{1/x}+125}{6}\right) = \frac{2x+1}{2x}$

$\Rightarrow 5^{\frac{2x+1}{2x}} = \frac{5^{1/x}+125}{6} \Rightarrow 6 \times 5^{1+\frac{1}{2x}} = 5^{\frac{1}{x}}+125$

$\Rightarrow (6 \times 5)\, 5^{\frac{1}{2x}} = 5^{\frac{1}{x}}+125 \Rightarrow 30(5^{\frac{1}{x}})^2 - 5^{\frac{1}{x}} - 125 = 0$

$a = 5^{1/x}$

$b^2 = a$
$b = a^{1/2}$

$30a^{1/2} - a - 125 = 0$

$-b^2 + 30b - 125 = 0$

$b^2 - 30b + 125 = 0 \Rightarrow (b-25)(b-5) = 0$

$b = 25 \quad \wedge \quad b = 5$

$(a^{1/2})^2 = (25)^2 \quad \wedge \quad (a^{1/2})^2 = (5)^2$

$a = 625 \quad \wedge \quad a = 25$

$\cancel{5}^{\frac{1}{x}} = \cancel{5}^4 \qquad \cancel{5}^{\frac{1}{x}} = \cancel{5}^2$

$\frac{1}{x} = 4 \qquad \frac{1}{x} = 2$

$x = \frac{1}{4} \qquad x = \frac{1}{2}$

g) $q(x): \log^2 x + \log x + 1 = \dfrac{7}{\log\left(\frac{x}{10}\right)}$

$\Rightarrow \log^2(x) + \log x + 1 - \dfrac{7}{\log x - \cancel{\log 10}^{1}} = 0$

$\Rightarrow \log^2(x) + \log x + 1 - \dfrac{7}{\log x - 1} = 0 \quad a = \log x$

$\Rightarrow a^2 + a + 1 - \dfrac{7}{a-1} = 0 \Rightarrow \dfrac{a^2(a-1) + (a+1)(a-1) - 7}{(a-1)} = 0$

$\Rightarrow a^3 \cancel{-a^2} \cancel{+a^2} - 1 - 7 = 0 \Rightarrow a^3 - 8 = 0$

$a^3 = 8$

$\boxed{a = 2}$

$\log x = 2$

$x = 10^2$

$x = 100$

h) $r(x): \log^2 x^3 - \log(0{,}1\, x^{10}) = 0$

$\Rightarrow (\log x^3)^2 - \log\left(\dfrac{x^{10}}{10}\right) = 0$

$\Rightarrow (3\log x)^2 - (\log x^{10} - \cancel{\log 10}^{1}) = 0$

$\Rightarrow 9\log^2 x - 10\log x + 1 = 0$

$a = \log x$

$\Rightarrow 9a^2 - 10a + 1 = 0$

$(9a - 9)(9a - 1) = 0$

$\boxed{a = 1} \quad \boxed{a = 1/9}$

$\log x = 1 \quad \log x = 1/9$

$x = 10 \quad x = 10^{1/9}$

i) $m(x)$: $\log_{\frac{1}{2}x} x^2 - 14\log_{16x} x^3 + 40\log_{4x}\sqrt{x} = 0$

$$\Rightarrow \frac{\log_x x^2}{\log_x\left(\frac{x}{2}\right)} - 14\frac{\log_x x^3}{\log_x(16x)} + 40\frac{\log_x\sqrt{x}}{\log_x(4x)} = 0$$

$$\Rightarrow \frac{\log_x x^2}{\log_x x - \log_x 2} - \frac{14\log_x x^3}{\log_x 16 + \log_x x} + 40\frac{\log_x x^{1/2}}{\log_x 4 + \log_x x} = 0$$

$$\Rightarrow \frac{2\log_x x}{\log_x x - \log_x 2} - \frac{42\log_x x}{\log_x 2^4 + \log_x x} + 40\cdot\frac{1}{2}\frac{\log_x x}{\log_x 2^2 + \log_x x} = 0$$

$$a = \log_x x \quad ; \quad b = \log_x 2$$

$$\Rightarrow \frac{2a}{a-b} - \frac{42a}{a+4b} + \frac{20a}{a+2b} = 0$$

$$\Rightarrow \frac{2a(a+2b)(a+4b) - 42a(a-b)(a+2b) + 20a(a-b)(a+4b)}{(a-b)(a+4b)(a+2b)} = 0$$

$$2a[(a+2b)(a+4b) - 21(a-b)(a+2b) + 10(a-b)(a+4b)] = 0$$

$$2a[a^2+4ab+2ab+8b^2 - 21(a^2+2ab-ab-2b^2) + 10(a^2+4ab-ab-4b^2)] = 0$$

$$2a[a^2+6ab+8b^2 - 21a^2 - 21ab + 42b^2 + 10a^2 + 30ab - 40b^2] = 0$$

$$2a(-10a^2+15ab+10b^2) = 0$$

$$-10a(2a^2-3ab-2b^2) = 0 \Rightarrow \boxed{a=0} \quad (2a-4b)(2a+b) = 0$$

$$(a=2b) \quad \boxed{a=-b/2}$$

$\log_x x = 0$

$x = 1$

$\log_x x = 2\log_x 2$

$\log_x x = \log_x 2^2$

$x = 4$

$\log_x x = -\frac{1}{2}\log_x 2$

$\cancel{\log_x} x = \cancel{\log_x} 2^{-1/2}$

$x = 2^{-1/2}$

$x = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$x = \frac{\sqrt{2}}{2}$

132.- $p(x): \log_{\frac{1}{4}} x - \frac{1}{\log_{\frac{1}{4}} x} - \frac{3}{2} = 0$

$a = \log_{\frac{1}{4}} x$

$\Rightarrow a - \frac{1}{a} - \frac{3}{2} = 0 \Rightarrow \frac{2a^2 - 2 - 3a}{2a} = 0$

$\Rightarrow 2a^2 - 3a - 2 = 0 \Rightarrow (2a-4)(2a+1) = 0$

$2a = 4$ $\qquad$ $2a = -1$

$a = 2$ $\qquad$ $a = -1/2$

$\log_{\frac{1}{4}} x = 2$ $\qquad$ $\log_{\frac{1}{4}} x = -\frac{1}{2}$

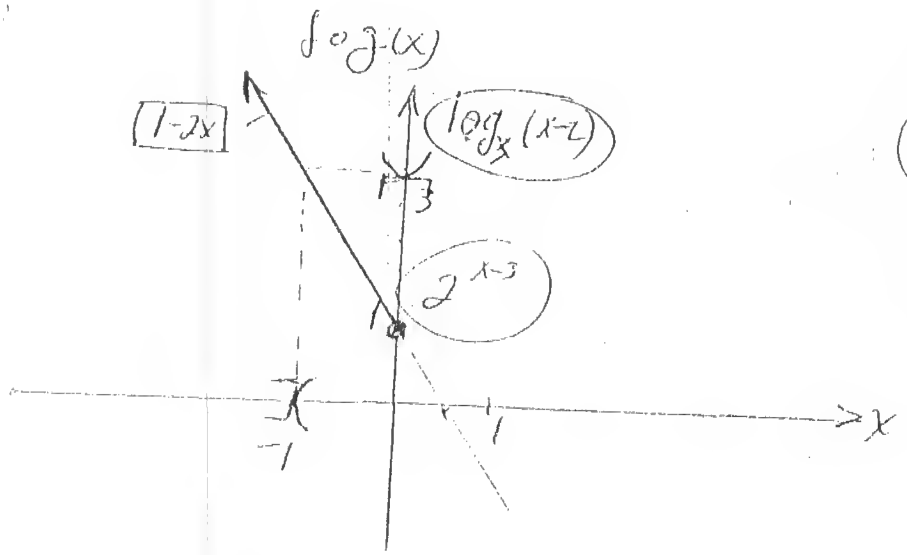
$x = \left(\frac{1}{4}\right)^2$ $\qquad$ $x = \left(\frac{1}{4}\right)^{-1/2} \Rightarrow x = (4)^{1/2}$

$x = \frac{1}{16}$ $\qquad$ $x = 2$

Suma : $\frac{33}{16}$

133.- $f(x) = \begin{cases} 2^{x-3} & ; x \leq 3 \\ \log_3(x-2) & ; x > 3 \end{cases}$ $\qquad g(x) = 1 - 2x \quad ; x \leq 0$

$f \circ g(x)$



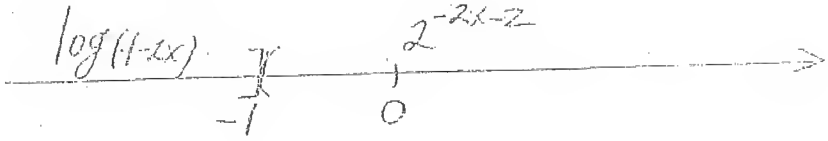
$y = 1 - 2x$

$x = 0$, $y = 1$ $\qquad$ $y = 0$, $x = 1/2$

$3 = 1 - 2x$

$2x = 1 - 3$

$x = -1$



$\log_3(1-2x-2) \Rightarrow 2^{1-2x-3}$

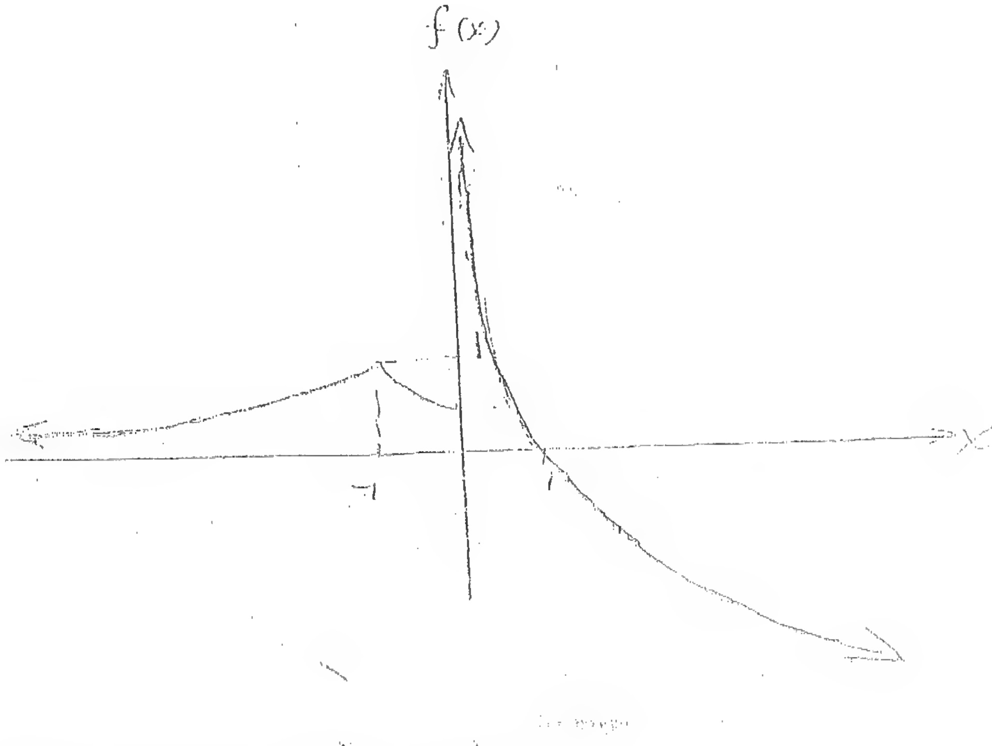
$\Rightarrow 2^{-2x-2}$

$\log_3(-2x-1)$

$$(f\circ g)(x) = \begin{cases} \log_3(-1-2x) & ; x \leq -1 \\ 2^{-2x-2} & ; -1 < x \leq 0 \end{cases}$$

b) correcta

134.- $f(x) = \begin{cases} 2^{-|x+1|} & ; x \leq 0 \\ \log_{\frac{1}{2}} |x| & ; x > 0 \end{cases}$



135.- $p(x)$: $\text{sgn}(\underbrace{\ln[|x|-1|}) = -1$

dominio signo $(x<0)$

$y = \ln||x|-1|$

Asintotas

$||x|-1| = 0$

$|x| = 1$

$\boxed{x = \pm 1}$

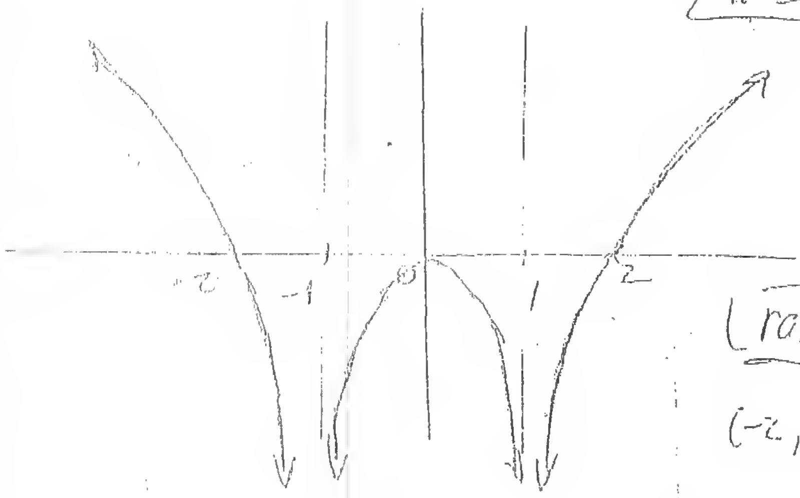
Raices

$||x|-1| = 1$

$|x|-1 = 1 \quad -|x|+1 = 1$

$|x| = 2 \quad |x| = 0$

$\boxed{x = \pm 2} \quad \boxed{x = 0}$



rango negativo

$(-2,-1) \cup (-1,0) \cup (0,1) \cup (1,2)$

136.-

$$\frac{f(-2) - f(a)}{f^{-1}(1)} \Rightarrow \frac{e^{-2}+1-\ln 3}{-\frac{1}{2}}$$

$y = 2x$

$x = \frac{y}{2}$

$f(x)^{-1} = \frac{x}{2}$

$\Rightarrow -2\left(e^{-2}+1-\ln 3\right)$

$\Rightarrow 2\ln 3 - 2 - 2e^{-2}$

$\Rightarrow \ln 3^2 - 2 - 2e^{-2} \Rightarrow \ln 9 - 2 - 2e^{-2}$

137.- a) $\log_{\frac{1}{2}}\left(\frac{2x^2-4x-6}{4x-11}\right) \leq -1$

$$\frac{2x^2-4x-6}{4x-11} \leq \left(\frac{1}{2}\right)^{-1}$$

P. critico

$4x-11=0$

$4x=11$

$x = \frac{11}{4}$

$$\frac{2x^2-4x-6}{(4x-11)} \leq 2$$

$2x^2-4x-6 \leq 2(4x-11)$

$2x^2-4x-6-8x+22 \leq 0$

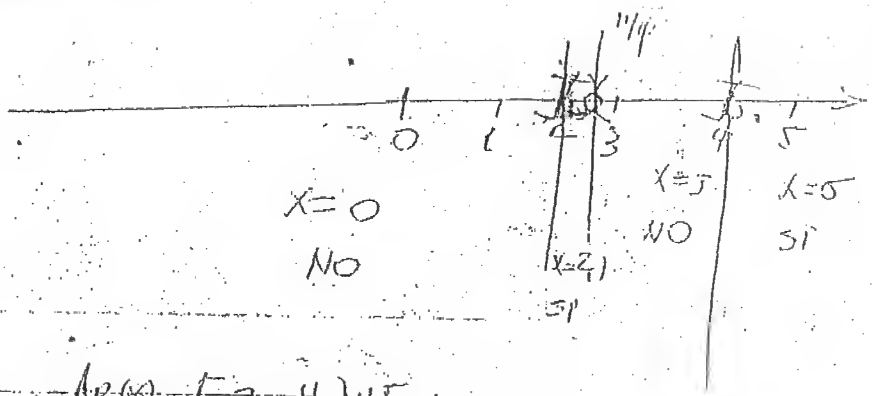
$2x^2-12x+16 \leq 0$

$2(x^2-6x+8) \leq 0$

$x^2-6x+8 \leq 0$

$(x-4)(x-2) \leq 0$

$\boxed{x \leq 4} \quad \boxed{x \leq 2}$



~~$Dp(x) = [2, \frac{11}{4}) \cup [4, +\infty)$~~

b) $\cancel{\log_{0.2}}\left(\frac{4}{x+3}\right) > \cancel{\log_{0.2}}(2-x)$

$\frac{4}{x+3} > 2-x \Rightarrow \frac{4}{x+3} - 2 + x > 0 \Rightarrow \frac{4+(-2+x)(x+3)}{x+3} > 0$

$\frac{4-2x-6+x^2+3x}{x+3} > 0 \Rightarrow \frac{x^2+x-2}{x+3} > 0$

Punto critico

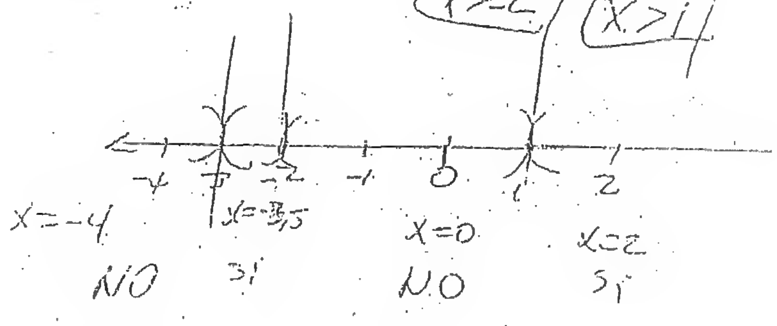
$x+3=0$

$x=-3$

$x^2+x-2>0$

$(x+2)(x-1)>0$

$x>-2$ / $x>1$



$Dp(x): (-3, 2) \cup (2, +\infty)$

) $\log_{\frac{1}{2}}(x^3+8) - \frac{1}{2}\log_{\frac{1}{2}}(x^2+4x+4) \leq \log_{\frac{1}{2}}(x+58)$

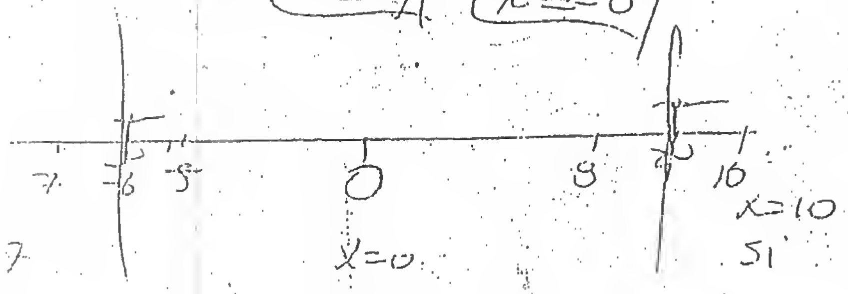
$\log_{\frac{1}{2}}(x^3+8) - \log_{\frac{1}{2}}\sqrt{(x+2)^{\cancel{2}}} \leq \log_{\frac{1}{2}}(x+58)$

$\cancel{\log_{\frac{1}{2}}}\left(\frac{x^3+8}{x+2}\right) \leq \cancel{\log_{\frac{1}{2}}}(x+58)$

$\Rightarrow \frac{\cancel{(x+2)}(x^2-2x+4)}{\cancel{(x+2)}} \leq x+58 \Rightarrow x^2-2x+4-58 \leq 0$

$\Rightarrow x^2-3x-54 \leq 0 \Rightarrow (x-9)(x+6) \leq 0$

$x \leq 9 \quad x \leq -6$



$Ap(x): [9, +\infty)$

) $\cancel{\log_{x-2}}(2x-3) \geq \cancel{\log_{x-2}}(24-6x)$

$2x-3 \geq 24-6x$

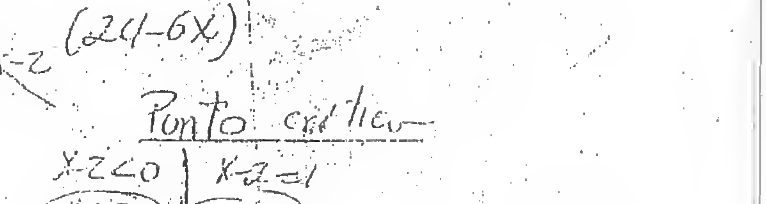
$2x+6x \geq 24+3$

$8x \geq 27$

$x \geq 27/8$

Punto critico

$x-2<0$	$x-2=1$
$x<2$	$x=3$



Sol $(2, 3) \cup (\frac{27}{8}, 4]$

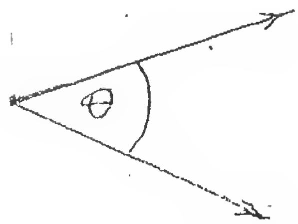
100

SOLUCIÓN EJERCICIOS PROPUESTOS

CAPÍTULO CUATRO

TRIGONOMETRÍA

1.-



ES LA REGION DE UN PLANO COMPRENDIDO ENTRE LOS SEGMENTOS DE RECTA.

b) CORRECTO

2.- b) CORRECTO, ESTA DEFINIDO POR EL VERTICE Y LOS SEGMENTOS DE RECTA

3.- a) CORRECTO

4.- $\alpha+\theta=180°$; NO PUEDEN SER AGUDOS.

b) CORRECTO

5.- DOS ANGULOS OPUESTOS POR EL VERTICE SON IGUALES

b) CORRECTO

6.- a) $\cancel{30}° \times \frac{\pi rad}{\cancel{180°}_{6}} = \frac{\pi}{6}$ radian

b) $\cancel{135}^{3}° \times \frac{\pi rad}{\cancel{180°}_{4}} = \frac{3\pi}{4}$ radian

c) $-\cancel{120}^{2}° \times \frac{\pi rad}{\cancel{180°}_{3}} = -\frac{2\pi}{3}$ radian

d) $\cancel{450}^{5}° \times \frac{\pi rad}{\cancel{180}_{2}°} = \frac{5\pi}{2}$ radian

e) $-\cancel{540}^{3}° \times \frac{\pi rad}{\cancel{180}_{1}°} = 3\pi$ radian

f) $\cancel{60}^{1}° \times \frac{\pi rad}{\cancel{180}_{3}°} = \frac{\pi}{3}$ radian

7.—

a) $\frac{\pi}{\cancel{6}_{1}} \times \frac{\cancel{180}^{30}°}{\cancel{\pi rad}} = 30°$

b) $-\frac{5\pi}{\cancel{4}_{1}} \times \frac{\cancel{180}^{45}°}{\cancel{\pi rad}} = -225°$

c) $\frac{4\pi}{\cancel{3}_{1}} \times \frac{\cancel{180}^{60}°}{\cancel{\pi rad}} = 240°$

d) $\frac{\pi}{\cancel{2}_{1}} \times \frac{\cancel{180}^{90}°}{\cancel{\pi rad}} = 90°$

e) $\frac{\pi}{\cancel{12}_{3/1}} \times \frac{\cancel{180}^{30}°}{\cancel{\pi rad}_{1}} = 15°$

f) $4\pi \times \frac{180°}{\pi rad} = 720°$

8.—

radian	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$\frac{2\pi}{3}$	$3\pi/4$	$\frac{28\pi}{45}$	$\frac{5\pi}{6}$	$\frac{\pi}{12}$
grado	0	30	45	60°	90	120	135	112°	150°	15°

$\cancel{112}^{28}° \times \frac{\pi rad}{\cancel{180}_{45}°} = \frac{28\pi}{45}$ | $\cancel{150}^{5}° \times \frac{\pi rad}{\cancel{180}_{6}°} = \frac{5\pi}{6}$

9.-

$$1\,hora \Rightarrow \frac{\overset{30}{\cancel{360}}}{\underset{1}{\cancel{12}}} = 30^\circ$$

ángulo $\Rightarrow$ 1hora = 5 minutos = 30°

$$1\,minuto = \frac{30}{5} = 6^\circ$$

$$3\,\cancel{minutos} \times \frac{6^\circ}{1\,\cancel{minutos}} = \overset{1}{\cancel{18^\circ}} \times \frac{\pi\,rad}{\underset{10}{\cancel{180^\circ}}} = \frac{\pi}{10}\,rad$$

$$l = \theta R$$

$$\frac{7\cancel{\pi}}{\cancel{10}} = \frac{\cancel{\pi}}{\cancel{10}} R$$

$$R = 7cm$$

10.- Si x y y son ángulos suplementarios

$$x + y = 180^\circ \quad (-1)$$

x → ángulo
y → suplemento

$$y = 4(90 - x)$$

$$y = 360 - 4x$$

$$4x + y = 360$$

$$-x - y = -180$$

$$4x + y = 360$$

$$3x = 180$$

$$x = \frac{180}{3}$$

$$x = 60^\circ$$

11.- ángulos congruentes (iguales)

$$8X = 180^\circ$$

$$\boxed{X = \frac{45^\circ}{2}}$$

$$\frac{\cancel{45^\circ}^{\,1}}{2} \times \frac{\pi\,\text{rad}}{\cancel{180^\circ}_{\,4}} = \frac{\pi}{8}\,\text{rad}$$

12.-

$$180 - X = 123$$

$$180 - 123 = X$$

a) $X = 57^\circ$

b) ángulo complementario $90 - 57$

$\Rightarrow 33^\circ$

13.-

a) $\text{sen}\,30^\circ \times \cancel{\cos 90}^{\,0} \left(-\cos\frac{7\pi}{6}\right)\tan\frac{3\pi}{4}$

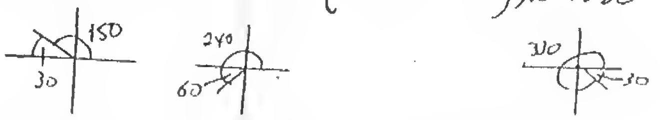
$\Rightarrow 0$

b) $\text{sen}\,\frac{5\pi}{6} \times \cos\frac{4\pi}{3}\left(-\tan\frac{\pi}{6}\right)\tan 330$

$\text{sen}\,150 \times \cos 240\,(-\tan 30)\tan 330$

$\frac{\pi}{6} = 30^\circ$

$\frac{5\pi}{6} = 150$



$\frac{\pi}{3} = 60$

$4\left(\frac{\pi}{3}\right) = 240$

$\text{sen}\,30 \times (-\cos 60)(-\tan 30)(-\tan 30)$

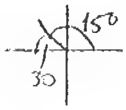
$\frac{1}{2} \times \left(-\frac{1}{2}\right)\left(-\frac{\sqrt{3}}{3}\right)\left(-\frac{\sqrt{3}}{3}\right)$

$\Rightarrow -\frac{3}{36} \Rightarrow -\frac{1}{12}$

c) $3\cos\left(\frac{\pi}{6}\right) + \sin\left(\frac{5\pi}{6}\right)\tan\frac{\pi}{3}$ $\qquad \frac{\pi}{6}=30$

$5\left(\frac{\pi}{6}\right)=150$

$3\cos 30 + \sin 150 \cdot \tan 60$



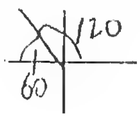
$3\cos 30 + \sin 30 \tan 60$

$3\frac{\sqrt{3}}{2} + \frac{1}{2}\times\sqrt{3} \Rightarrow \frac{4}{2}\sqrt{3}$

$\Rightarrow 2\sqrt{3}$

d) $\tan\left(\frac{\pi}{6}\right) - \cos\frac{2\pi}{3} - \tan\frac{3\pi}{4}$ $\qquad \frac{\pi}{3} = 60^\circ$

$\tan 30 - \cos 120 - \tan 135$ $\qquad 2\left(\frac{\pi}{3}\right)=120^\circ$



$\frac{\pi}{4} = 45$

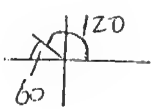
$3\left(\frac{\pi}{4}\right) = 135^\circ$

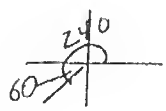
$\tan 30 - (-\cos 60) - (-\tan 45)$

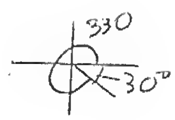
$\frac{\sqrt{3}}{3} + \frac{1}{2} + 1$

$\Rightarrow \frac{2\sqrt{3}+3+6}{6} \Rightarrow \frac{9+2\sqrt{3}}{6}$

e) $\frac{\sin 120 + \cos 240}{\tan 60 + \tan 330}$







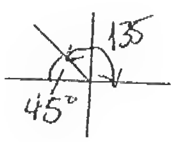
$\Rightarrow \frac{\sin 60 + (-\cos 60)}{\tan 60 + (-\tan 30)} \Rightarrow \frac{\frac{\sqrt{3}}{2} - \frac{1}{2}}{\sqrt{3} - \frac{\sqrt{3}}{3}} \Rightarrow \frac{\frac{\sqrt{3}-1}{2}}{\frac{3\sqrt{3}-\sqrt{3}}{3}}$

$\Rightarrow \frac{3(\sqrt{3}-1)}{2(2\sqrt{3})} \Rightarrow \frac{3(\sqrt{3}-1)}{4\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \Rightarrow \frac{3(\sqrt{3})^2 - \sqrt{3}}{4(\sqrt{3})^2} \Rightarrow \frac{9-\sqrt{3}}{12}$

f) $\frac{2 \cdot \sin^2\left(\frac{\pi}{6}\right)\cos^2 \pi}{4\tan\left(\frac{\pi}{4}\right)\sin^2\left(\frac{3\pi}{4}\right)}$

$\frac{\pi}{4} = 45$

$3\left(\frac{\pi}{4}\right) = 135$



$$\frac{2\sin^2 30\cos^2 180}{4\tan 45\sin^2 135} \Rightarrow \frac{2(\sin 30)^2(\cos 180)^2}{4\times(\sin 135)^2}$$

$$\Rightarrow \frac{2\left(\frac{1}{2}\right)^2(-1)^2}{4(1)\times\left(\frac{\sqrt{2}}{2}\right)^2} \Rightarrow \frac{\frac{2}{4}}{\frac{4\times 2}{4}} \Rightarrow \frac{1}{4}$$

14:-

a) $\tan\pi + \sin\pi$

$\Rightarrow \tan 180 + \sin 180$

$\Rightarrow 0$

b) $\frac{\sin 50}{\cos 40} \Rightarrow \frac{\sin 50}{\sin 50} \Rightarrow 1$

c). $3\sin 45 - 4\tan\left(\frac{\pi}{6}\right)$

$3\sin 45 - 4\tan(30)$

$3\left(\frac{\sqrt{2}}{2}\right) - 4\left(\frac{\sqrt{3}}{3}\right) \Rightarrow \frac{3\sqrt{2}}{2} - \frac{4\sqrt{3}}{3}$

$\Rightarrow \frac{9\sqrt{2}-8\sqrt{3}}{6}$

d) $\frac{\sin(-40)}{\cos 50} \Rightarrow \frac{-\sin 40}{\cos 50}$

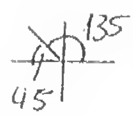
$\Rightarrow \frac{-\cos 50}{\cos 50}$

$\Rightarrow -1$

e) $6\cos\left(\frac{3\pi}{4}\right) + 2\tan\left(-\frac{\pi}{3}\right)$

$\frac{\pi}{4} = 45^\circ$

$3\left(\frac{\pi}{4}\right) = 135^\circ$



$\Rightarrow 6\cos 135 + 2(-\tan 60)$

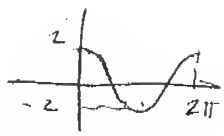
$\Rightarrow 6(-\cos 45) - 2\tan 60$

$\Rightarrow -6\cdot\frac{\sqrt{2}}{2} - 2\sqrt{3} \Rightarrow 3\sqrt{2} - 2\sqrt{3}$

6

15.- ~~dom~~ rango $\underbrace{[-1,3]}_{4}$

Amplitud: $\frac{4}{2} \Rightarrow 2$; $q=2$



$p=1$

$y = 1+2\cos x$

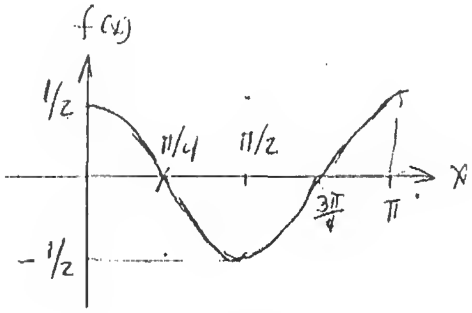
$p^2 - q^2 \Rightarrow -3$ e) CORRECTO

16.- $f(x) = \frac{1}{2}\cos(2x)$

$T = \frac{2\pi}{a} \Rightarrow \frac{\cancel{2}\pi}{\cancel{2}}$

$T = \pi$

$A = 1/2$



e) CORRECTO

17.- $f(x) = p + q\,\text{sen}(kx)$;

$T = 4\pi$

$T = \frac{\cancel{2\pi}}{k} = 4\cancel{\pi}$

$k = 1/2$

Amplitud $= \frac{(11-e)}{2}$

$q = \frac{11-e}{2}$

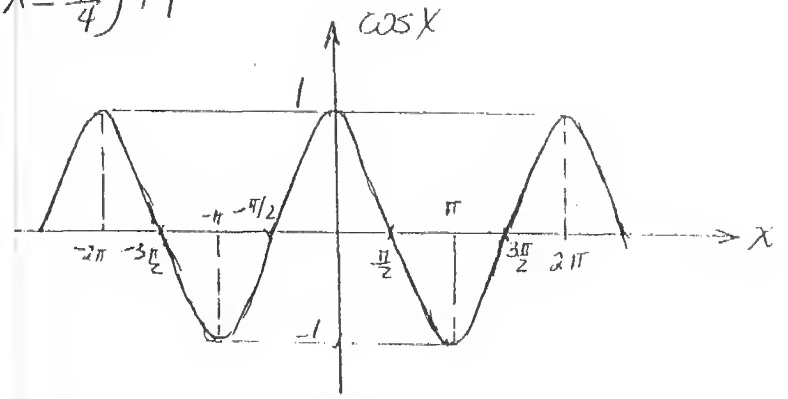
$p = e +$ Amplitud

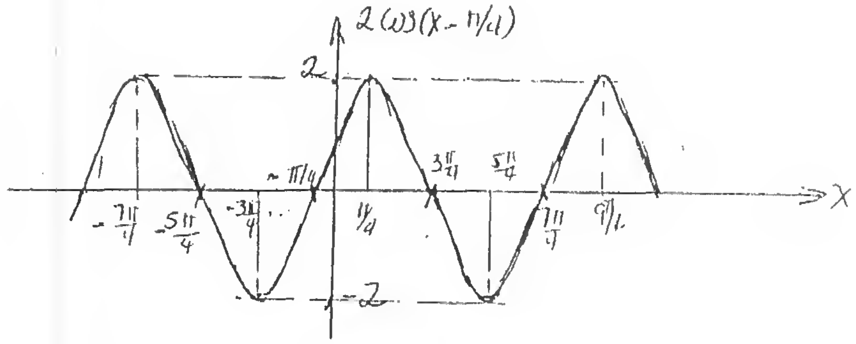
$p = e + \frac{11-e}{2} \Rightarrow \frac{2e+11-e}{2}$

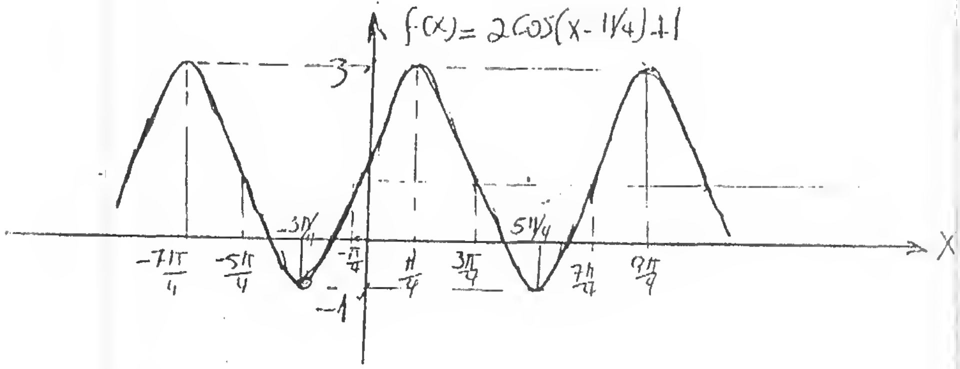
$p = \frac{e+11}{2}$

18.—

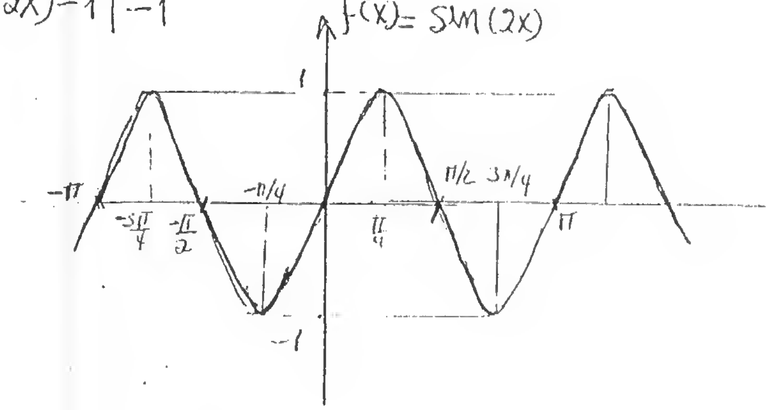
a) $y = 2\cos\left(x - \frac{\pi}{4}\right) + 1$







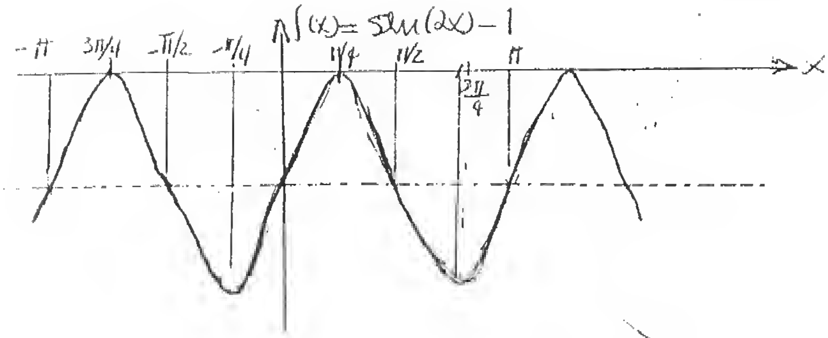
b) $y = |\sin(2x) - 1| - 1$

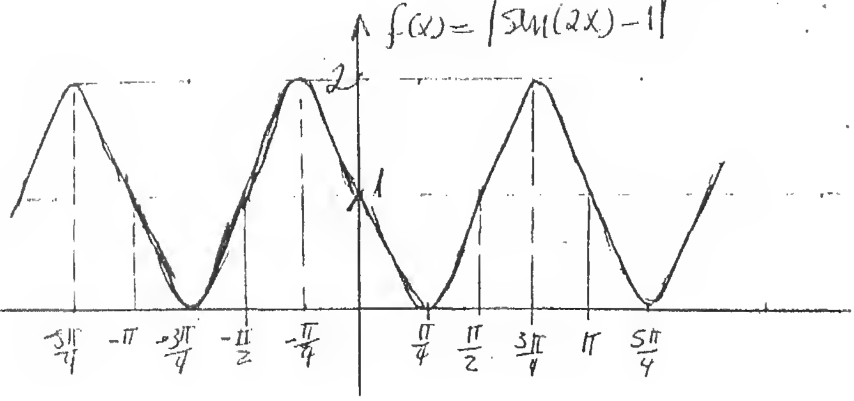


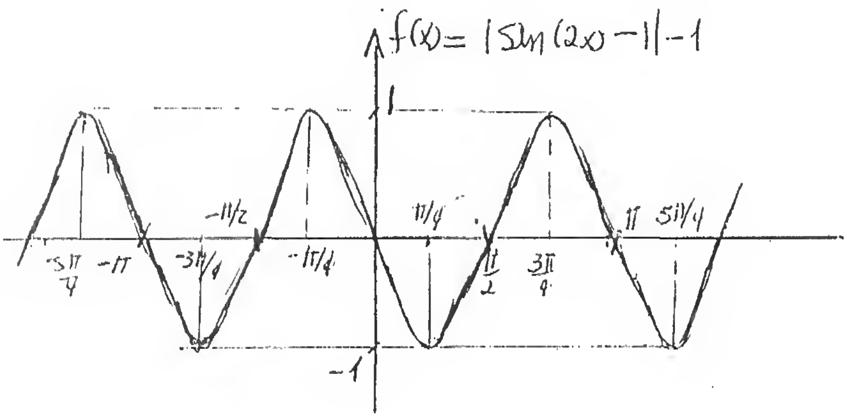
$T = \frac{2\pi}{a}$

$T = \frac{2\pi}{2}$

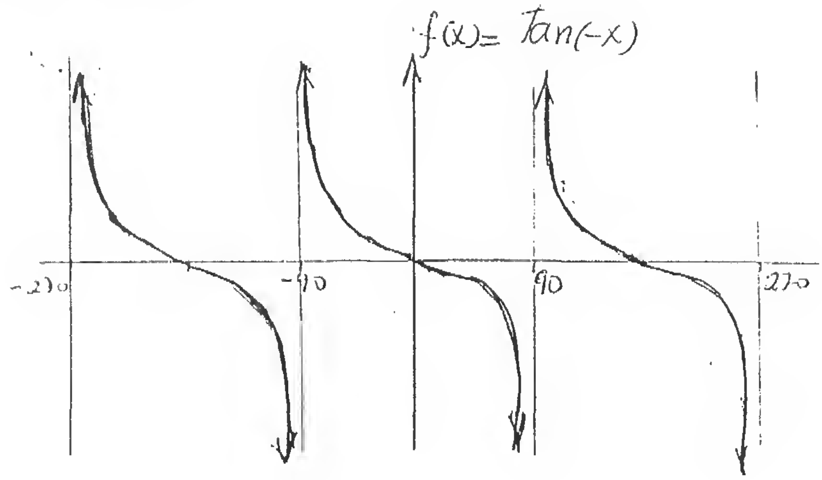
$T = \pi$


f(x)= sin(2x) − 1
−π 3π/4 −π/2 −π/4 π/4 π/2 π
2π/4
x


f(x)= |sin(2x) − 1|
2
1
−5π/4 −π −3π/4 −π/2 −π/4 π/4 π/2 3π/4 π 5π/4


f(x)= |sin(2x) − 1| − 1
1
−1
−5π/4 −π −3π/4 −π/2 −π/4 π/4 π/2 3π/4 π 5π/4

c) $y = 1 - \tan(\pi - x)$

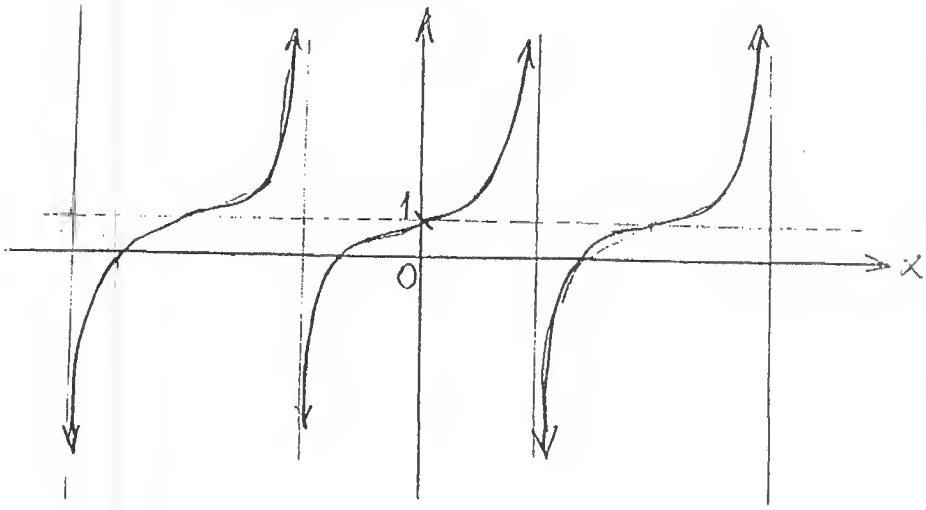

f(x)= tan(−x)
−270 −90 90 270

$$f(x)=\tan(180-x) \Rightarrow \frac{\tan 180^\circ-\tan x}{1+\tan 180^\circ \tan x}$$

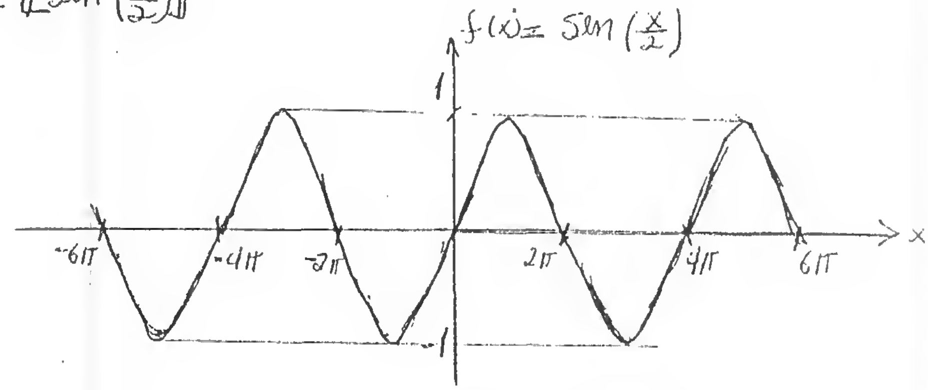
$$f(x)=\tan(180-x)=-\tan x=\tan(-x)$$

$$f(x)=1-\underbrace{\tan(180-x)}_{-\tan x}$$

$$f(x)=1+\tan x$$



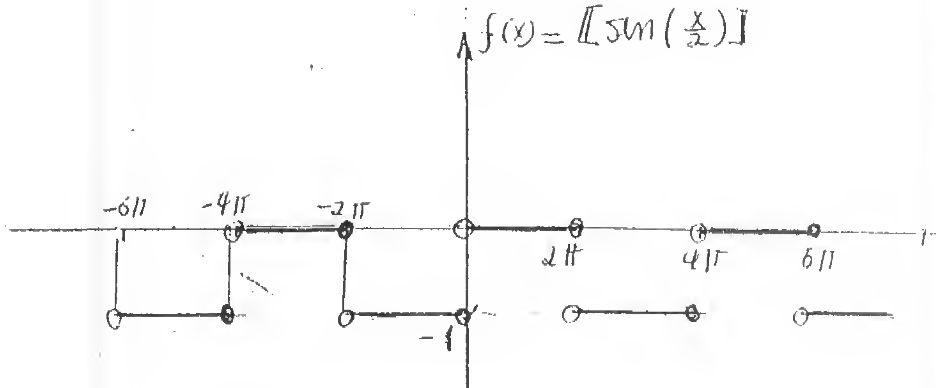
d) $y=\frac{1}{2}-[\![\operatorname{sen}(\frac{x}{2})]\!]$

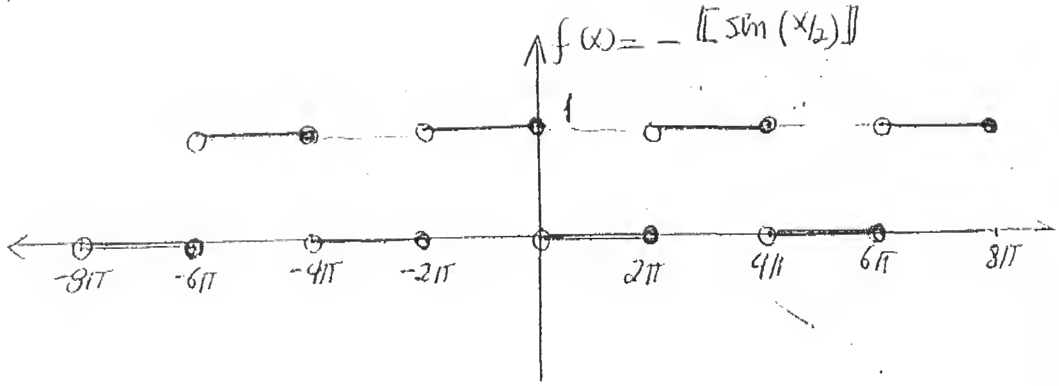


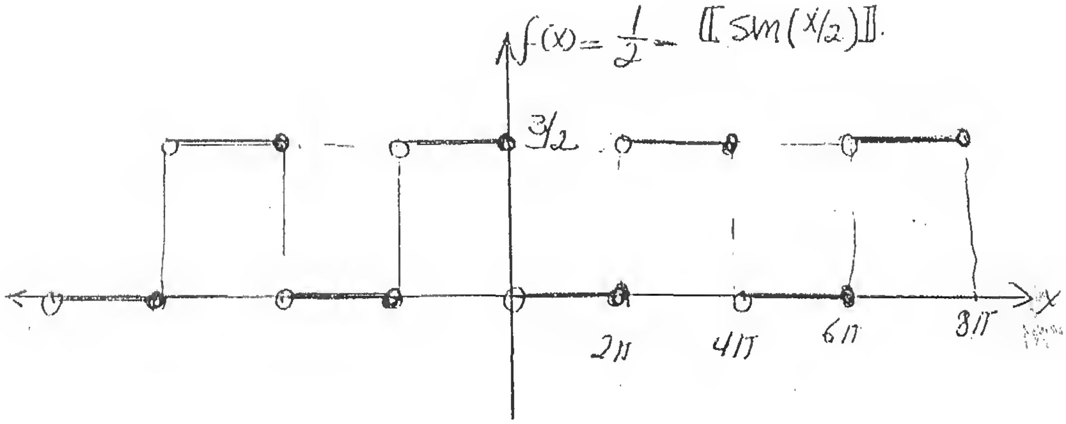
$T=\frac{2\pi}{a}$

$T=\frac{2\pi}{\frac{1}{2}}$

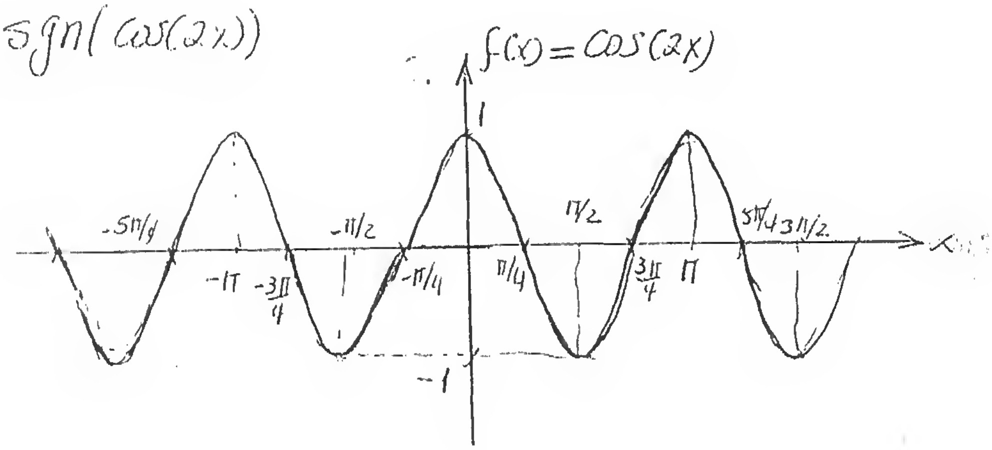
$T=4\pi$

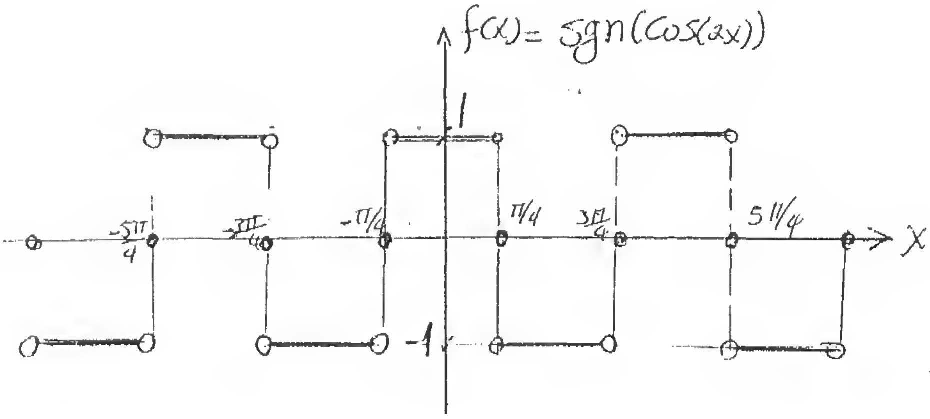



f(x) = − ⟦sin(x/2)⟧
1
−8π
−6π
−4π
−2π
2π
4π
6π
8π


f(x) = 1/2 − ⟦sin(x/2)⟧.
3/2
2π
4π
6π
8π
x

e) $y = sgn(\cos(2x))$


f(x) = cos(2x)
1
−5π/4
−π/2
π/2
5π/4
3π/2
−π
−3π/4
−π/4
π/4
3π/4
π
−1
x


f(x) = sgn(cos(2x))
1
−5π/4
−3π/4
−π/4
π/4
3π/4
5π/4
−1
x

19.— Aproximadamente $T = 7{,}5h \Rightarrow \frac{15}{2}$

a) $\cancel{T} = \frac{15}{2} \Rightarrow \frac{2\pi}{b} = \frac{15}{2}$

$b = \frac{4\pi}{15}$

$2\text{Amplitud}: (4{,}5 - 1{,}5)$

$\text{Amplitud} = \frac{3}{2}$

$a = 3/2$

b) $h(t) = \frac{3}{2}\cos\left(\frac{4\pi}{15}t\right)$

$h(13) = \frac{3}{2}\cos\left(\frac{4\pi}{15}(13)\right)$

c) mínimo

$\cos\left(\frac{4\pi}{15}t\right) = -1$

$\left(\frac{4\pi}{15}t\right) = \cos^{-1}(-1)$

$\frac{4\cancel{\pi}}{15}t = \cancel{\pi}$

$t = \frac{15}{4} + 7{,}5n \;,\; n \in \mathbb{N}$

$t_1 = \frac{15}{4} + \frac{15}{2} \Rightarrow \cancel{\frac{15 + 30}{4}} \Rightarrow \frac{45}{4}h$

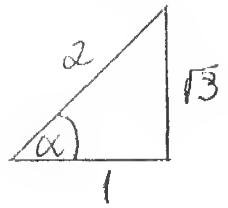
12

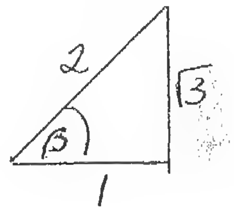
20.- $\alpha = \arccos(-1/2)$; $\frac{\pi}{2} < \alpha < \pi$

$\beta = \arcsen(-\sqrt{3}/2)$; $\frac{3\pi}{2} < \beta < 2\pi$

$\cos\alpha = -\frac{1}{2}$ (2^{do} C)

$\sen\beta = -\frac{\sqrt{3}}{2}$ (4^{to} C)





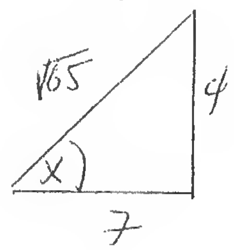
$\sen\alpha + \tan\beta$.

$$\frac{\sqrt{3}}{2} + \left(-\frac{\sqrt{3}}{1}\right)$$

$$\frac{\sqrt{3} - 2\sqrt{3}}{2} \Rightarrow -\frac{\sqrt{3}}{2}$$

21.- $x = \arctan\left(\frac{4}{7}\right)$; $\pi \leq x \leq \frac{3\pi}{2}$

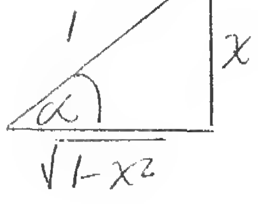
$\tan x = \frac{4}{7}$ (3^{ro} C)



$$\cos x = -\frac{7}{\sqrt{65}} \cdot \frac{\sqrt{65}}{\sqrt{65}}$$

$$\cos x = -\frac{7\sqrt{65}}{65}$$

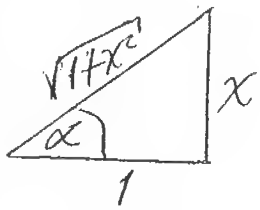
22.- a) $\cos(\underbrace{\arcsen(x)}_{\alpha})$ $\Rightarrow \sen\alpha = x$



$$\cos\alpha = \frac{\sqrt{1-x^2}}{1}$$

b) $\cos(\underbrace{\arctan(x)}_{\alpha})$

$\alpha \Rightarrow \tan(\alpha) = \frac{x}{1}$



$\cos\alpha = \frac{1}{\sqrt{1+x^2}}$

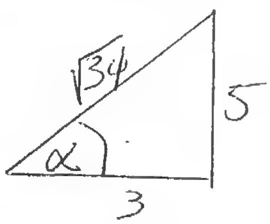
c) $\arccos\left[\cos\left(-\frac{17}{5}\pi\right)\right]$

$\cancel{\cos^{-1}}\left[\cancel{\cos}\left(-\frac{17}{5}\pi\right)\right]$

$\Rightarrow -\frac{17}{5}\pi$

d) $\operatorname{sen}\underbrace{\left[\arctan\left(-\frac{5}{3}\right)\right]}_{\alpha}$

$\tan\alpha = -\frac{5}{3}$



$\operatorname{sen}\alpha = \frac{5}{\sqrt{34}}$

23.- $\forall x, y \in \mathbb{R} : (\operatorname{sen}^{-1}(x+y)) = \operatorname{sen}^{-1}x + \operatorname{sen}^{-1}y$

b) CORRECTO

24.- $8 \cos 10 \cdot \cos 20 \cdot \cos 40$

$\Rightarrow \cos 20 = \cos(30-10)$

$\Rightarrow \cos 30 \cos 10 + \operatorname{sen} 30 \operatorname{sen} 10$

$\Rightarrow \boxed{\frac{\sqrt{3}}{2} \cos 10 + \frac{1}{2} \operatorname{sen} 10}$

$\Rightarrow \cos 40 = \cos(30+10)$

$\Rightarrow \cos 30 \cos 10 - \operatorname{sen} 30 \operatorname{sen} 10$

$\Rightarrow \boxed{\frac{\sqrt{3}}{2} \cos 10 - \frac{1}{2} \operatorname{sen} 10°}$

$\therefore 8 \cos 10 \times \cos 20 \times \cos 40$

$\Rightarrow 8 \cos 10 \left(\frac{\sqrt{3}}{2} \cos 10 + \frac{1}{2} \operatorname{sen} 10\right)\left(\frac{\sqrt{3}}{2} \cos 10 - \frac{1}{2} \operatorname{sen} 10\right)$

$\Rightarrow 8 \cos 10 \left(\frac{3}{4} \cos^2 10 - \frac{1}{4} \operatorname{sen}^2 10\right) \Rightarrow \frac{8}{4} \cos 10 (3 \cos^2 10 - \operatorname{sen}^2 10)$

$\Rightarrow 2 \cos 10 (3 \cos^2 10 - (1 - \cos^2 10)) \Rightarrow 2 \cos 10 \underbrace{(4 \cos^2 10 - 1)}_{\frac{1}{2 \operatorname{sen} 10}}$

$\Rightarrow \frac{2 \cos 10}{2 \operatorname{sen} 10} \Rightarrow \cot 10°$

d) correcto

15

25.- $\cos\left(\frac{\pi}{12}\right) \Rightarrow \frac{\pi}{12} \times \frac{180}{\pi rad} = 15^{\circ}$

$\cos(15) \Rightarrow \cos(45-30)$

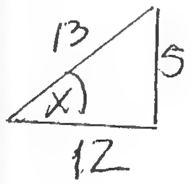
$\Rightarrow \cos 45 \cos 30 + \operatorname{sen} 45 \operatorname{sen} 30$

$\Rightarrow \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2}$

$\Rightarrow \frac{\sqrt{6} + \sqrt{2}}{4}$

d) CORRECTO

26.- $\sin x = \frac{5}{13}$ 2do cuadrante



$C = \sqrt{13^2 - 5^2}$

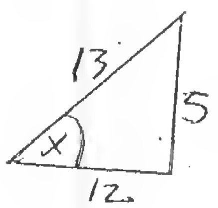
$C = 12$

$\cos(x+60) \Rightarrow \cos x \cos 60 - \operatorname{sen} x \operatorname{sen} 60$

$\Rightarrow -\frac{12}{13} \times \frac{1}{2} - \frac{5}{13} \times \frac{\sqrt{3}}{2}$

$\Rightarrow \frac{-12 + 5\sqrt{3}}{26}$

27.- $\operatorname{sen} x = \frac{5}{13}$ 2do cuadrante



$\sin(2x) = 2 \operatorname{sen} x \cos x$

$\Rightarrow 2 \times \frac{5}{13} \times -\frac{12}{13}$

$\Rightarrow \frac{120}{169}$ incorrecto

28.—

$$\sqrt{\frac{2\sec(3x)}{1+\sec(3x)}}$$

$$\Rightarrow \sqrt{\frac{\frac{2}{\cos(3x)}}{1+\frac{1}{\cos(3x)}}} \Rightarrow \sqrt{\frac{\frac{2}{\cancel{\cos(3x)}}}{\frac{\cos(3x)+1}{\cancel{\cos(3x)}}}} \Rightarrow \sqrt{\frac{2}{1+\cos(3x)}}$$

$$\Rightarrow \cancel{\sqrt{\frac{2}{1+\cos(3x)} \times \frac{1-\cos(3x)}{1-\cos(3x)}} \Rightarrow \sqrt{\frac{2(1-\cos(3x))}{1-\cos^2(3x)}}}$$

$$\cancel{\sqrt{\frac{2(1-\cos(3x))}{\operatorname{sen}^2(3x)}} \Rightarrow \frac{\sqrt{2(1-\cos(3x))}}{\sqrt{\operatorname{sen}^2(3x)}} \Rightarrow \frac{\sqrt{2(1-\cos(3x))}}{\operatorname{sen}(3x)}}$$

$$\Rightarrow \left(\sqrt{\frac{1+\cos(3x)}{2}}\right)^{-1} = \left(\cos\left(\frac{3x}{2}\right)\right)^{-1} \Rightarrow \frac{1}{\cos\left(\frac{3x}{2}\right)}$$

$$\Rightarrow \sec\left(\frac{3x}{2}\right)$$ e) correcto

29.— a) $\operatorname{sen}\left(\frac{x}{2}+\frac{x}{2}\right) = 2\operatorname{sen}\frac{x}{2}\cos\frac{x}{2}$ CORRECTO

$\frac{1}{2}\operatorname{sen} x = \operatorname{sen}\frac{x}{2}\cdot\cos\frac{x}{2}$

b) $\cos(4x) = \cos^2(2x) - \operatorname{sen}^2(2x)$ CORRECTO

c) $\tan x \cdot \cos x = \frac{1}{\csc x}$

$$\frac{\operatorname{sen} x}{\cancel{\cos x}}\cdot\cancel{\cos x} \Rightarrow \operatorname{sen} x \Rightarrow \left(\frac{1}{\operatorname{sen} x}\right)^{-1} \Rightarrow (\csc x)^{-1}$$

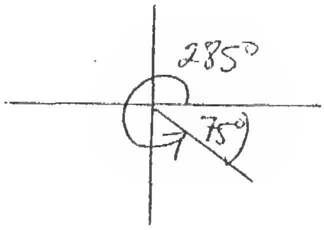
$$\Rightarrow \frac{1}{\csc x}$$ CORRECTO

17

d) $sen^2(2x) + \cos(2x) = 1$; FALSO

30. $\tan\left(\frac{19\pi}{12}\right)$ $\qquad \frac{\cancel{19\pi}}{\cancel{12}} \times \frac{\cancel{180°}}{\cancel{\pi rad}} = 285$

$\Rightarrow +\tan 285$



$\Rightarrow -\tan 75$

$\Rightarrow -\tan(45+30)$

$$\Rightarrow -\frac{sen(45+30)}{\cos(45+30)} \Rightarrow -\frac{sen 45 \cos 30 + sen 30 \cos 45}{\cos 45 \cos 30 - sen 45\, sen 30}$$

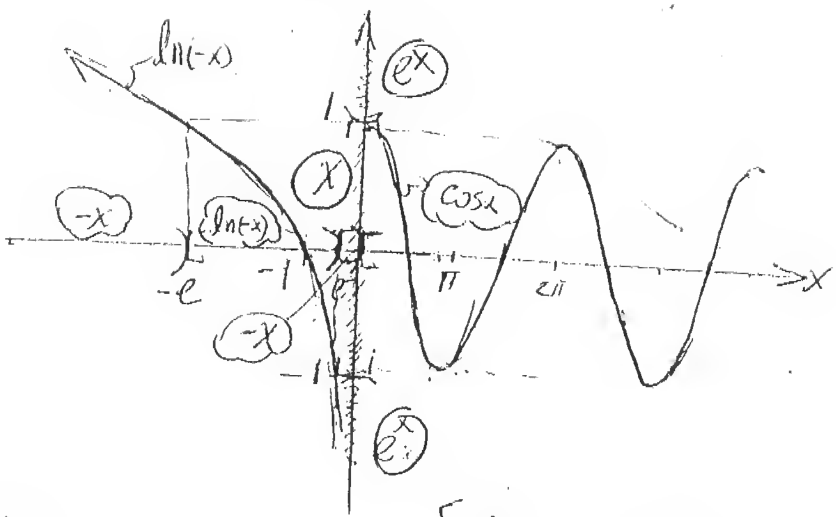
$$\Rightarrow -\frac{\frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \times \frac{1}{2}} \Rightarrow -\frac{\frac{\sqrt{3}}{2} + \frac{1}{2}}{\frac{\sqrt{3}}{2} - \frac{1}{2}} \Rightarrow$$

$$\frac{\frac{\sqrt{3}+1}{\cancel{2}}}{\frac{1-\sqrt{3}}{\cancel{2}}} \Rightarrow \frac{1+\sqrt{3}}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}} \Rightarrow \frac{1+2\sqrt{3}+3}{1-(\sqrt{3})^2} \Rightarrow$$

$$\frac{4+2\sqrt{3}}{-2} \Rightarrow \frac{\cancel{2}(2+\sqrt{3})}{-\cancel{2}} \Rightarrow -(2+\sqrt{3})$$

31.- $f(x) = \begin{cases} \cos x & ; x \geq 0 \\ \ln(-x) & ; x < 0 \end{cases}$ $\qquad g(x) = \begin{cases} x & ; |x| \leq 1 \\ e^x & ; |x| > 1 \end{cases}$

$g \circ f$



$$e^{\ln(-x)} \Rightarrow -x$$

$$(g \circ f)(x) \begin{cases} -x & ; x < -e \\ \ln(-x) & ; -e \leq x < -e^{-1} \\ -x & ; -e^{-1} \leq x < 0 \\ \cos x & ; x \geq 0 \end{cases}$$

32.- $a = \tan 25°$

$$\frac{\tan 205 - \tan 115}{\tan 245 + \tan 335}$$

$$\tan 205 = \tan(180 + 25) \Rightarrow \frac{\tan 180 + \tan 25}{1 - \tan 180 \tan 25} \Rightarrow -a$$

$$\tan 115 = \frac{\sin(90+25)}{\cos(90+25)} \Rightarrow \frac{\sin 90 \cos 25 + \sin 25 \cos 90}{\cos 90 \cos 25 - \sin 90 \sin 25}$$

$$\Rightarrow \frac{\cos 25}{-\sin 25} \Rightarrow \left(-\frac{\sin 25}{\cos 25}\right)^{-1} \Rightarrow -(a)^{-1} \Rightarrow -\frac{1}{a}$$

$$\tan 245 = \tan(270 - 25) \Rightarrow \frac{\sin(270 - 25)}{\cos(270-25)}$$

$$\Rightarrow \frac{\sin 270 \cos 25 - \sin 25 \cos 270}{\cos 270 \cos 25 + \sin 270 \sin 25} \Rightarrow \frac{-\cos 25}{-\sin 25} \Rightarrow \frac{1}{a}$$

$$\tan 335 = \tan(360-25) \Rightarrow \frac{\overset{0}{\cancel{\tan 360}} - \overset{a}{\cancel{\tan 25}}}{1+\overset{0}{\cancel{\tan 360}}\tan 25} = -a$$

$$\Rightarrow \frac{-a-\left(-\frac{1}{a}\right)}{\frac{1}{a}+(-a)} \Rightarrow \frac{\frac{1}{a}-a}{\frac{1}{a}-a} = 1$$

33.—

a) $\frac{1}{2\,\text{sen}10} - 2\,\text{sen}70$

$$\text{sen}10 = \text{sen}(30-20) \Rightarrow \text{sen}30\cos 20 - \text{sen}20\cos 30$$

$$\underbrace{\left(\frac{1}{2}\cos 20 - \frac{\sqrt{3}}{2}\text{sen}20\right)}_{\frac{\cos 20-\sqrt{3}\,\text{sen}20}{2}}$$

$$\text{sen}10\,\text{sen}70 \Rightarrow \frac{1}{2}(\cos(60) - \cos 80)$$

$$\therefore \frac{1}{2\,\text{sen}10} - 2\cdot\text{sen}70 = \frac{1-4\,\text{sen}10\,\text{sen}70}{2\,\text{sen}10}$$

$$\Rightarrow \frac{1-4\left(\frac{1}{2}(\cos 60 - \cos 80)\right)}{2\,\text{sen}10} \Rightarrow \frac{1-2\overset{1}{\cos 60}+2\cos 80}{2\,\text{sen}10} \Rightarrow \frac{1-1+2\,\text{sen}10}{2\,\text{sen}10} = 1$$

b.) $\sin\left(\frac{\pi}{12}\right)\cos\left(\frac{\pi}{12}\right)$ $\qquad \frac{\cancel{\pi}}{\cancel{12}} \times \frac{\cancel{180}}{\cancel{\pi}\,rad} = 15^\circ$

$\Rightarrow \sin 15^\circ \times \cos(15)$

$\Rightarrow \sin(45-30)\times\cos(45-30)$

$\Rightarrow (\sin 45\cos 30 - \sin 30\cos 45)(\cos 45\cos 30 + \sin 45\sin 30)$

$\Rightarrow \left(\frac{\sqrt{2}}{2}\times\frac{\sqrt{3}}{2} - \frac{1}{2}\,\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\,\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\times\frac{1}{2}\right)$

$\Rightarrow \left(\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}\right)\left(\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}\right) \Rightarrow \left(\frac{\sqrt{6}}{4}\right)^2 - \left(\frac{\sqrt{2}}{4}\right)^2$

$\Rightarrow \frac{6}{16} - \frac{2}{16} \Rightarrow \frac{4}{16} \Rightarrow \frac{1}{4}$

c.) $\tan 55 - \tan 35$

$\tan(55-35) = \frac{\tan 55 - \tan 35}{1 + \tan 55\tan 35}$

$\Rightarrow \tan 20 = \frac{\tan 55 - \tan 35}{1 + \tan 55 \times \cot 55} \Rightarrow \tan 20 = \frac{\tan 55 - \tan 35}{1 + \underbrace{\frac{\cancel{\sin 55}}{\cancel{\cos 55}} \times \frac{\cancel{\cos 55}}{\cancel{\sin 55}}}_{1}}$

$\tan 55 - \tan 35 = 2\tan 20$

d.) $\cos\left(\frac{\pi}{5}\right)\cos\left(\frac{3\pi}{5}\right)$ $\qquad \frac{\cancel{\pi}}{5} \times \frac{180}{\cancel{\pi}} = 36^\circ$

$3\left(\frac{\pi}{5}\right) = 108^\circ$

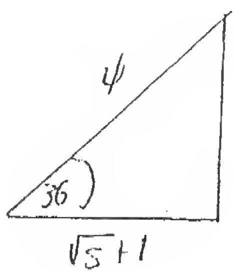
$\Rightarrow \cos(36)\cos(108)$

$\Rightarrow \cos 36\,(4\cos^3(36) - 3\cos 36)$

$\Rightarrow 4\cos^4 36 - 3\cos^2 36$

$\Rightarrow 4\left(\frac{\sqrt{5}+1}{4}\right)^4 - 3\left(\frac{\sqrt{5}+1}{4}\right)^2$

$\Rightarrow -1/4$



$\cos(2x) = 2\cos^2 x - 1$

$\cos(3x) = 4\cos^3 x - 3\cos x$

e.) $$\frac{\sin(\frac{3\pi}{2}+\alpha)\tan(\frac{\pi}{2}+\beta)\ \sin(\frac{3\pi}{2}-\beta)\cot(\frac{\pi}{2}+\alpha)}{\cos(\pi-\alpha)\cot(\frac{3\pi}{2}-\beta)\ \cos(2\pi-\beta)\tan(\pi-\alpha)}$$

$\frac{\pi}{2} = 90$

$3(\frac{\pi}{2}) = 270$

$$\Rightarrow \frac{\sin(90+\alpha)\tan(90+\beta)\ \sin(270-\beta)\cot(90+\alpha)}{\cos(180-\alpha)\cot(270-\beta)\ \cos(360-\beta)\tan(\pi-\alpha)}$$

$\sin(90+\alpha) = \cancel{\sin 90}^{1}\cos\alpha + \sin\alpha\cancel{\cos 90}^{0} \Rightarrow \cos\alpha$

$\tan(90+\beta) = \frac{\sin(90+\beta)}{\cos(90+\beta)} = \frac{\cancel{\sin 90}^{1}\cos\beta + \sin\beta\cancel{\cos 90}^{0}}{\cancel{\cos 90}^{0}\cos\beta - \cancel{\sin 90}^{1}\sin\beta} \Rightarrow \frac{\cos\beta}{-\sin\beta} \Rightarrow -\cot\beta$

$\sin(270-\beta) = \cancel{\sin 270}^{-1}\cos\beta - \sin\beta\cancel{\cos 270}^{0} = -\cos\beta$

$\cot(90+\alpha) = \frac{\cos(90+\alpha)}{\sin(90+\alpha)} = \frac{\cancel{\cos 90}^{0}\cos\alpha - \cancel{\sin 90}^{1}\sin\alpha}{\cancel{\sin 90}\cos\alpha + \sin\alpha\cancel{\cos 90}^{0}} = \frac{-\sin\alpha}{\cos\alpha} \Rightarrow -\tan\alpha$

$\cos(180-\alpha) = \cancel{\cos 180}^{-1}\cos\alpha + \cancel{\sin 180}^{0}\sin\alpha \Rightarrow -\cos\alpha$

$\cot(270-\beta) = \frac{\cos(270-\beta)}{\sin(270-\beta)} = \frac{\cancel{\cos 270}^{0}\cos\beta + \cancel{\sin 270}^{-1}\sin\beta}{\cancel{\sin 270}^{-1}\cos\beta - \sin\beta\cancel{\cos 270}^{0}} \Rightarrow \frac{-\sin\beta}{-\cos\beta} = \tan\beta$

$\cos(360-\beta) = \cancel{\cos 360}^{1}\cos\beta + \cancel{\sin 360}^{0}\sin\beta \Rightarrow \cos\beta$

$\tan(180-\alpha) = \frac{\cancel{\tan 180}^{0} - \tan\alpha}{1 + \cancel{\tan 180}^{0}\tan\alpha} \Rightarrow -\tan\alpha$

$$\Rightarrow \frac{\cancel{\cos\alpha}(-\cot\beta)\ (+\cancel{\cos\beta})(\cancel{-\tan\alpha})}{+\cancel{\cos\alpha}\ \tan\beta\ (\cancel{\cos\beta})(\cancel{-\tan\alpha})} \Rightarrow -\frac{\cot\beta}{\tan\beta} \Rightarrow -\frac{\frac{\cos\beta}{\sin\beta}}{\frac{\sin\beta}{\cos\beta}} \Rightarrow -\frac{\cos^2\beta}{\sin^2\beta}$$

F.) $\cos\left(\frac{\pi}{65}\right)\cos\left(\frac{2\pi}{65}\right)\cos\left(\frac{4\pi}{65}\right)\cos\left(\frac{8\pi}{65}\right)\cos\left(\frac{16\pi}{65}\right)\cos\left(\frac{32\pi}{65}\right)$

$$a = \frac{\pi}{65}$$

$\Rightarrow \cos(a)\cos(2a)\cos(4a)\cos(8a)\cos(16a)\cos(32a)$

$\Rightarrow E = \cos(a)\cos(2a)\cos(4a)\cos(8a)\cos(16a)\cos(32a)$

$2\operatorname{sen}(a)E = \underbrace{2\operatorname{sen}(a)\cos(a)}_{\operatorname{sen}(2a)}\cdot\cos(2a)\cos(4a)\cos(8a)\cos(16a)\cos(32a)$

$2\cdot 2\operatorname{sen}(a)E = \underbrace{2\operatorname{sen}(2a)\cdot\cos(2a)}_{\operatorname{sen}(4a)}\cos(4a)\cos(8a)\cos(16a)\cos(32a)$

$2\cdot 4\operatorname{sen}(a)E = \underbrace{2\operatorname{sen}(4a)\cos(4a)}_{\operatorname{sen}(8a)}\cos(16a)\cos(32a)\cos(8a)$

$2\cdot 8\operatorname{sen}(a)E = \underbrace{2\operatorname{sen}(8a)\cos(8a)}_{\operatorname{sen}(16a)}\cos(16a)\cos(32a)$

$2\cdot 16\operatorname{sen}(a)E = \underbrace{2\operatorname{sen}(16a)\cos(16a)}\cos(32a)$

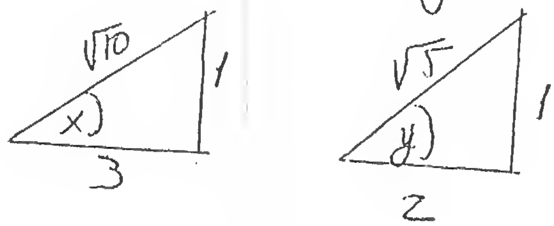
$2\cdot 32\operatorname{sen}(a)E = 2\operatorname{sen}(32a)\cos(32a)$

$64\operatorname{sen}(a)E = \operatorname{sen}(64a)$

$$E = \frac{\operatorname{sen}(64a)}{64\operatorname{sen} a} \Rightarrow \frac{\operatorname{sen}\left(64\frac{\pi}{65}\right)}{64\operatorname{sen}\left(\frac{\pi}{65}\right)}$$

$$\cos\left(\frac{\pi}{65}\right)\cos\left(\frac{2\pi}{65}\right)\cos\left(4\frac{\pi}{65}\right)\cos\left(8\frac{\pi}{65}\right)\cos\left(16\frac{\pi}{65}\right)\cos\left(32\frac{\pi}{65}\right) \Rightarrow \frac{\operatorname{sen}\left(\frac{64\pi}{65}\right)}{64\operatorname{sen}\left(\frac{\pi}{65}\right)}$$

34.– $\underbrace{\cos^{-1}\left(\frac{3}{\sqrt{10}}\right)}_{x} + \underbrace{\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)}_{y} = \frac{\pi}{4}$



$$x+y=\pi/4$$

$$\cos(x+y)=\cos\pi/4$$

$$\cos x\cos y-\sin x\sin y=\frac{\sqrt{2}}{2}$$

$$\Rightarrow \frac{3}{\sqrt{10}}\cdot\frac{2}{\sqrt{5}}-\frac{1}{\sqrt{10}}\cdot\frac{1}{\sqrt{5}}$$

$$\frac{6}{\sqrt{50}}-\frac{1}{\sqrt{50}} \Rightarrow \frac{5}{\sqrt{50}} \Rightarrow \frac{\cancel{5}}{\cancel{5}\sqrt{2}} \Rightarrow \frac{1}{\sqrt{2}} \Rightarrow \frac{\sqrt{2}}{2}$$ demostrado

35.– a) $$\frac{\sin^3(\alpha-270)\cos(360-\alpha)}{\tan^3\left(\alpha-\frac{\pi}{2}\right)\cos^3\left(\alpha-\frac{3\pi}{2}\right)}=\cos\alpha$$

$$\sin(\alpha-270)=\sin\alpha\overset{0}{\cos 270}-\overset{-1}{\sin 270}\cos\alpha=\cos\alpha$$

$\cos(360-\alpha) = \cancel{\cos 360}^{1}\cos\alpha + \cancel{\operatorname{sen}360}^{0}\operatorname{sen}\alpha = \cos\alpha$

$\tan(\alpha-90) = \dfrac{\operatorname{sen}(\alpha-90)}{\cos(\alpha-90)} = \dfrac{\operatorname{sen}\alpha\cancel{\cos 90}^{0} - \cancel{\operatorname{sen}90}^{1}\cos\alpha}{\cos\alpha\cancel{\cos 90}^{0} + \cancel{\operatorname{sen}90}^{1}\operatorname{sen}\alpha} \Rightarrow -\dfrac{\cos\alpha}{\operatorname{sen}\alpha}$

$\cos(\alpha-270) = \cos\alpha\cancel{\cos 270}^{0} + \operatorname{sen}\alpha\cancel{\operatorname{sen}270}^{-1} = -\operatorname{sen}\alpha$

$\Rightarrow \dfrac{(\cos\alpha)^3\cdot\cos\alpha}{\left(-\dfrac{\cos\alpha}{\operatorname{sen}\alpha}\right)^3(-\operatorname{sen}\alpha)^3} \Rightarrow \dfrac{\cos^4\alpha}{\dfrac{\cos^3\alpha}{\cancel{\operatorname{sen}^3\alpha}}\cdot\cancel{\operatorname{sen}^3\alpha}} \Rightarrow \cos\alpha$

demostrado

b) $\dfrac{\cos^2\alpha}{\cot\left(\frac{\alpha}{2}\right)-\tan\left(\frac{\alpha}{2}\right)} = \dfrac{1}{4}\operatorname{sen}(2\alpha)$

$\cot\left(\frac{\alpha}{2}\right) = \dfrac{\cos(\alpha/2)}{\operatorname{sen}(\alpha/2)} \Rightarrow \dfrac{\sqrt{\frac{\cos\alpha+1}{2}}}{\sqrt{\frac{-\cos\alpha+1}{2}}} \Rightarrow \sqrt{\dfrac{\frac{\cos\alpha+1}{\cancel{2}}}{\frac{-\cos\alpha+1}{\cancel{2}}}} \Rightarrow \sqrt{\dfrac{\cos\alpha+1}{-\cos\alpha+1}}$

$\dfrac{\cos^2\alpha}{\sqrt{\frac{\cos\alpha+1}{-\cos\alpha+1}} - \sqrt{\frac{-\cos\alpha+1}{\cos\alpha+1}}} \Rightarrow \dfrac{\cos^2\alpha}{\frac{\sqrt{\cos\alpha+1}}{\sqrt{-\cos\alpha+1}} - \frac{\sqrt{-\cos\alpha+1}}{\sqrt{\cos\alpha+1}}} \Rightarrow \dfrac{\cos^2\alpha}{\frac{(\sqrt{\cos\alpha+1})^2-(\sqrt{-\cos\alpha+1})^2}{\sqrt{-\cos\alpha+1}\sqrt{\cos\alpha+1}}}$

$\Rightarrow \dfrac{\cos^2\alpha}{\frac{\cos\alpha+\cancel{1}+\cos\alpha-\cancel{1}}{\sqrt{(1-\cos\alpha)(\cos\alpha+1)}}} \Rightarrow \dfrac{\cos^2\alpha}{\frac{2\cos\alpha}{\sqrt{1-\cos^2\alpha}}} \Rightarrow \dfrac{\cos^2\alpha}{\frac{2\cos\alpha}{\sqrt{1-\cos^2\alpha}}} \Rightarrow \dfrac{\cos\alpha\sqrt{1-\cos^2\alpha}}{2}$

$\Rightarrow \dfrac{\cos\alpha\sqrt{\operatorname{sen}^2\alpha}}{2} \Rightarrow \dfrac{1}{2}\operatorname{sen}\alpha\cdot\cos\alpha \Rightarrow \dfrac{1}{2}\cdot\dfrac{\overbrace{2\operatorname{sen}\alpha\cos\alpha}^{\operatorname{sen}(2\alpha)}}{2}$

$\Rightarrow \dfrac{1}{4}\operatorname{sen}(2\alpha)$ demostrado

c) $\dfrac{\tan^2(2x)-\tan^2(x)}{1-\tan^2(2x)\tan^2(x)} = \tan(3x)\tan x.$

$\Rightarrow \dfrac{(\tan(2x)+\tan x)(\tan(2x)-\tan x)}{(1+\tan(2x)\tan x)(1-\tan(2x)\tan x)} \Rightarrow \underbrace{\dfrac{\tan(2x)+\tan x}{1-\tan(2x)\tan x}}_{\tan(2x+x)} \cdot \underbrace{\dfrac{\tan(2x)-\tan x}{1+\tan(2x)\tan x}}_{\tan(2x-x)}$

$\Rightarrow \tan(3x)\tan x$

d) $\sin w \, \sin(60-w)\, \sin\left(\overset{60}{\frac{\pi}{3}}+w\right) = \sin(3w)$

$\sin w \cdot (\sin 60 \cos w - \sin w \cos 60)(\sin 60 \cos w + \sin w \cos 60)$

$\sin w\left(\frac{\sqrt{3}}{2}\cos w - \frac{1}{2}\sin w\right)\left(\frac{\sqrt{3}}{2}\cos w + \frac{1}{2}\sin w\right)$

$\sin w\left(\frac{3}{4}\cos^2 w - \frac{1}{4}\sin^2 w\right) \Rightarrow \frac{1}{4}\sin w(3\cos^2 w - \sin^2 w)$

$\Rightarrow \frac{1}{4}\underbrace{(3\sin w\cos^2 w - \sin^3 w)}_{\sin(3w)} \Rightarrow \frac{1}{4}\sin(3w)$

$\sin(3w) = \sin(2w+w) \Rightarrow \sin(2w)\cos w + \sin w\cos(2w)$

$\Rightarrow 2\sin w\cos w \cdot \cos w + \sin w(\cos^2 w - \sin^2 w)$

$\Rightarrow 2\sin w\cos^2 w + \sin w\cos^2 w - \sin^3 w$

$\Rightarrow \boxed{3\sin w\cos^2 w - \sin^3 w.}$

e) $\sin 47 + \sin 61 - \sin 11 - \sin 25 = \cos 7$

$\sin 47 + \sin 61 - (\sin 25 + \sin 11)$

$2\sin 54 \cos 7 - (2\sin 18\cos 7)$

$\cos 7\,(2\sin 54 - 2\sin 18)$

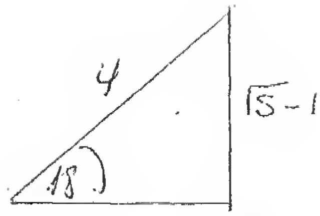
$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\sin(3x) = 3\sin x - 4\sin^3 x$

$$\Rightarrow \cos 7\,\big(2(3\,\text{sen}\,18 - 4\,\text{sen}^3 18) - 2\,\text{sen}\,18\big)$$

$$\Rightarrow \cos 7\,(6\,\text{sen}\,18 - 8\,\text{sen}^3 18 - 2\,\text{sen}\,18)$$

$$\Rightarrow \cos 7\,(4\,\text{sen}\,18 - 8\,\text{sen}^3 18)$$



$$\Rightarrow \cos 7\left(\cancel{4}\,\frac{(\sqrt{5}-1)}{\cancel{4}} - 8\left(\frac{\sqrt{5}-1}{4}\right)^3\right)$$

$$\Rightarrow \cos 7\left(\sqrt{5}-1 - \frac{\cancel{8}(\cancel{8}\sqrt{5}-\cancel{16})}{\cancel{64}}\right) \Rightarrow \cos 7\,(\cancel{\sqrt{5}}-1-\cancel{\sqrt{5}}+2)$$

$$\Rightarrow \cos 7$$ demostrado

f) $$\frac{1+\text{sen}\,x+\cos x}{1+\text{sen}\,x-\cos x} = \frac{1+\cos x}{\text{sen}\,x}$$

$$\Rightarrow \frac{1+(\text{sen}\,x+\cos x)}{1+(\text{sen}\,x-\cos x)} \times \frac{1-(\text{sen}\,x-\cos x)}{1-(\text{sen}\,x-\cos x)} \Rightarrow \frac{1-(\text{sen}\,x-\cos x)+(\text{sen}\,x+\cos x)+\text{sen}^2x+\cos^2x}{1-(\text{sen}\,x-\cos x)^2}$$

$$\Rightarrow \frac{1-\cancel{\text{sen}\,x}+\cos x+\cancel{\text{sen}\,x}+\cos x-\text{sen}^2x+\cos^2x}{1-(\text{sen}^2x+2\,\text{sen}\,x\cos+\cos^2x)} \Rightarrow \frac{1+2\cos x-\text{sen}^2x+\cos^2x}{1-(1-2\,\text{sen}\,x\cos x)}$$

$$\Rightarrow \frac{1+2\cos x-(1-\cos^2x)+\cos^2x}{\cancel{1}-\cancel{1}+2\,\text{sen}\,x\cos x} \Rightarrow \frac{\cancel{1}+2\cos x-\cancel{1}+\cos^2x+\cos^2x}{2\,\text{sen}\,x\cos x}$$

$$\Rightarrow \frac{2\cos x+2\cos^2x}{2\,\text{sen}\,x\cos x} \Rightarrow \frac{\cancel{2}\cancel{\cos x}(1+\cos x)}{\cancel{2}\,\text{sen}\,x\cancel{\cos x}}$$

$$\Rightarrow \frac{1+\cos x}{\text{sen}\,x}$$ demostrado

36.—

a) $\operatorname{sen}^2\theta(1+\cot^2\theta)=1$

$$\operatorname{sen}^2\theta\left(1+\frac{\cos^2\theta}{\operatorname{sen}^2\theta}\right) \Rightarrow \cancel{\operatorname{sen}^2\theta}\left(\frac{\operatorname{sen}^2\theta+\cos^2\theta}{\cancel{\operatorname{sen}^2\theta}}\right)$$

$$\Rightarrow \underbrace{\operatorname{sen}^2\theta+\cos^2\theta}_{1}$$

$$\Rightarrow 1$$ VERDADERO

b) $1-\csc^2\theta=-\cot^2\theta$

$$\Rightarrow 1-\frac{1}{\operatorname{sen}^2\theta} \Rightarrow \frac{\operatorname{sen}^2\theta-1}{\operatorname{sen}^2\theta} \Rightarrow \frac{-(1-\operatorname{sen}^2\theta)}{\operatorname{sen}^2\theta}$$

$$\Rightarrow -\frac{\cos^2\theta}{\operatorname{sen}^2\theta} \Rightarrow -\cot^2\theta$$ VERDADERO

c) $\operatorname{sen}\theta(\cot\theta+\tan\theta)=\sec\theta$

$$\Rightarrow \operatorname{sen}\theta\left(\frac{\cos\theta}{\operatorname{sen}\theta}+\frac{\operatorname{sen}\theta}{\cos\theta}\right) \Rightarrow \cancel{\operatorname{sen}\theta}\left(\frac{\overbrace{\cos^2\theta+\operatorname{sen}^2\theta}^{1}}{\cancel{\operatorname{sen}\theta}\cos\theta}\right)$$

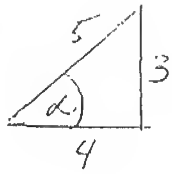
$$\Rightarrow \frac{1}{\cos\theta} \Rightarrow \sec\theta$$ VERDADERO

d) $(1-\operatorname{sen}^2\theta)(1+\tan^2\theta)=-1$

$$\cos^2\theta\left(1+\frac{\operatorname{sen}^2\theta}{\cos^2\theta}\right) \Rightarrow \cancel{\cos^2\theta}\left(\frac{\cos^2\theta+\operatorname{sen}^2\theta}{\cancel{\cos^2\theta}}\right)$$

$\Rightarrow 1$ VERDADERO

37.— $\operatorname{sen}\alpha = -\frac{3}{5}$ 3er cuadrante

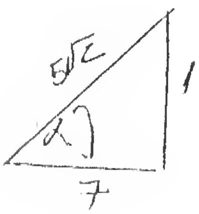


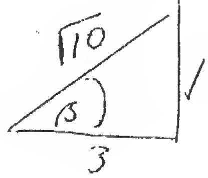
$$\tan(2\alpha) = \frac{\tan\alpha + \tan\alpha}{1-\tan\alpha\cdot\tan\alpha}$$

$$\tan(2\alpha) = \frac{2\cdot\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2} \Rightarrow \frac{\frac{3}{2}}{1-\frac{9}{16}} \Rightarrow \frac{\frac{3}{2}}{\frac{16-9}{16}}$$

$$\Rightarrow \frac{3/1}{\frac{7}{8}} \Rightarrow \frac{24}{7}$$

38.— $\tan\alpha = \frac{1}{7}$ $\quad \operatorname{sen}\beta = \frac{1}{\sqrt{10}}$ $\quad$ 1er cuadrante





$$(\alpha+2\beta) = \pi/4$$

$$\cos(\alpha+2\beta) = \cos\pi/4$$

$$\Rightarrow \cos\alpha\cos(2\beta) - \operatorname{sen}\alpha\operatorname{sen}(2\beta)$$

$$\Rightarrow \cos\alpha\cdot(\cos^2\beta-\operatorname{sen}^2\beta) - \operatorname{sen}\alpha\cdot 2\operatorname{sen}\beta\cos\beta$$

$$\Rightarrow \cos\alpha(\cos^2\beta-\operatorname{sen}^2\beta) - 2\operatorname{sen}\alpha\cos\beta\operatorname{sen}\beta$$

$$\Rightarrow \frac{7}{5\sqrt{2}}\left(\left(\frac{3}{\sqrt{10}}\right)^2-\left(\frac{1}{\sqrt{10}}\right)^2\right)-2\times\frac{1}{5\sqrt{2}}\cdot\frac{3}{\sqrt{10}}\cdot\frac{1}{\sqrt{10}}$$

$$\Rightarrow \frac{7}{5\sqrt{2}}\left(\frac{9}{10}-\frac{1}{10}\right)-\frac{6}{50\sqrt{2}} \Rightarrow \frac{7}{5\sqrt{2}}\left(\frac{\cancel{8}^{4}}{\cancel{10}_{5}}\right)-\frac{3}{25\sqrt{2}}$$

$$\Rightarrow \frac{28}{25\sqrt{2}}-\frac{3}{25\sqrt{2}} \Rightarrow \frac{\cancel{25}^{1}}{\cancel{25}_{1}\sqrt{2}} \Rightarrow \frac{\sqrt{2}}{2}=\cos\left(\frac{\pi}{4}\right)$$

demostrado.

39.— $2\tan\left(\frac{x}{2}\right)$ $\quad x\in[0,90]$

$$\frac{2\cancel{\pi}^{1}}{\cancel{3}_{1}}\times\frac{\cancel{180°}^{60}}{\cancel{\pi\,rad}_{1}} \Rightarrow 120°$$

$$\frac{\pi}{2}=90°$$

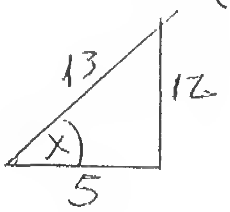
$$2\tan\left(\frac{120}{2}\right)-2\tan\left(\frac{90}{2}\right)$$

$$2\tan 60-2\cancel{\tan 45}^{1}$$

$$2\sqrt{3}-1$$

40.— $\operatorname{sen}(x)=-\frac{12}{13}$ $\quad$ 4º cuadrante.

$$\cos(x+60°)=\cos x\cos 60-\operatorname{sen}x\operatorname{sen}60$$



$$\Rightarrow \frac{5}{13}\times\frac{1}{2}-\frac{12}{13}\times\frac{\sqrt{3}}{2}$$

$$\Rightarrow \frac{5}{26}-\frac{12\sqrt{3}}{26}$$

$$\Rightarrow \frac{5-12\sqrt{3}}{26}$$

41.- $\tan(3x) = ?$

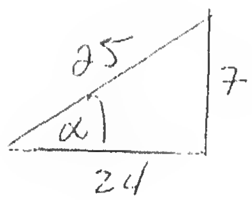
$$\tan(2x) = \tan(x+x) \Rightarrow \frac{\tan x + \tan x}{1 - \tan x \cdot \tan x} \Rightarrow \boxed{\frac{2\tan x}{1-\tan^2 x}}$$

$$\tan(2x+x) = \frac{\tan(2x) + \tan x}{1 - \tan(2x)\cdot\tan x}$$

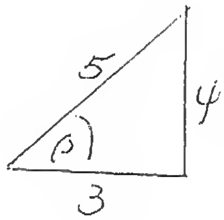
$$\Rightarrow \frac{\dfrac{2\tan x}{1-\tan^2 x} + \tan x}{1 - \dfrac{2\tan x}{1-\tan^2 x}\cdot\tan x} \Rightarrow \frac{\dfrac{2\tan x + \tan x(1-\tan^2 x)}{1-\tan^2 x}}{1 - \dfrac{2\tan^2 x}{1-\tan^2 x}}$$

$$\Rightarrow \frac{\dfrac{2\tan x + \tan x - \tan^3 x}{\cancel{1-\tan^2 x}}}{\dfrac{1-\tan^2 x - 2\tan^2 x}{\cancel{1-\tan^2 x}}} \Rightarrow \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$

42.-



(2do cuadrante)



(3er cuadrante)

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \operatorname{sen}\alpha\operatorname{sen}\beta$$

$$\Rightarrow \frac{24}{25}\times\frac{3}{5} - \frac{7}{25}\times\frac{4}{5}$$

$$\Rightarrow \frac{72}{75} - \frac{28}{75} \Rightarrow \frac{44}{75}$$

43.- a) $\dfrac{(2\operatorname{sen}^2\theta - 1)^2}{\operatorname{sen}^4\theta - \cos^4\theta} = 1 - 2\cos^2\theta$

$$\Rightarrow \frac{(2(1-\cos^2\theta)-1)^2}{(\cancel{\operatorname{sen}^2\theta+\cos^2\theta})(\operatorname{sen}^2\theta - \cos^2\theta)} \Rightarrow \frac{(2-2\cos^2\theta-1)^2}{\operatorname{sen}^2\theta - \cos^2\theta}$$

$$\Rightarrow \frac{(1-2\cos^2\theta)^2}{1-\cos^2\theta-\cos^2\theta} \Rightarrow \frac{(1-2\cos^2\theta)^{\cancel{2}}}{\cancel{(1-2\cos^2\theta)}} \Rightarrow 1-2\cos^2\theta$$

b.) $\dfrac{2\tan x}{1-\tan^2 x}+\dfrac{1}{2\cos^2 x-1}=\dfrac{\cos x+\operatorname{sen} x}{\cos x-\operatorname{sen} x}$

$\Rightarrow \dfrac{2\,\dfrac{\operatorname{sen} x}{\cos x}}{1-\dfrac{\operatorname{sen}^2 x}{\cos^2 x}}+\dfrac{1}{2\cos^2 x-1} \Rightarrow \dfrac{\dfrac{2\operatorname{sen} x}{\cos x}}{\dfrac{\cos^2 x-\operatorname{sen}^2 x}{\cos^2 x}}+\dfrac{1}{2\cos^2 x-1}$

$\Rightarrow \dfrac{2\operatorname{sen} x\cos x}{\cos^2 x-\operatorname{sen}^2 x}+\dfrac{1}{2\cos^2 x-1} \Rightarrow \dfrac{2\operatorname{sen} x\cos x}{\cos^2 x-(1-\cos^2 x)}+\dfrac{1}{2\cos^2 x-1}$

$\Rightarrow \dfrac{2\operatorname{sen} x\cos x}{\cos^2 x-1+\cos^2 x}+\dfrac{1}{2\cos^2 x-1} \Rightarrow \dfrac{2\operatorname{sen} x\cos x}{2\cos^2 x-1}+\dfrac{1}{2\cos^2 x-1}$

$\Rightarrow \dfrac{2\operatorname{sen} x\cos x+1}{2\cos^2 x-1} \Rightarrow \dfrac{2\operatorname{sen} x\cos x+\operatorname{sen}^2 x+\cos^2 x}{\cos^2 x-\operatorname{sen}^2 x} \Rightarrow \dfrac{(\operatorname{sen} x+\cos x)^2}{(\cos x+\operatorname{sen} x)(\cos x-\operatorname{sen} x)}$

$\dfrac{\operatorname{sen} x+\cos x}{\cos x-\operatorname{sen} x}$

c.) $\tan\alpha+\dfrac{1}{\cos^3 x}-\dfrac{1}{\sec\alpha-\tan\alpha}=\dfrac{\operatorname{sen}^2\alpha}{\cos^3\alpha}$

$\Rightarrow \dfrac{\operatorname{sen}\alpha}{\cos\alpha}+\dfrac{1}{\cos^3 x}-\dfrac{1}{\dfrac{1}{\cos\alpha}-\dfrac{\operatorname{sen}\alpha}{\cos\alpha}} \Rightarrow$

$\Rightarrow \dfrac{\operatorname{sen}\alpha}{\cos\alpha}+\dfrac{1}{\cos^3\alpha}-\dfrac{1}{\dfrac{1-\operatorname{sen}\alpha}{\cos\alpha}}$

$$\Rightarrow \frac{\sin\alpha}{\cos\alpha} + \frac{1}{\cos^3\alpha} - \frac{\cos\alpha}{1-\sin\alpha}$$

$$\Rightarrow \frac{\sin\alpha\cos^2\alpha(1-\sin\alpha) + 1 - \sin\alpha - \cos^4\alpha}{\cos^3\alpha(1-\sin\alpha)}$$

$$\Rightarrow \frac{\sin\alpha\cos^2\alpha - \sin^2\cos^2\alpha + 1 - \sin\alpha - \cos^4\alpha}{\cos^3\alpha(1-\sin\alpha)}$$

$$\Rightarrow \frac{\sin\alpha\cos^2\alpha - \sin^2\alpha\cos^2\alpha + \sin^2\alpha + \cos^2\alpha - \sin\alpha - \cos^4\alpha}{\cos^3\alpha(1-\sin\alpha)}$$

$$\Rightarrow \frac{(\sin\alpha\cos^2\alpha - \sin\alpha) - (\sin^2\alpha\cos^2\alpha - \sin^2\alpha) + (\cos^2\alpha - \cos^4\alpha)}{(\cos^3\alpha)(1-\sin\alpha)}$$

$$\Rightarrow \frac{\sin\alpha(\cos^2\alpha - 1) - \sin^2\alpha(\cos^2\alpha - 1) + \cos^2\alpha(1-\cos^2\alpha)}{\cos^3\alpha(1-\sin\alpha)}$$

$$\Rightarrow \frac{-\sin\alpha(1-\cos^2\alpha) + \sin^2\alpha(1-\cos^2\alpha) + \cos^2\alpha(1-\cos^2\alpha)}{\cos^2\alpha(1-\sin\alpha)}$$

$$\Rightarrow \frac{(1-\cos^2\alpha)(-\sin\alpha + \overset{1}{\cancel{\sin^2\alpha + \cos^2\alpha}})}{\cos^3\alpha(1-\sin\alpha)} \Rightarrow \frac{\sin^2\alpha\,(\cancel{1-\sin\alpha})}{\cos^3\alpha\,(\cancel{1-\sin\alpha})}$$

$$\Rightarrow \frac{\sin^2\alpha}{\cos^3\alpha}$$

d) $$\frac{3\cos^2 z + 5\sin z - 5}{\cos^2 z} = \frac{3\sin z - 2}{1+\sin z}$$

$$\Rightarrow \frac{3(1-\sin^2 z) + 5(\sin z - 1)}{1-\sin^2 z} \Rightarrow \frac{3(1-\sin z)(1+\sin z) - 5(1-\sin z)}{(1+\sin z)(1-\sin z)}$$

$$\Rightarrow \frac{\cancel{(1-\sin z)}(3(1+\sin z)-5)}{(1+\sin z)\cancel{(1-\sin z)}} \Rightarrow \frac{3+3\sin z-5}{1+\sin z}$$

$$\Rightarrow \frac{3\sin z-2}{1+\sin z}$$

e.) $$\frac{2\sin^2 w+3\cos w-3}{\sin^2 w} = \frac{2\cos w-1}{1+\cos w}$$

$$\Rightarrow \frac{2(1-\cos^2 w)+3(\cos w-1)}{1-\cos^2 w} \Rightarrow \frac{2(1+\cos w)(1-\cos w)-3(1-\cos w)}{(1+\cos w)(1-\cos w)}$$

$$\Rightarrow \frac{\cancel{(1-\cos w)}(2(1+\cos w)-3)}{(1+\cos w)\cancel{(1-\cos w)}} \Rightarrow \frac{2+2\cos w-3}{1+\cos w}$$

$$\Rightarrow \frac{2\cos w-1}{1+\cos w}$$

F.) $$\frac{\sin^2 t+4\sin t+3}{\cos^2 t} = \frac{3+\sin t}{1-\sin t}$$

$$\Rightarrow \frac{\sin^2 t+\sin t+3\sin t+3}{1-\sin^2 t} \Rightarrow \frac{(\sin^2 t+\sin t)+(3\sin t+3)}{(1+\sin t)(1-\cos t)}$$

$$\Rightarrow \frac{\sin t(\sin t+1)+3(1+\sin t)}{(1+\sin t)(1-\sin t)} \Rightarrow \frac{\cancel{(1+\sin t)}(\sin t+3)}{\cancel{(1+\sin t)}(1-\sin t)}$$

$$\Rightarrow \frac{\sin t+3}{1-\sin t}$$

9.) $\sec y - \dfrac{\cos y}{1+\operatorname{sen} y} = \tan y$

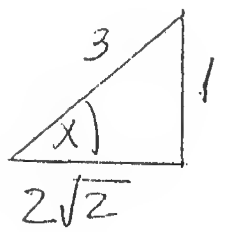
$$\frac{1}{\cos y} - \frac{\cos y}{1+\operatorname{sen} y} \Rightarrow \frac{1+\operatorname{sen} y - \cos^2 y}{(1+\operatorname{sen} y)\cos y} \Rightarrow \frac{1+\operatorname{sen} y - (1-\operatorname{sen}^2 y)}{(1+\operatorname{sen} y)\cos y}$$

$$\Rightarrow \frac{(1+\operatorname{sen} y) - (1+\operatorname{sen} y)(1-\operatorname{sen} y)}{\cos y\,(1+\operatorname{sen} y)} \Rightarrow \frac{\cancel{(1+\operatorname{sen} y)}(1-1+\operatorname{sen} y)}{\cos y\,\cancel{(1+\operatorname{sen} y)}}$$

$$\Rightarrow \frac{\operatorname{sen} y}{\cos y}$$

$$\Rightarrow \tan y$$

44.-



$C = \sqrt{3^2 - 1}$

$C = \sqrt{8}$

$C = 2\sqrt{2}$

a) $\cos x = \dfrac{2\sqrt{2}}{3}$

b) $\cos(2x) = \cos^2 x - \operatorname{sen}^2 x$

$$\Rightarrow \left(\frac{2\sqrt{2}}{3}\right) - \left(\frac{1}{3}\right)^2 \Rightarrow \frac{8}{9} - \frac{1}{9}$$

$$\Rightarrow \frac{7}{9}$$

c) $\operatorname{sen}(2x) = 2\operatorname{sen} x \cos x$

$$\Rightarrow 2\left(\frac{1}{3}\right)\left(\frac{2\sqrt{2}}{3}\right)$$

$$\Rightarrow \frac{4\sqrt{2}}{9}$$

45.- $p(x): 2\operatorname{sen}^2(x) - 7\operatorname{sen}(x) + 3 = 0 \qquad x \in [0, \pi]$

$a = \operatorname{sen} x$

$2a^2 - 7a + 3 = 0$

$(2a - 6)(2a - 1) = 0$

$2a - 6 = 0 \qquad 2a - 1 = 0$

$2a = 6 \qquad 2a = 1$

$\boxed{a = 3} \qquad \boxed{a = 1/2}$

$\Downarrow \qquad \Downarrow$

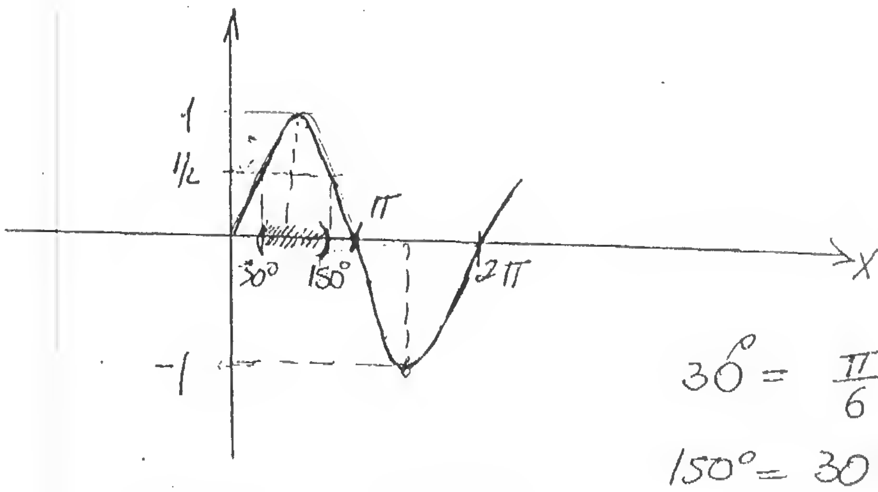
$\cancel{\operatorname{sen} x = 3}$ NO $\qquad \operatorname{sen} x = 1/2$

$x = 30°, 150°$

Suma: $30 + 150$

$\Rightarrow 180° \Rightarrow \pi \text{ rad}$ a) CORRECTO

46.- $p(x): \operatorname{sen} x > 1/2 \quad ; x \in [0, 2\pi)$



$30° = \frac{\pi}{6}$

$150° = 30(5)$

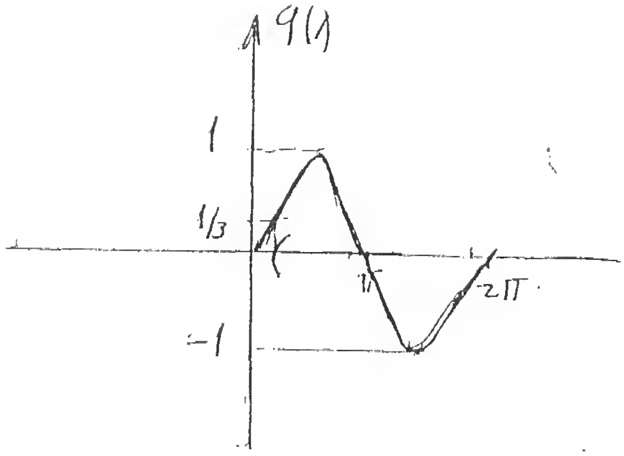
$\Rightarrow \frac{\pi}{6}(5)$

$\Rightarrow \frac{5\pi}{6}$

$Ap(x): (30°, 150°)$

$\Rightarrow \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$

47.- $g(x)$: $\cos x < 1/3$, $x \in (0, 2\pi)$



$\cos x = 1/3$

$x = 70{,}52^\circ$

$Ap(x): (70{,}52^\circ, 360^\circ)$

48.- $f(x) = \cos x + \sqrt{3}\,\text{sen}\,x$

a) $f(x) = R\cos(x - \alpha) \Rightarrow R\cos\alpha\cancel{\cos x} + R\cancel{\text{sen}\,x}\,\text{sen}\,\alpha$

$\cancel{\cos x} + \sqrt{3}\,\cancel{\text{sen}\,x}$

$R\cos\alpha = 1$ $\qquad$ $R\,\text{sen}\,\alpha = \sqrt{3}$

$R = \dfrac{1}{\cos\alpha}$ $\qquad$ $R = \dfrac{\sqrt{3}}{\text{sen}\,\alpha}$

$$\frac{1}{\cos\alpha} = \frac{\sqrt{3}}{\text{sen}\,\alpha}$$

$$\frac{\text{sen}\,\alpha}{\cos\alpha} = \sqrt{3}$$

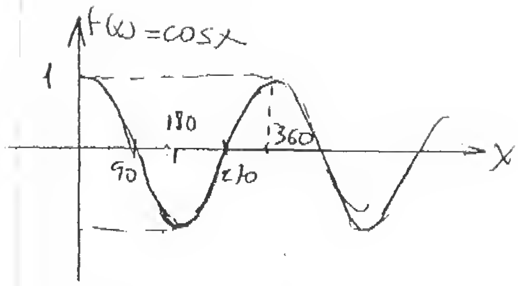
$$\tan\alpha = \sqrt{3}$$

$$\alpha = 60^\circ$$

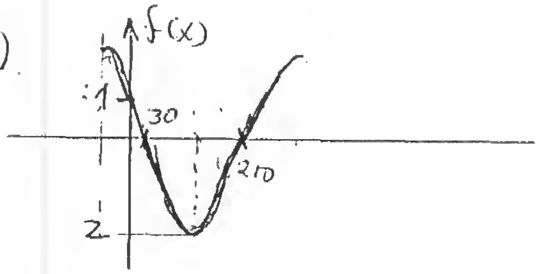
$$R = \frac{1}{\cos\alpha} = \sec 60^\circ = 2$$

b) Rango: $[-2, 2]$

c) $f(x) = 2\cos(x - 60)$



$f(x) = 2\cos(x-60)$



Si es inversible, porque es biyectiva en el intervalo $[0, 90]$

d) $f(x) = 2\cos(x-60) = \sqrt{2}$

$\cos(x-60) = \frac{\sqrt{2}}{2}$

$(x-60) = 45°$

$x = 105°$

49.- $P(x) = 2\,\text{sen}^2 x = 1 - \cos x$

$\Rightarrow 2(1-\cos^2 x) = 1 - \cos x$

$\Rightarrow 2 - 2\cos^2 x = 1 - \cos x$

$\Rightarrow 2\cos^2 x - \cos x - 1 = 0$

$a = \cos x$

$\Rightarrow 2a^2 - a - 1 = 0$

$(2a-2)(2a+1) = 0$

$2a - 2 = 0 \qquad 2a + 1 = 0$

$2a = 2 \qquad 2a = -1$

$a = 1 \qquad a = -1/2$

$\cos x = 1$

$x = 0°; 360°$

$\cos x = -1/2$

$x = 135°; 225°$

Suma $= 0 + 135 + 225 + 360 = 720° \Rightarrow 2\pi$

d) CORRECTO

50.- $2\cos^2 x = \operatorname{sen}(2x)$; $x \in [0, \pi]$

$\Rightarrow 2\cos^2 x = 2\operatorname{sen}x\cos x$

$\Rightarrow 2\cos^2 x - 2\operatorname{sen}x\cos x = 0$

$\Rightarrow 2\cos x(\cos x - \operatorname{sen}x) = 0$

$\Rightarrow 2\cos x = 0$ $\qquad$ $\cos x - \operatorname{sen}x = 0$

$x = 0$ $\qquad$ $\cos x = \operatorname{sen}x$

$\frac{\operatorname{sen}x}{\cos x} = 1$

$\tan x = 1$

$x = 45°$

4.15

Solución
Ejercicios Propuestos
CAPÍTULO CINCO
MATEMÁTICAS

MATRICES y SISTEMAS DE ECUACIONES E INECUACIONES

1= $A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $B=\begin{pmatrix} e & f \\ g & h \end{pmatrix}$

$$A+B=\begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}$$

$$(A+B)^2=\begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}\times\begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}$$

$$(A+B)^2 \Rightarrow \begin{pmatrix} (a+e)^2+(b+f)(c+g) & \text{———} \\ \text{——— ———} & \text{———} \end{pmatrix}$$

$$(a+e)^2+(b+f)(c+g)=\boxed{a^2+2ae+e^2+bc+bg+cf+fg}$$

$$A^2=\begin{pmatrix} a & b \\ c & d \end{pmatrix}\times\begin{pmatrix} a & b \\ c & d \end{pmatrix}=\begin{pmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{pmatrix}$$

$$B^2=\begin{pmatrix} e & f \\ g & h \end{pmatrix}\times\begin{pmatrix} e & f \\ g & h \end{pmatrix}=\begin{pmatrix} e^2+fg & ef+fh \\ eg+hg & fg+h^2 \end{pmatrix}$$

$$2AB=2\begin{pmatrix} a & b \\ c & d \end{pmatrix}\times\begin{pmatrix} e & f \\ g & h \end{pmatrix}=\begin{pmatrix} 2ae+2bg & 2af+2bh \\ 2ce+2dg & 2cf+2dh \end{pmatrix}$$

$$A^2+2AB+B^2=a^2+bc+2ae+2bg+e^2+fg$$

$$(A+B)^2 \neq A^2+B^2+2AB \qquad b)\ \text{FALSO}$$

2-

$$\begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix}_{2\times2} \cdot \underset{2\times3}{X} = \begin{pmatrix} 8 & -1 & 7 \\ 4 & 8 & -3 \end{pmatrix}_{2\times3}$$

$$\begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} = \begin{pmatrix} 8 & -1 & 7 \\ 4 & 8 & -3 \end{pmatrix}$$

$$\begin{cases} 4a+5d = 8 \quad (3) \\ 3a+4d = 4 \quad (-4) \end{cases}$$

$$\begin{cases} 4b+5e = -1 \quad (3) \\ 3b+4e = 8 \quad (-4) \end{cases}$$

$$\begin{cases} 3c+4f = -3 \quad (-4) \\ 4c+5f = 7 \quad (3) \end{cases}$$

$$\begin{array}{r} 12a+15d=24 \\ -12a-16d=-16 \\ \hline -d=8 \end{array}$$

$d=-8$

$4a+5d=8$

$4a=8-5(-8)$

$a=\frac{48}{4}$

$a=12$

$$\begin{array}{r} 12b+15e=-3 \\ -12b-16e=-32 \\ \hline -e=-35 \end{array}$$

$e=35$

$4b+5e=-1$

$4b=-1+5e$

$b=\frac{-1-5(35)}{4}$

$b=-44$

$$\begin{pmatrix} 12 & -44 & 43 \\ -8 & 35 & -33 \end{pmatrix}$$

$$\begin{array}{r} -12c-16f=12 \\ 12c+15f=21 \\ \hline -f=33 \end{array} \Rightarrow f=-33$$

$3c+4f=-3$

$3c=-3-4f$

$c=\frac{-3-4(-33)}{3}$

$c=43$

3.- a)

$$A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \qquad a_{ij}=i+j-2$$

$$A=\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$$

b) $$B=\begin{pmatrix} 0 & 0 \\ 0 & -2 \end{pmatrix}$$

Si es matriz diagonal

c) $A+B=C$

$$\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}+\begin{pmatrix} 0 & 0 \\ 0 & -2 \end{pmatrix}=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

NO ES MATRIZ IDENTIDAD

d) $$\begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \\ d_{31} & d_{32} \end{pmatrix} \qquad d_{ij}=2i+3j-4$$

$$\Rightarrow \begin{pmatrix} 2(1)+3(1)-4 & 2(1)+3(2)-4 \\ 2(2)+3(1)-4 & 2(2)+3(2)-4 \\ 2(3)+3(1)-4 & 3(2)+3(2)-4 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 4 \\ 3 & 6 \\ 5 & 8 \end{pmatrix}$$

e) $$\begin{pmatrix} 1 & 4 \\ 3 & 6 \\ 5 & 8 \end{pmatrix} \times \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 9 \\ 6 & 15 \\ 8 & 21 \end{pmatrix}$$

ii) $\begin{pmatrix} 1 & 4 \\ 3 & 6 \\ 5 & 8 \end{pmatrix} \times \begin{pmatrix} 0 & 0 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 0 & -8 \\ 0 & -12 \\ 0 & -16 \end{pmatrix}$

iii) $\begin{pmatrix} 1 & 4 \\ 3 & 6 \\ 5 & 8 \end{pmatrix} \times \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 6 & 3 \\ 8 & 5 \end{pmatrix}$

4.- $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{pmatrix}$

$A = \begin{pmatrix} 0 & 3 & 4 & 5 \\ 3 & 0 & 5 & 6 \\ 4 & 5 & 0 & 7 \end{pmatrix}$

5.- $f(x,y) = x + y \in M_{3\times 3}$ $\quad A = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 1 & -1 \\ 1 & 1 & 4 \end{pmatrix}$

$A^t = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 4 & -1 & 4 \end{pmatrix}$

$$\begin{array}{c} (-2) \\ (3) \\ (-3) \end{array} \left(\begin{array}{ccc|ccc} 3 & 2 & 4 & 1 & 0 & 0 \\ 2 & 1 & -1 & 0 & 1 & 0 \\ 1 & 1 & 4 & 0 & 0 & 1 \end{array}\right) \Rightarrow (+1)(2) \left(\begin{array}{ccc|ccc} 3 & 2 & 4 & 1 & 0 & 0 \\ 0 & -1 & -11 & -2 & 3 & 0 \\ 0 & -1 & -8 & 1 & 0 & -3 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 3 & 0 & -18 & -3 & 6 & 0 \\ 0 & -1 & -11 & -2 & 3 & 0 \\ 0 & 0 & 3 & 3 & -3 & -3 \end{array}\right) (-11)(6) \left(\begin{array}{ccc|ccc} 1 & 0 & -6 & -1 & 2 & 0 \\ 0 & 1 & 11 & 2 & -3 & 0 \\ 0 & 0 & 1 & 1 & -1 & -1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 0 & 0 & 1 & 5 & -4 & -6 \\ 0 & 1 & 0 & -9 & 8 & 11 \\ 0 & 0 & 1 & 1 & -1 & -1 \end{array}\right) \qquad A = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 1 & -1 \\ 1 & 1 & 4 \end{pmatrix}$$

$$A^t = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 4 & -1 & 4 \end{pmatrix} \qquad A^{-1} = \begin{pmatrix} 5 & -4 & -6 \\ -9 & 8 & 11 \\ 1 & -1 & -1 \end{pmatrix}$$

a) $f(A, A^t) = \begin{pmatrix} 6 & 4 & 5 \\ 4 & 2 & 0 \\ 5 & 0 & 8 \end{pmatrix}$

b) $f(A^T, A^{-1}) = \begin{pmatrix} 8 & -2 & 5 \\ -7 & 9 & 12 \\ 5 & -2 & 3 \end{pmatrix}$

c) $f(A^{-1}, \underbrace{A \cdot A^{-1}}_{I}) = \begin{pmatrix} 5 & -4 & -6 \\ -9 & 8 & 11 \\ 1 & -1 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} 6 & -4 & -6 \\ -9 & 9 & 11 \\ 1 & -1 & 0 \end{pmatrix}$$

d) $f(A^{-1}, A^{-1} A^t)$

$$\begin{pmatrix} 5 & -4 & -6 \\ -9 & 8 & 11 \\ 1 & -1 & -1 \end{pmatrix} \times \begin{pmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 4 & -1 & 4 \end{pmatrix} = \begin{pmatrix} -17 & 12 & -23 \\ 33 & -21 & 43 \\ -3 & 2 & -4 \end{pmatrix}$$

$$\begin{pmatrix} 5 & -4 & -6 \\ -9 & 8 & 11 \\ 1 & -1 & -1 \end{pmatrix} + \begin{pmatrix} -17 & 12 & -23 \\ 33 & -21 & 43 \\ -3 & 2 & -4 \end{pmatrix} = \begin{pmatrix} -12 & 8 & -29 \\ 24 & -13 & 54 \\ -2 & 1 & -5 \end{pmatrix}$$

6.-

a) $B-2C = \begin{pmatrix} -3 & 5 \\ -6 & 2 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} -14 & -8 \\ 6 & 2 \\ 12 & -4 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} -17 & -3 \\ 0 & 4 \\ 12 & -5 \end{pmatrix}$

b.) $9\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 9 & 0 \\ 0 & 9 \end{pmatrix}$

c.-) $BC^t \Rightarrow \begin{pmatrix} -3 & 5 \\ -6 & 2 \\ 0 & -1 \end{pmatrix} \times \begin{pmatrix} 7 & -3 & -6 \\ 4 & -1 & 2 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} -1 & 4 & 28 \\ -34 & 16 & 40 \\ -4 & 1 & -2 \end{pmatrix}$

d.-) $C^t \times B \Rightarrow \begin{pmatrix} 7 & -3 & -6 \\ 4 & -1 & 2 \end{pmatrix} \begin{pmatrix} -3 & 5 \\ -6 & 2 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -3 & 35 \\ -6 & 16 \end{pmatrix}$

7.- $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & 4 \\ -1 & 0 & -2 \end{pmatrix}_{3\times3} \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}_{3\times2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}_{3\times2}$

$$\left.\begin{array}{l} a+2c+3e=0 \\ a+4c+4e=0 \\ -a \quad -2e=0 \end{array}\right\}$$

SISTEMA HOMOGENEO, SOLUCION TRIVIAL

$a=0$
$c=0$
$e=0$

$b+2d+3f=0$
$b+4d+4f=1$
$-b \quad 0 \quad -2f=1$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 1 & 4 & 4 & 1 \\ -1 & 0 & -2 & 1 \end{array}\right) \xrightarrow{(-2)} \left(\begin{array}{ccc|c} 0 & 2 & 1 & 1 \\ 0 & 4 & 2 & 2 \\ -1 & 0 & -2 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 4 & 2 & 2 \\ -1 & 0 & -2 & 1 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 4 & 2 & 2 \\ 1 & 0 & 2 & -1 \end{array}\right)$$

$4d+2f=2$

$\boxed{2d+f=1}$ $\quad b+2f=-1$

$2d=1-f$ $\quad \boxed{b=-1-2f}$

$\boxed{d=\frac{1-f}{2}}$

$f=-1$
$d=1$
$b=1$

$$x=\begin{pmatrix} 0 & 1 \\ 0 & 1 \\ 0 & -1 \end{pmatrix}$$

6) $\begin{pmatrix} 0 & -2 & 2 \\ 3 & 1 & 3 \\ 1 & -2 & 3 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} = \begin{pmatrix} 3 & 6 \\ 6 & 12 \\ 0 & 0 \end{pmatrix}$

$-2c+2e = 3$

$3a+c+3e = 6$

$a-2c+3e = 0$

$(-3)\left(\begin{array}{ccc|c} 0 & -2 & 2 & 3 \\ 3 & 1 & 3 & 6 \\ 1 & -2 & 3 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 0 & -2 & 2 & 3 \\ 0 & 7 & -6 & 6 \\ 1 & -2 & 3 & 0 \end{array}\right) \xRightarrow{(7)(-1)} \left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & -2 & 2 & 3 \\ 0 & 7 & -6 & 6 \end{array}\right)(2)$

$\Rightarrow 2\left(\begin{array}{ccc|c} 1 & 0 & 1 & -3 \\ 0 & -2 & 2 & 3 \\ 0 & 0 & -2 & 33 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 2 & 0 & 0 & 27 \\ 0 & -2 & 0 & 36 \\ 0 & 0 & -2 & 33 \end{array}\right)$

$2a=27$

$\boxed{a=\frac{27}{2}}$

$-2c=36$

$\boxed{c=-18}$

$-2e=33$

$\boxed{e=-33/2}$

$-2d+2f=6$

$3b+d+3f=12$

$b-2d+3f=0$

$\Rightarrow (-3)\left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 3 & 1 & 3 & 12 \\ 0 & -2 & 2 & 6 \end{array}\right)$

$\Rightarrow \left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 7 & -6 & 12 \\ 0 & -1 & 1 & 3 \end{array}\right) \left(\begin{smallmatrix} 7 \\ -2 \end{smallmatrix}\right)\left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & -1 & 1 & 3 \\ 0 & 7 & -6 & 12 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 1 & -6 \\ 0 & -1 & 1 & 3 \\ 0 & 0 & -1 & -33 \end{array}\right)$

$\Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & -39 \\ 0 & -1 & 0 & -30 \\ 0 & 0 & 1 & 33 \end{array}\right)$

$\boxed{b=-39}$

$\boxed{d=30}$

$\boxed{f=33}$

$X=\frac{1}{2}\begin{pmatrix} 27 & -78 \\ -36 & 60 \\ -33 & 66 \end{pmatrix}$

c) $\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} \times \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 1 \end{pmatrix} = \begin{pmatrix} 6 & 9 & 8 \\ 0 & 1 & 6 \end{pmatrix}$

$a + 2b + 3c = 6$

$2a + 3b + 4c = 9$

$3a + 4b + c = 8$

$$(-3)(-2)\left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 2 & 3 & 4 & 9 \\ 3 & 4 & 1 & 8 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & -1 & -2 & -3 \\ 0 & -2 & -8 & -10 \end{array}\right) \Rightarrow (2)\left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & -1 & -2 & -3 \\ 0 & +1 & +4 & +5 \end{array}\right)$$

$$(2)\left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 2 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 2 & 0 & 0 & 2 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & 2 & 2 \end{array}\right)$$

$a=1$

$b=1$

$c=1$

$d + 2e + 3f = 0$

$2d + 3e + 4f = 1$

$3d + 4e + f = 6$

$$(-3)(-2)\left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 6 \end{array}\right) \Rightarrow (2)\left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -1 & -2 & 1 \\ 0 & +2 & +8 & -6 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & -1 & -2 & 1 \\ 0 & 0 & 4 & -4 \end{array}\right)$$

$$(2)\left(\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & -1 & -2 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$d=1$

$e=1$

$f=-1$

$$X = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$

d) $X\begin{pmatrix}3 & 2\\ -2 & -1\end{pmatrix}=\begin{pmatrix}-1 & 2\\ -1 & 1\end{pmatrix}$

$$\begin{pmatrix}a & b\\ c & d\end{pmatrix}\times\begin{pmatrix}3 & 2\\ -2 & -1\end{pmatrix}=\begin{pmatrix}-1 & 2\\ -1 & 1\end{pmatrix}$$

$$(-2)\begin{cases}3a-2b=-1\\ 2a-b=2\end{cases}$$

$\Rightarrow -b=2-2a$

$b=2a-2$

$b=2(5)-2$

$b=8$

$$\begin{array}{r}3a-2b=-1\\ -4a+2b=-4\\ \hline -a=-5\end{array}$$

$a=5$

$$(-2)\begin{cases}3c-2d=-1\\ 2c-d=1\end{cases}$$

$$\begin{array}{r}3c-2d=-1\\ -4c+2d=-2\\ \hline -c=-3\end{array}$$

$c=3$

$b=2c-1$

$b=2(3)-1$

$b=5$

$$X=\begin{pmatrix}5 & 8\\ 3 & 5\end{pmatrix}$$

8.-

$$A=\begin{pmatrix}4 & 0 & 0\\ 1 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}\quad A^2=\begin{pmatrix}16 & 0 & 0\\ 4 & 0 & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$A^3=\begin{pmatrix}16 & 0 & 0\\ 4 & 0 & 0\\ 0 & 0 & 1\end{pmatrix}\times\begin{pmatrix}4 & 0 & 0\\ 1 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}=\begin{pmatrix}64 & 0 & 0\\ 16 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}$$

$$B=\begin{pmatrix}1&1&1\\1&0&-1\\0&0&1\end{pmatrix}$$

$$B^2=\begin{pmatrix}1&1&1\\1&0&-1\\0&0&1\end{pmatrix}\times\begin{pmatrix}1&1&1\\1&0&-1\\0&0&1\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}2&1&1\\1&1&0\\0&0&1\end{pmatrix}$$

$$A^3-3B^2$$

$$\begin{pmatrix}64&0&0\\16&0&0\\0&0&-1\end{pmatrix}+\begin{pmatrix}-6&-3&-3\\-3&-3&0\\0&0&-3\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}58&-3&-3\\13&-3&0\\0&0&-4\end{pmatrix}$$

b)

$$A=\begin{pmatrix}a&b\\c&d\end{pmatrix}\qquad B=\begin{pmatrix}x&y\\z&w\end{pmatrix}\qquad A^t=\begin{pmatrix}a&c\\b&d\end{pmatrix}\qquad B^t=\begin{pmatrix}x&z\\y&w\end{pmatrix}$$

$$A\times B=\begin{pmatrix}ax+bz&ay+bw\\cx+dz&cy+dw\end{pmatrix}\qquad B^t\times A^t=\begin{pmatrix}ax+bz&cx+zd\\ay+bw&cy+dw\end{pmatrix}$$

$$(A\times B)^t=\begin{pmatrix}ax+bz&cx+dz\\ay+bw&cy+dw\end{pmatrix}$$

demostrado.

9.-

$$A=\begin{pmatrix}\cos\beta & 0 & \operatorname{sen}\beta\\ 0 & 1 & 0\\ -\operatorname{sen}\beta & 0 & \cos\beta\end{pmatrix}$$

$$\begin{array}{c}(\operatorname{sen}\beta)\\ \\ (\cos\beta)\end{array}\left(\begin{array}{ccc|ccc}\cos\beta & 0 & \operatorname{sen}\beta & 1 & 0 & 0\\ 0 & 1 & 0 & 0 & 1 & 0\\ -\operatorname{sen}\beta & 0 & \cos\beta & 0 & 0 & 1\end{array}\right)\Rightarrow \begin{array}{c}\\ \\ (-\operatorname{sen}\beta)\end{array}\left(\begin{array}{ccc|ccc}\cos\beta & 0 & \operatorname{sen}\beta & 1 & 0 & 0\\ 0 & 1 & 0 & 0 & 1 & 0\\ 0 & 0 & 1 & \operatorname{sen}\beta & 0 & \cos\beta\end{array}\right)$$

$$\left(\begin{array}{ccc|ccc}\cos\beta & 0 & 0 & 1-\operatorname{sen}^2\beta & 0 & -\operatorname{sen}\beta\cos\beta\\ 0 & 1 & 0 & 0 & 1 & 0\\ 0 & 0 & 1 & \operatorname{sen}\beta & 0 & \cos\beta\end{array}\right)\Rightarrow\left(\begin{array}{ccc|ccc}\frac{\cos\beta}{\cos\beta} & 0 & 0 & \frac{\cos^2\beta}{\cos\beta} & 0 & -\frac{\operatorname{sen}\beta\cos\beta}{\cos\beta}\\ 0 & 1 & 0 & 0 & 1 & 0\\ 0 & 0 & 1 & \operatorname{sen}\beta & 0 & \cos\beta\end{array}\right)$$

$$\Rightarrow\left(\begin{array}{ccc|ccc}1 & 0 & 0 & \cos\beta & 0 & -\operatorname{sen}\beta\\ 0 & 1 & 0 & 0 & 1 & 0\\ 0 & 0 & 1 & \operatorname{sen}\beta & 0 & \cos\beta\end{array}\right) \qquad A^T=\begin{pmatrix}\cos\beta & 0 & -\operatorname{sen}\beta\\ 0 & 1 & 0\\ \operatorname{sen}\beta & 0 & \cos\beta\end{pmatrix}$$

A^{-1}

DEMOSTRADO

10.- $A=\begin{pmatrix}3 & 1\\ -5 & 6\end{pmatrix}$ $\quad B=\begin{pmatrix}4 & 8\\ 0 & -3\end{pmatrix}$ $\quad X=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$

$$AX+X=B$$

$$AX=\begin{pmatrix}3a+c & 3b+d\\ -5a+6c & -5b+6d\end{pmatrix}+\begin{pmatrix}a & b\\ c & d\end{pmatrix}=\begin{pmatrix}4 & 8\\ 0 & -3\end{pmatrix}$$

$$\begin{cases}4a+c=4 & (-7)\\ -5a+7c=0\end{cases}\qquad \begin{cases}4b+d=8 & (-7)\\ -5b+7d=-3\end{cases}$$

$$\begin{array}{r}-28b-7d=-56\\ -5b+7d=-3\\ \hline -33b\quad=-59\end{array}\qquad b=\frac{59}{33}$$

$$\begin{array}{r}-28a-7c=-28\\ -5a+7c=0\\ \hline -33a\quad=-28\end{array}\qquad a=\frac{28}{33}$$

$c=4-4a$

$c=4-4\left(\frac{28}{33}\right)$

$c=\frac{20}{33}$

$$\Rightarrow \frac{1}{33}\begin{pmatrix}28 & 59\\ 20 & 28\end{pmatrix}$$

$d=8-4b\Rightarrow 4\left(2-\frac{59}{33}\right)$

$d=\frac{28}{33}$

11.— $A=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ $B=\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$ $C=\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$

$A \times B=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $B \times C = \left(\begin{array}{l}\text{NO SE}\\ \text{PUEDE}\\ \text{REALIZAR}\\ \text{LA OPERACION.}\end{array}\right)$

12.— $M=\begin{pmatrix} a & 2 \\ 2 & -1 \end{pmatrix}$

a)

$$M^2=\begin{pmatrix} a & 2 \\ 2 & -1 \end{pmatrix} \times \begin{pmatrix} a & 2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} a^2+4 & 2a-2 \\ 2a-2 & 5 \end{pmatrix}$$

b) $M^2=\begin{pmatrix} 5 & -4 \\ -4 & 5 \end{pmatrix}$

$2a-2=-4$

$2a=-2$

$a=-1$

13.— $A=\begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix}$ $B=\begin{pmatrix} 5 & 1 & 0 & -3 \end{pmatrix}$ $C=\begin{pmatrix} 2 & 0 \\ 1 & -4 \\ 3 & 1 \\ 0 & -1 \end{pmatrix}$ $D=\begin{pmatrix} 10 & 15 & -5 \\ 11 & 10 & 10 \end{pmatrix}$

a)

$A \cdot D=\begin{pmatrix} 52 & 65 & 5 \\ 73 & 90 & 10 \end{pmatrix}$

b) $A^t=\begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix} \times \begin{pmatrix} 2 & 1 & 3 & 0 \\ 0 & -4 & 1 & -1 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 6 & -13 & 7 & -4 \\ 4 & -8 & 9 & -3 \end{pmatrix}$

c) $CD = \begin{pmatrix} 2 & 0 \\ 1 & -4 \\ 3 & 1 \\ 0 & -1 \end{pmatrix} \times \begin{pmatrix} 10 & 15 & -5 \\ 11 & 10 & 10 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 20 & 30 & -10 \\ -34 & -25 & -45 \\ 41 & 55 & -5 \\ -11 & -10 & -10 \end{pmatrix}$

d) $C^t B^t$

$\begin{pmatrix} 2 & 1 & 3 & 0 \\ 0 & -4 & 1 & -1 \end{pmatrix} \times \begin{pmatrix} 5 \\ 1 \\ 0 \\ -3 \end{pmatrix} = \begin{pmatrix} 11 \\ -1 \end{pmatrix}$

14.- $B = \begin{pmatrix} 2 & 1 \\ 3 & 1 \\ 0 & 6 \end{pmatrix}$ $A = \begin{pmatrix} 1 & 2 \\ 5 & 3 \end{pmatrix}$

$B \times A = \begin{pmatrix} 7 & 7 \\ 8 & 9 \\ 30 & 18 \end{pmatrix} \Rightarrow (B \times A)^t = \begin{pmatrix} 7 & 8 & 30 \\ 7 & 9 & 18 \end{pmatrix}$ DEMOSTRADO

$\begin{pmatrix} 1 & 5 \\ 2 & 3 \end{pmatrix} \times \begin{pmatrix} 2 & 3 & 0 \\ 1 & 1 & 6 \end{pmatrix} = \begin{pmatrix} 7 & 8 & 30 \\ 7 & 9 & 18 \end{pmatrix}$

15.- $A = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$

$\left(\begin{array}{cc|cc} a & 0 & 1 & 0 \\ 0 & b & 0 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{cc|cc} 1 & 0 & 1/a & 0 \\ 0 & 1 & 0 & 1/b \end{array}\right)$

(I)

$A^{-1} = \begin{pmatrix} 1/a & 0 \\ 0 & 1/b \end{pmatrix} \Rightarrow$ MATRIZ IDENTIDAD, (-I)

16.—

$$A^2 = 2A + I$$

$$A^2 - 2A - I = 0$$

$$(A - 2I)(A + I) = 0$$

$$A - 2I = 0 \qquad A = -I$$

$$A = 2I$$

17.—

$$B = \begin{pmatrix} 1+m & 0 \\ 0 & 1-m \end{pmatrix}$$

$$B^2 = 2B + I$$

$$B^2 = \begin{pmatrix} 1+m & 0 \\ 0 & 1-m \end{pmatrix} \times \begin{pmatrix} 1+m & 0 \\ 0 & 1-m \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} (1+m)^2 & 0 \\ 0 & (1-m)^2 \end{pmatrix} = \begin{pmatrix} 2+2m & 0 \\ 0 & 2-2m \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} (1+m)^2 & 0 \\ 0 & (1-m)^2 \end{pmatrix} = \begin{pmatrix} 3+2m & 0 \\ 0 & 3-2m \end{pmatrix}$$

$$(1+m)^2 = 3+2m$$

$$1 + \cancel{2m} + m^2 = 3 + \cancel{2m}$$

$$m^2 = 2$$

$$m = \pm\sqrt{2}$$

$ad - \cancel{bc} - ab + \cancel{bc}$

18.—

$$A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad P = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\cancel{\begin{array}{c} (-c) \\ (a) \end{array} \left(\begin{array}{cc|cc} a & b & 1 & 0 \\ c & d & 0 & 1 \end{array}\right) \Rightarrow \begin{array}{c} (ad-bc) \\ -b \end{array} \left(\begin{array}{cc|cc} a & b & 1 & 0 \\ 0 & ad-bc & a-c & a \end{array}\right) \left(\begin{array}{cc|cc} a^2d-abc & 0 & ad-ab & ab \\ 0 & ad-bc & a-c & d \end{array}\right)}$$

$A \sim B$

$$\begin{array}{c} \\ (-2) \end{array}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

NO SON SEMEJANTES

19.- $\det(A) = 0$

$2|x| - |x-2| = 0$

$2|x| = |x-2|$

$2x = |x-2|$ $\qquad$ $-2x = |x-2|$

$2x = x-2 \ \wedge \ 2x = -x+2$ $\qquad$ $-2x = x-2 \ \wedge \ -2x = x+2$

$\boxed{x=-2}$ ✓ $\quad 3x = 2$ $\qquad$ $-3x = -2$ $\qquad$ $\boxed{x=-2}$

$\boxed{x = 2/3}$ $\qquad$ $\boxed{x = 2/3}$

Sol: $\{-2, 2/3\}$

20.-

a) $$\begin{array}{c} (4)(3) \\ \\ \\ \end{array}\left(\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 4 & -8 & 0 & 0 & 1 & 0 \\ 3 & 1 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\begin{array}{c} (1) \\ \\ \\ \end{array}\left(\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 0 & -4 & 8 & 4 & 1 & 0 \\ 0 & 4 & 6 & 3 & 0 & 1 \end{array}\right) \Rightarrow \begin{array}{c} (7) \\ 7 \\ (-8)(-4) \end{array}\left(\begin{array}{ccc|ccc} -4 & 0 & 16 & 8 & 1 & 0 \\ 0 & -4 & 8 & 4 & 1 & 0 \\ 0 & 0 & 14 & 7 & 1 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} -28 & 0 & 0 & 0 & -1 & -8 \\ 0 & -28 & 0 & 0 & 3 & -4 \\ 0 & 0 & 14 & 7 & 1 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1/28 & 8/28 \\ 0 & 0 & 0 & 0 & -3/28 & 4/28 \\ 0 & 0 & 1 & 7/14 & 1/14 & 1/14 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} 0 & 1/28 & 8/28 \\ 0 & -3/28 & 4/28 \\ 7/14 & 1/14 & 1/14 \end{pmatrix}$$

$$A^{-1} = \frac{1}{28}\begin{pmatrix} 0 & 1 & 8 \\ 0 & -3 & 4 \\ 14 & 2 & 2 \end{pmatrix}$$

b)

$$B = \begin{pmatrix} 4 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$B^{-1} \underset{(-2)}{=} \left(\begin{array}{ccc|ccc} 4 & 0 & 2 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \Rightarrow (2)\left(\begin{array}{ccc|ccc} 4 & 0 & 2 & 1 & 0 & 0 \\ 0 & -2 & 2 & 1 & -2 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\begin{array}{c}(-2)\\ \Rightarrow(-2)\\ \\ \end{array}\left(\begin{array}{ccc|ccc} 4 & 0 & 2 & 1 & 0 & 0 \\ 0 & -2 & 2 & 1 & -2 & 0 \\ 0 & 0 & 4 & 1 & -2 & 2 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} -8 & 0 & 0 & -1 & -2 & 2 \\ 0 & 4 & 0 & -1 & 2 & 2 \\ 0 & 0 & 4 & 1 & -2 & 2 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & +1/8 & +1/4 & -1/4 \\ 0 & 1 & 0 & -1/4 & 1/2 & 1/2 \\ 0 & 0 & 1 & 1/4 & -1/2 & 1/2 \end{array}\right)$$

$$AB^{-1} \Rightarrow \begin{pmatrix} -1 & 1 & 2 \\ 4 & -8 & 0 \\ 3 & 1 & 0 \end{pmatrix} \times \begin{pmatrix} 1/8 & 1/4 & -1/4 \\ -1/4 & 1/2 & 1/2 \\ 1/4 & -1/2 & 1/2 \end{pmatrix} = \begin{pmatrix} 1/8 & -3/4 & 7/4 \\ 5/2 & -3 & -5 \\ 1/8 & 5/4 & -1/4 \end{pmatrix}$$

$-\frac{1}{8} - \frac{1}{4} + \frac{1}{2} = \frac{1}{8}$ $\quad \frac{1}{2} + 2 = 5/2$

$-\frac{1}{4} + \frac{1}{2} - 1 = -3/4$ $\quad 1 - 4 = -3$ $\quad \frac{3}{8} - \frac{1}{4} = \frac{1}{8}$ $\quad -\frac{3}{4} + \frac{1}{2} = -\frac{1}{4}$

$\frac{1}{4} + \frac{1}{2} + 1 = 7/4$ $\quad -1 - 4 = -5$ $\quad \frac{3}{4} + \frac{1}{2} = 5/4$

c)

$$A \times B \Rightarrow \begin{pmatrix} -1 & 1 & 2 \\ 4 & -8 & 0 \\ 3 & 1 & 0 \end{pmatrix} \times \begin{pmatrix} 4 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 3 & 0 \\ 0 & -8 & 8 \\ 14 & 1 & 6 \end{pmatrix}$$

$$-2 \begin{pmatrix} 4 & 1 & 3 \\ 0 & -4 & 4 \\ 2 & 8 & 2 \end{pmatrix} \Rightarrow \begin{pmatrix} -8 & -2 & -6 \\ 0 & 8 & -8 \\ -4 & -16 & -4 \end{pmatrix}$$

$$A \times B - 2C^t \Rightarrow \begin{pmatrix} -2 & 3 & 0 \\ 0 & -8 & 8 \\ 14 & 1 & 6 \end{pmatrix} + \begin{pmatrix} -8 & -2 & -6 \\ 0 & 8 & -8 \\ -4 & -16 & -4 \end{pmatrix}$$

$$\det. \begin{pmatrix} -10 & 1 & -6 \\ 0 & 0 & 0 \\ 10 & -15 & 2 \end{pmatrix} = 0$$

d)

$$\det. \begin{vmatrix} -1 & 1 & 2 \\ 4 & -8 & 0 \\ 3 & 1 & 0 \end{vmatrix} = 2 \begin{vmatrix} 4 & -8 \\ 3 & 1 \end{vmatrix} \Rightarrow 2(4-(-24))$$

$$\Rightarrow 56$$

$$\det \begin{vmatrix} 4 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} \Rightarrow 4 \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} + 2 \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix}$$

$$\Rightarrow 4(1-0) + 2(2-0)$$

$$\Rightarrow 4 + 4$$

$$\Rightarrow 8$$

$$\det \begin{vmatrix} 4 & 0 & 2 \\ 1 & -4 & 8 \\ 3 & 4 & 2 \end{vmatrix} \Rightarrow 4 \begin{vmatrix} -4 & 8 \\ 4 & 2 \end{vmatrix} + 2 \begin{vmatrix} 1 & -4 \\ 3 & 4 \end{vmatrix}$$

$\Rightarrow 4(-8-32)+2(4-(-12))$

$\Rightarrow -160+32$

$\Rightarrow -128.$

$\det A + \det B - \det C$

$56 + 8 + 128$

$\Rightarrow 192$

e)
$$C=\begin{pmatrix}4 & 0 & 2\\ 1 & -4 & 8\\ 3 & 4 & 2\end{pmatrix}$$

$$\Rightarrow C^{-1} = \begin{array}{c} (-3)\\ (-4)\\ (4)\end{array}\left(\begin{array}{ccc|ccc}4 & 0 & 2 & 1 & 0 & 0\\ 1 & -4 & 8 & 0 & 1 & 0\\ 3 & 4 & 2 & 0 & 0 & 1\end{array}\right) \Rightarrow (-1)\left(\begin{array}{ccc|ccc}4 & 0 & 2 & 1 & 0 & 0\\ 0 & 16 & -30 & 1 & -4 & 0\\ 0 & 16 & 2 & -3 & 0 & 4\end{array}\right)$$

$$\left(\begin{array}{ccc|ccc}4 & 0 & 2 & 1 & 0 & 0\\ 0 & 16 & -30 & 1 & -4 & 0\\ 0 & 0 & 32 & -4 & 4 & 4\end{array}\right) \Rightarrow \begin{array}{c} (-4)\\ (4)\\ (15)\end{array}\left(\begin{array}{ccc|ccc}4 & 0 & 2 & 1 & 0 & 0\\ 0 & 16 & -30 & 1 & -4 & 0\\ 0 & 0 & 8 & -1 & 1 & 1\end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccc|ccc}16 & 0 & 0 & -5 & 1 & 1\\ 0 & 64 & 0 & -11 & -1 & 15\\ 0 & 0 & 8 & -1 & 1 & 1\end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc}1 & 0 & 0 & -5/16 & 1/16 & 1/16\\ 0 & 1 & 0 & -11/64 & -1/64 & 15/64\\ 0 & 0 & 1 & -1/8 & 1/8 & 1/8\end{array}\right)$$

$$B^{-1} \times C^{-1} \Rightarrow \begin{pmatrix}1/8 & 1/4 & -1/4\\ -1/4 & 1/2 & 1/2\\ 1/4 & -1/2 & 1/2\end{pmatrix} \times \begin{pmatrix}-5/16 & 1/16 & 1/16\\ -11/64 & -1/64 & 15/64\\ -1/8 & 1/8 & 1/8\end{pmatrix} = \begin{pmatrix}-\frac{13}{256} & -7/256 & 9/256\\ -58/256 & 10/256 & 42/256\\ -14/256 & 22/256 & -10/256\end{pmatrix}$$

21.- $\begin{pmatrix} -1 & 0 & -2 \\ k & -k & 3 \\ -\frac{k^2}{2} & -3 & -2 \end{pmatrix} \times \begin{pmatrix} -2 & -10 & 1 \\ -k & -\frac{k^3}{3} & k^5 \\ -1 & -2k & 3 \end{pmatrix}$

$-2k \quad +k^2 - 3$

$-10k \quad +\frac{k^4}{3} - 6k$

$k - k^6 - 6$

$\Rightarrow \begin{pmatrix} 4 & 10+4k & -7 \\ k^2-2k-3 & \frac{k^4}{3}-16k & -k^6+k-6 \\ k^2+3k+2 & k^3+5k^2+4k & -3k^5-\frac{k^2}{2}-6 \end{pmatrix}$

$k^2+3k \; 2$

$5k^2+k^3+4k$

$-\frac{k^2}{2} - 3k^5 - 6$

$k^2-2k-3=0$ | $k^2+3k+2=0$ | $k^3+5k^2+4k=0$
$(k-3)(k+1)=0$ | $(k+2)(k+1)=0$ | $k(k^2+5k+4)=0$
$k=3$, $k=-1$ | $k=-2$, $k=-1$ | $k(k+4)(k+1)=0$
 | | $k=0$, $k=-4$, $k=-1$

Sol: $k=-1$

22.-

a) $\begin{vmatrix} x & 1 \\ -1 & 1/x \end{vmatrix}$ $1-(-1) \Rightarrow 2$; si es inversible

$\begin{vmatrix} e^{x} & e^{-2x} \\ e^{2x} & e^{3x} \end{vmatrix} \Rightarrow e^{4x} - e^{-4x} \neq 0$; si es inversible

$\begin{vmatrix} 1 & e^{x} & 0 \\ e^{x} & -e^{2x} & 0 \\ 0 & 0 & 0 \end{vmatrix}$; det = 0 ; NO es inversible

$\begin{vmatrix} \operatorname{sen} x & \cos x \\ -\cos x & \operatorname{sen} x \end{vmatrix}$ det $= \operatorname{sen}^2 x + \cos^2 x \Rightarrow 1$; si es inversible

b) i) $\begin{array}{c} \\ (x) \end{array}\left(\begin{array}{cc|cc} x & 1 & 1 & 0 \\ -1 & 1/x & 0 & 1 \end{array}\right) \Rightarrow \begin{array}{c}(-2) \\ \\ \end{array}\left(\begin{array}{cc|cc} x & 1 & 1 & 0 \\ 0 & 2 & 1 & x \end{array}\right)$

$\left(\begin{array}{cc|cc} -2x & 0 & -1 & x \\ 0 & 2 & 1 & x \end{array}\right) \Rightarrow \left(\begin{array}{cc|cc} 1 & 0 & 1/2x & -1/2 \\ 0 & 1 & 1/2 & x/2 \end{array}\right) \Rightarrow \begin{pmatrix} \frac{1}{2x} & -\frac{1}{2} \\ \frac{1}{2} & \frac{x}{2} \end{pmatrix}$

ii) $\begin{array}{c} -e^x \\ \\ \end{array}\left(\begin{array}{cc|cc} e^x & e^{-2x} & 1 & 0 \\ e^{2x} & e^{3x} & 0 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{cc|cc} e^x & e^{-2x} & 1 & 0 \\ 0 & e^{3x}-e^{-x} & -e^x & 1 \end{array}\right)\begin{array}{c}(e^{3x}-e^{-x}) \\ (-e^{-2x})\end{array}$

$\left(\begin{array}{cc|cc} e^{4x}-1 & 0 & e^{3x}-e^{-x} & -e^{-2x} \\ 0 & e^{3x}-e^{-x} & -e^x & 1 \end{array}\right) \Rightarrow \left(\begin{array}{cc|cc} 1 & 0 & \frac{e^{3x}-e^{-x}}{e^{4x}-1} & -\frac{e^{-2x}}{e^{4x}-1} \\ 0 & 1 & \frac{-e^x}{e^{3x}-e^{-x}} & \frac{1}{e^{3x}-e^{-x}} \end{array}\right)$

$\frac{-e^x}{e^{3x}-e^{-x}} \to \dfrac{\frac{-e^x}{e^{-x}}}{\frac{e^{3x}}{e^{-x}}-\frac{e^{-x}}{e^{-x}}} \Rightarrow \frac{-e^{2x}}{e^{4x}-1}$

$\frac{1}{e^{3x}-e^{-x}} \to \dfrac{\frac{1}{e^{-x}}}{\frac{e^{3x}}{e^{-x}}-\frac{e^{-x}}{e^{-x}}} \Rightarrow \frac{e^x}{e^{4x}-1}$

$A^{-1} = \frac{1}{e^{4x}-1}\begin{pmatrix} e^{3x}-e^{-x} & -e^{-2x} \\ -e^{2x} & e^x \end{pmatrix}$

iii) $\left(\begin{array}{cc|cc} \operatorname{sen} x & \cos x & 1 & 0 \\ -\cos x & \operatorname{sen} x & 0 & 1 \end{array}\right)\begin{array}{c}(\cos x) \\ (\operatorname{sen} x)\end{array}$

$\Rightarrow \begin{array}{c} \\ -\cos x \end{array}\left(\begin{array}{cc|cc} \operatorname{sen} x & \cos x & 1 & 0 \\ 0 & 1 & \cos x & \operatorname{sen} x \end{array}\right) \Rightarrow \left(\begin{array}{cc|cc} \operatorname{sen} x & 0 & -\cos^2 x & -\operatorname{sen} x\cos x \\ 0 & 1 & \cos x & \operatorname{sen} x \end{array}\right)$

$\Rightarrow \left(\begin{array}{cc|cc} 1 & 0 & -\frac{\cos^2 x}{\operatorname{sen} x} & -\cos x \\ 0 & 1 & \cos x & \operatorname{sen} x \end{array}\right) \Rightarrow \begin{pmatrix} -\cot x \cdot \cos x & -\cos x \\ \cos x & \operatorname{sen} x \end{pmatrix}$

23. a) $\det(A) = \begin{vmatrix} 0 & 1 \\ 0 & 1 \end{vmatrix} = 0$

$\det(B) = \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} = 0$

b)

$A \times B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ $B \times A = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}$

$A \times B \neq B \times A$.

c) $C \times D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $D \times C = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

$C \times D \neq D \times C$

24.

$$\begin{pmatrix} a & 0 \\ -a & a \end{pmatrix} + \begin{pmatrix} b & 0 \\ b & b \end{pmatrix} + \begin{pmatrix} -2c & c \\ 2c & 3c \end{pmatrix} = \begin{pmatrix} 8 & -2 \\ -7 & -6 \end{pmatrix}$$

$c = -2$

$a + b - 2c = 8$

$a + b = 8 + 2c$

$a + b = 4$ $\Rightarrow$ $\underbrace{a + b}_{4} + c$

$-a + b + 2c = -7$ $4 - 2$

$-a + b = -7 - 2c$ $\Rightarrow 2$

$-a + b = -3$

25.- $A=\begin{pmatrix} 3 & -2 \\ -3 & 4 \end{pmatrix}$ $\quad -\lambda I=\begin{pmatrix} -\lambda & 0 \\ 0 & -\lambda \end{pmatrix}$

$A-\lambda I=\begin{pmatrix} 3-\lambda & -2 \\ -3 & 4-\lambda \end{pmatrix}$

Matriz singular $\Rightarrow$ $\det(M)=0$

$$\begin{vmatrix} 3-\lambda & -2 \\ -3 & 4-\lambda \end{vmatrix}=0$$

$(3-\lambda)(4-\lambda)-6=0$

$12-3\lambda-4\lambda+\lambda^2-6=0$

$\lambda^2-7\lambda+6=0$ $\Rightarrow$ suma de raices $-\frac{b}{a}$

$(\lambda-6)(\lambda-1)=0$

$\boxed{\lambda=6}$ $\boxed{\lambda=1}$ $\qquad \Rightarrow 7$

C) correcto

26.- $\begin{vmatrix} \log_2 8 & \log_2 4 & -1 \\ \log_3 81 & 3 & -1 \\ \log_2(1/2) & 2 & -4 \end{vmatrix}$

$\log_2 8 = \log_2 2^3 \Rightarrow 3\cancel{\log_2 2}^{1}$

$\log_2(4) \Rightarrow \log_2 2^2 \Rightarrow 2\cancel{\log_2 2}^{1}$

$\log_3 81 \Rightarrow \log_3 3^4 \Rightarrow 4\cancel{\log_3 3}^{1}$

$\log_2(1/2) = \cancel{\log_2 1}^{0} - \cancel{\log_2 2}^{1} \Rightarrow -1$

$$\begin{array}{c} \\ \\ (3)(4) \end{array}\begin{vmatrix} 3 & 2 & -1 \\ 4 & 3 & -1 \\ -1 & 2 & -4 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 0 & 8 & -13 \\ 0 & 11 & -17 \\ -1 & 2 & -4 \end{vmatrix} \Rightarrow - \begin{vmatrix} 8 & -13 \\ 11 & -17 \end{vmatrix} \Rightarrow -(-130-(-143))$$

$$\Rightarrow -7$$

e) CORRECTO

27.- $XA + B = C$

$$X = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} 5 & -2 \\ 7 & 1 \end{pmatrix} + \begin{pmatrix} 6 & 7 \\ 5 & -2 \end{pmatrix} = \begin{pmatrix} -5 & 0 \\ -8 & 7 \end{pmatrix}$$

$$\begin{pmatrix} 5a+7b & -2a+b \\ 5c+7d & -2c+d \end{pmatrix} = \begin{pmatrix} -5 & 0 \\ -8 & 7 \end{pmatrix} + \begin{pmatrix} -6 & -7 \\ -5 & 2 \end{pmatrix}$$

$$(-7)\cdot \begin{cases} 5a+7b = -11 \\ -2a+b = -7 \end{cases} \qquad (-7)\begin{cases} 5c+7d = -13 \\ -2c+d = 9 \end{cases}$$

$$\begin{array}{r} 5a+7b = -11 \\ 14a-7b = 49 \\ \hline 19a = 38 \end{array} \qquad \begin{array}{r} 5c+7d = -13 \\ +14c-7d = -63 \\ \hline 19c = -76 \end{array}$$

$a = 2$

$-2a+b = -7$

$b = -7+2a$

$b = -3$

$c = -4$

$-2c+d = 9$

$d = 9+2c$

$d = 1$

$$X = \begin{pmatrix} 2 & -3 \\ -4 & 1 \end{pmatrix}$$

$$\det(X) = (2-12)$$

$$\Rightarrow -10$$

28!

$$P(x) = \begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix} + \begin{vmatrix} 8x & 2 \\ -1 & x \end{vmatrix} - \begin{vmatrix} 4 & 1 \\ -2 & x \end{vmatrix} = 0$$

$$\Rightarrow (4-6) + (8x^2+2) - (4x+2) = 0$$

$$\Rightarrow -2 + 8x^2 \cancel{+2} - 4x - 2 = 0$$

$$\Rightarrow 8x^2 - 4x - 2 = 0$$

$$\Rightarrow 4x^2 - 2x - 1 = 0$$

suma de raices : $\frac{-b}{a} \Rightarrow \frac{-(-2)}{4}$

$\Rightarrow \frac{1}{2}$ a) CORRECTO

29-

$$A = \begin{pmatrix} \sin 90 & \sec 60 & \sin 90 \\ \cos 360 & \sin 180 & \cos 60 \\ -\cos 180 & \sin 360 & \cos 360 \end{pmatrix} \quad B = \begin{pmatrix} \log 1 & \log_2\left(\frac{1}{4}\right) & 0 \\ \log_2\left(\frac{1}{4}\right) & \log_3 9 & 0 \\ \log_3 9 & \ln e & \ln e \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 0 & 1/2 \\ 1 & 1 & 1/2 \end{pmatrix} \quad B = \begin{pmatrix} 0 & -2 & 0 \\ -2 & 2 & 0 \\ 2 & 1 & 1 \end{pmatrix}$$

$$A^t = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 1 & 1/2 & 1/2 \end{pmatrix} ; \frac{B}{2} = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 1 & 0 \\ 1 & 1/2 & 1/2 \end{pmatrix}$$

$+$

$(-2)\ (-1)$

$$R = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{pmatrix} ; \det(R) = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix}$$

$\Rightarrow -1$ d) CORRECTO

30.- $A=\begin{vmatrix} m & h & g \\ f & e & d \\ c & b & a \end{vmatrix}=10$

$$B=\begin{vmatrix} a & b & c \\ d-3a & e-3b & f-3c \\ 2g & 2h & 2m \end{vmatrix}$$

$$\begin{vmatrix} c & b & a \\ f & e & d \\ m & h & g \end{vmatrix}=-10 \Rightarrow \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & m \end{vmatrix}=10$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ 2g & 2h & 2m \end{vmatrix}=20 \Rightarrow \begin{vmatrix} a & b & c \\ d-3a & e-3b & f-3c \\ 2g & 2h & 2m \end{vmatrix}=20$$

$$\frac{det(A)}{det(B)}=\frac{10}{20}=\frac{1}{2}$$ d) CORRECTO

31.- $det(A)=\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}=n$

$$B=\begin{vmatrix} 6d & 4e & 2f \\ 3g & 2h & i \\ 9a & 6b & 3c \end{vmatrix}$$

$$\begin{vmatrix} g & h & i \\ d & e & f \\ a & b & c \end{vmatrix}=-n \Rightarrow \begin{array}{c} \begin{array}{ccc} (3) & (2) & \\ \downarrow & \downarrow & \end{array} \\ \begin{vmatrix} d & e & f \\ g & h & i \\ a & b & c \end{vmatrix} \end{array}=n$$

$$\overset{(2)}{\Rightarrow}\begin{vmatrix} 3d & 2e & f \\ 3g & 2h & i \\ 3a & 2b & c \end{vmatrix} = 6n \Rightarrow \begin{vmatrix} 6d & 4e & 2f \\ 3g & 2h & i \\ 9a & 6b & 3c \end{vmatrix} = 36n$$

(3)

b)

$$\begin{vmatrix} d+f & e & f+e \\ a+c & b & c+b \\ g+i & h & i+h \end{vmatrix}$$

$$\begin{vmatrix} d & e & f \\ a & b & c \\ g & h & i \end{vmatrix} = -n \Rightarrow \begin{vmatrix} d+f & e & f+e \\ a+c & b & c+b \\ g+i & h & i+h \end{vmatrix} = -n$$

32 –

$P(X)$:

$$(-x)(-1)\begin{vmatrix} x+2 & 1 & 1 & 1 \\ 1 & x+2 & 1 & 1 \\ 1 & 1 & x+2 & 1 \\ x & x & x & x \end{vmatrix} = 0 \quad (-x-2)$$

$$\begin{vmatrix} 0 & -x-1 & -(x+2)^2 & -x-1 \\ 0 & x+1 & -x-1 & 0 \\ 1 & 1 & x+2 & 1 \\ 0 & 0 & -x^2-x & 0 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} -x-1 & -(x+2)^2 & -x-1 \\ x+1 & -x-1 & 0 \\ 0 & -x^2-x & 0 \end{vmatrix} = 0$$

$$(x^2+x)\begin{vmatrix} -x-1 & -x-1 \\ x+1 & 0 \end{vmatrix} = 0 \Rightarrow (x^2+x)(x+1)^2 = 0$$

$x(x+1)(x+1)^2 = 0$

$x(x+1)^3 = 0$

$x = 0$, $x = -1$

33.—

$$f(x)=\begin{vmatrix} a & b & -2a & 3b \\ -1 & x & 0 & 0 \\ 0 & -1 & x & 0 \\ 0 & 0 & -1 & x \end{vmatrix} \overset{(a)}{\Rightarrow} -\begin{vmatrix} -1 & x & 0 & 0 \\ a & b & -2a & 3b \\ 0 & -1 & x & 0 \\ 0 & 0 & -1 & x \end{vmatrix}$$

$$\Rightarrow -\begin{vmatrix} -1 & x & 0 & 0 \\ 0 & b+ax & -2a & 3b \\ 0 & -1 & x & 0 \\ 0 & 0 & -1 & x \end{vmatrix} \overset{(b+ax)}{\Rightarrow} -\begin{vmatrix} -1 & x & 0 & 0 \\ 0 & -1 & x & 0 \\ 0 & (b+ax) & -2a & 3b \\ 0 & 0 & -1 & x \end{vmatrix}$$

$$\Rightarrow -\begin{vmatrix} -1 & x & 0 & 0 \\ 0 & -1 & x & 0 \\ 0 & 0 & bx+ax^2-2a & 3b \\ 0 & 0 & -1 & x \end{vmatrix} \Rightarrow -\begin{vmatrix} -1 & x & 0 & 0 \\ 0 & -1 & x & 0 \\ 0 & 0 & -1 & x \\ 0 & 0 & ax^2+bx-2a & 3b \end{vmatrix} (ax^2+bx-2a)$$

$$\Rightarrow -\begin{vmatrix} -1 & x & 0 & 0 \\ 0 & -1 & x & 0 \\ 0 & 0 & -1 & x \\ 0 & 0 & 0 & \boxed{} \end{vmatrix} \Rightarrow \det = ax^3+bx^2-2ax+3b = f(x)$$

$\boxed{} \downarrow ax^3+bx^2-2ax+3b$

$\Rightarrow f(0) = 0+0-0+3b = -3$

$3\cdot b = -3$

$b=-1$

$f(1)=f(-1)$

$a+\cancel{b}-2a+\cancel{3b} = -a+\cancel{b}+2a+\cancel{3b}$

$-a = a$

$2a=0$

$a=0$

34.- a) CORRECTO, por propiedades

b) CORRECTO

$$-\begin{vmatrix} c & d \\ a & b \end{vmatrix} \Rightarrow \begin{vmatrix} d & c \\ b & a \end{vmatrix}$$

c) CORRECTO

$$K^2A = \begin{vmatrix} K^2a & b \\ K^2c & d \end{vmatrix} \Rightarrow K^3A = \begin{vmatrix} K^2a & Kb \\ K^2c & Kd \end{vmatrix}$$

d) CORRECTO

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} \Rightarrow ad-bc$$

$$\begin{vmatrix} a & c \\ b & d \end{vmatrix} \Rightarrow ad-bc$$

e) FALSO

$$\begin{vmatrix} Ka & Kb \\ Kc & Kd \end{vmatrix} = K^2\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

35.- a) $\det = \begin{vmatrix} 5 & 0 \\ 1 & 3 \end{vmatrix} = \underline{\underline{15}}$

b) $\det = \begin{vmatrix} 2 & 1 \\ 0 & 4 \end{vmatrix} = \underline{\underline{8}}$

c) $\begin{pmatrix} 5 & 0 \\ 1 & 3 \end{pmatrix} \times \begin{pmatrix} 2 & 1 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 10 & 5 \\ 2 & 13 \end{pmatrix}$

d) $\begin{pmatrix} 2 & 1 \\ 0 & 4 \end{pmatrix} \times \begin{pmatrix} 5 & 0 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 11 & 3 \\ 4 & 12 \end{pmatrix}$

e) $\det(AB) = (130-10) = 120$

$\det(A) \times \det(B) = 15 \times 8 = 120$

e.1) $\det(B \times A) = 132 - 12 = 120$

$\det(A) \times \det(B) = 15 \times 8 = 120$

36.— $A = \begin{pmatrix} \cos\alpha & \operatorname{sen}\alpha \\ 1 & \cos\alpha \end{pmatrix}$

$\det(A) = \cos^2\alpha - \operatorname{sen}\alpha = 1$

$\Rightarrow \cancel{1} - \operatorname{sen}^2\alpha - \operatorname{sen}\alpha - \cancel{1} = 0$

$-\operatorname{sen}\alpha(\operatorname{sen}\alpha + 1) = 0$

$\operatorname{sen}\alpha = 0$

$\alpha = 0°, 180°, 360° \ldots$ $\quad \operatorname{sen}\alpha = -1$

$\alpha = 270°$

Sol: $\{ n\pi, -\frac{3\pi}{2} \} \cdot n \in \mathbb{Z}$.

37.— $(-1)\begin{vmatrix} 1 & a & b+c \\ 1 & b & a+c \\ 1 & c & a+b \end{vmatrix} = 0$

$\begin{vmatrix} 1 & a & b+c \\ 0 & b-a & a-b \\ 0 & c-a & a-c \end{vmatrix} = 0 \; (b-a)(a-c) - (a-b)(c-a)$

$\Rightarrow ab - bc - a^2 + ac - (ac - a^2 - bc + ab)$

$\Rightarrow \cancel{ab} - \cancel{bc} - \cancel{a^2} + \cancel{ac} - \cancel{ac} + \cancel{a^2} + \cancel{bc} - \cancel{ab}$

$\det(A) = 0$

demostrado

38.-
$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 3$$

a)
$$2*3 = \begin{vmatrix} a & b & c \\ d & e & f \\ 2g & 2h & 2i \end{vmatrix}$$

$\Rightarrow \underline{6}$

b)
$$\begin{array}{ccc} & & *-1 \\ \end{array}$$
$$\begin{vmatrix} c & b & a \\ f & e & d \\ i & h & g \end{vmatrix} = -3 \Rightarrow \begin{vmatrix} c & b & -a \\ f & e & -d \\ i & h & -g \end{vmatrix} = -1(-3) \Rightarrow 3$$

c)
$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 3 \Rightarrow \begin{vmatrix} a-4c & b & c \\ d-4f & e & f \\ g-4i & h & i \end{vmatrix}$$
$*4$

39.-
$$\begin{vmatrix} \operatorname{sen} x & \cos x & \operatorname{sen} x \\ \cos x - \operatorname{sen} x & -\operatorname{sen} x & \cos x \\ 1+\cos x+\operatorname{sen} x & \cos x & \operatorname{sen} x \end{vmatrix}$$

$$(-1)\begin{array}{c}(-\cos x) \\ \Rightarrow \frac{\operatorname{sen} x}{\ }\end{array}\begin{vmatrix} \operatorname{sen} x & \cos x & \operatorname{sen} x \\ \cos x & -\operatorname{sen} x & \cos x - \operatorname{sen} x \\ \operatorname{sen} x & \cos x & 1+\cos x+\operatorname{sen} x \end{vmatrix} \Rightarrow -\frac{1}{\operatorname{sen} x}\begin{vmatrix} \operatorname{sen} x & \cos x & \operatorname{sen} x \\ 0 & -1 & -\operatorname{sen}^2 x \\ 0 & 0 & 1+\cos x \end{vmatrix}$$

$$\Rightarrow -\frac{\cancel{\operatorname{sen} x}}{\cancel{\operatorname{sen} x}}\begin{vmatrix} -1 & -\operatorname{sen}^2 x \\ 0 & 1+\cos x \end{vmatrix} \Rightarrow -(-1)(1+\cos x)$$
$$\Rightarrow 1+\cos x$$

40.- $(-1)\begin{vmatrix} 1 & \operatorname{sen} a & \cos a \\ 1 & \operatorname{sen} b & \cos b \\ 1 & \operatorname{sen} c & \cos c \end{vmatrix} = \operatorname{sen}(b-c) + \operatorname{sen}(a-b) + \operatorname{sen}(a-c)$

$\Rightarrow \begin{vmatrix} 1 & \operatorname{sen} a & \cos a \\ 0 & \operatorname{sen} b - \operatorname{sen} a & \cos b - \cos a \\ 0 & \operatorname{sen} c - \operatorname{sen} a & \cos c - \cos a \end{vmatrix} \Rightarrow (\operatorname{sen} b - \operatorname{sen} a)(\cos c - \cos a) - (\cos b - \cos a)(\operatorname{sen} c - \operatorname{sen} a)$

$\Rightarrow \operatorname{sen} b \cos c - \operatorname{sen} b \cos a - \operatorname{sen} a \cos c + \operatorname{sen} a \cos a -$

$(\operatorname{sen} c \cos b - \operatorname{sen} a \cos b - \operatorname{sen} c \cos a + \operatorname{sen} a \cos a)$

$\Rightarrow \operatorname{sen} b \cos c - \operatorname{sen} b \cos a - \operatorname{sen} a \cos c + \cancel{\operatorname{sen} a \cos a} - \operatorname{sen} c \cos b + \operatorname{sen} a \cos b + \operatorname{sen} c \cos a - \cancel{\operatorname{sen} a \cos a}$

$\Rightarrow \underbrace{(\operatorname{sen} b \cos c - \operatorname{sen} c \cos b)}_{\operatorname{sen}(b-c)} + \underbrace{(\operatorname{sen} a \cos b - \operatorname{sen} b \cos a)}_{\operatorname{sen}(a-b)} + \underbrace{(\operatorname{sen} c \cos a - \operatorname{sen} a \cos c)}_{\operatorname{sen}(c-a)}$

$\operatorname{sen}(b-c) \quad + \quad \operatorname{sen}(a-b) \quad + \quad \operatorname{sen}(c-a)$

41.-

a) $\begin{vmatrix} x+4 & x+14 \\ x & 2x+1 \end{vmatrix} \Rightarrow (x+4)(2x+1) - x(x+14)$

$\Rightarrow 2x^2 + x + 8x + 4 - x^2 - 14x$

$\Rightarrow x^2 - 5x + 4$

$\Rightarrow (x-4)(x-1)$

b) $\begin{array}{l} -(x+1) \\ -(x+2) \end{array} \begin{vmatrix} 1 & -1 & 9 \\ x+1 & x & 2 \\ x+2 & x+1 & x-1 \end{vmatrix} \Rightarrow \begin{vmatrix} 1 & -1 & 9 \\ 0 & 2x+1 & -9x-7 \\ 0 & 2x+3 & -8x-19 \end{vmatrix}$

$\Rightarrow (2x+1)(-8x-19) - (-9x-7)(2x+3)$

$\Rightarrow -16x^2 - 38x - 8x - 19 - (-18x^2 - 27x - 14x - 21)$

$\Rightarrow -16x^2 - 46x - 19 + 18x^2 + 41x + 21$

$\Rightarrow 2x^2-5x+2$

$\Rightarrow \frac{1}{\cancel{2}}(\cancel{2}x-\cancel{4})(2x-1)$

$\Rightarrow (x-2)(2x-1)$

-3) (C) $\Rightarrow \begin{vmatrix} 1 & -2 & 2 \\ x & -4 & 4 \\ 3 & x & 6 \end{vmatrix} \Rightarrow \begin{vmatrix} 1 & -2 & 2 \\ 0 & 2x-4 & -2x+4 \\ 0 & x+6 & 0 \end{vmatrix} \Rightarrow -(x+6)(-2x+4)$

$\Rightarrow (2x-4)(x+6)$

42.-

$$p(x): \begin{vmatrix} 2\sin^2(2x) & 2\cos(x) \\ 3\sin(x) & 1 \end{vmatrix} = -1$$

$2\sin^2(2x) - 6\sin x\cos x = -1$

$4\sin x\cos x - 6\sin x\cos x = -1$

$-2\sin x\cos x = -1$

$2\sin x\cos x = 1$

$\sin(2x) = 1$

$2x = 90°$

$x = 45°$

$\boxed{S.P: \frac{\pi}{4}}$

$$43.- \begin{vmatrix} x^2 & x & 1 \\ y^2 & y & 1 \\ z^2 & z & 1 \end{vmatrix} (-1) \Rightarrow \begin{vmatrix} x^2 & x & 1 \\ y^2-x^2 & y-x & 0 \\ z^2-x^2 & z-x & 0 \end{vmatrix} \Rightarrow (y^2-x^2)(z-x)-(y-x)(z^2-x^2)$$

$$\Rightarrow (y+x)(y-x)(z-x)-(y-x)(z-x)(z+x)$$

$$\Rightarrow (y-x)(z-x)(y+\cancel{x}-z-\cancel{x})$$

$$\Rightarrow (y-x)(z-x)(y-z)$$ Demostrado

$$44.- \text{(2)}\begin{vmatrix} x & y & z \\ 5 & 0 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 1$$

$$\begin{vmatrix} x & y & z \\ 2x+5 & 2y & 2z+3 \\ x+1 & y+1 & z+1 \end{vmatrix} = 1$$

$$45.- \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 5$$

$$\begin{vmatrix} a_{31} & a_{32} & a_{33} \\ a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \end{vmatrix} = -5 \Rightarrow \begin{vmatrix} -a_{31} & -a_{32} & -a_{33} \\ a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \end{vmatrix} = 5$$

$$\Rightarrow \begin{vmatrix} -a_{31} & -2a_{32} & a_{33} \\ a_{21} & 2a_{22} & a_{23} \\ a_{11} & 2a_{12} & a_{13} \end{vmatrix} = 10$$

$$\Rightarrow \begin{vmatrix} -a_{31} & -2a_{32} & -a_{33} \\ 3a_{21} & -6a_{22} & -3a_{23} \\ a_{11} & 2a_{12} & a_{13} \end{vmatrix} = 30.$$

A 6.- $\begin{vmatrix} a_1 & b_1+b_2 \\ c_1 & d_1+d_2 \end{vmatrix} \Rightarrow \begin{vmatrix} a_1 & b_1 \\ c_1 & d_1 \end{vmatrix} + \begin{vmatrix} a_1 & b_2 \\ c_1 & d_2 \end{vmatrix}$

$$a_1(d_1+d_2) - c_1(b_1+b_2)$$

$$a_1d_1 - b_1c_1 + a_1d_2 - b_2c_1$$

$$a_1(d_1+d_2) - c_1(b_1+b_2)$$

demostrado.

47.-

$$\begin{cases} x + 2y + z = k \\ 2x + y + 4z = 6 \\ x - 4y + 5z = 9 \end{cases}$$

$$\begin{matrix}(-1)(-2)\end{matrix}\left(\begin{array}{ccc|c} 1 & 2 & 1 & k \\ 2 & 1 & 4 & 6 \\ 1 & -4 & 5 & 9 \end{array}\right) \Rightarrow (-2)\left(\begin{array}{ccc|c} 1 & 2 & 1 & k \\ 0 & -3 & 2 & 6-2k \\ 0 & -6 & 4 & 9-k \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 1 & k \\ 0 & -3 & 2 & 6-2k \\ 0 & 0 & 0 & -3+3k \end{array}\right)$$

Para que el sistema de ecuaciones sea consistente $-3+3k=0$

$k=1$

$k=1$; soluciones infinitas

$$-3y+2z=4 \qquad\qquad x+2y+z=1$$

$$-3y=4-2z \qquad\qquad x=1-z-2y$$

$$y=\frac{2z-4}{3} \qquad\qquad x=1-z-\frac{2(2z-4)}{3}$$

$$x=\frac{3-3z-4z+8}{3}$$

$$x=\frac{11-7z}{3}$$

Sol: $\left\{\frac{11-7a}{3}, \frac{2a-4}{3}, a\right\}$

48:

$$A=\begin{pmatrix}1&0&2\\2&1&0\\3&1&-1\end{pmatrix}$$

$$\begin{array}{l}(-3)(-2)\\ \Rightarrow\end{array}\left(\begin{array}{ccc|ccc}1&0&2&1&0&0\\2&1&0&0&1&0\\3&1&-1&0&0&1\end{array}\right)\Rightarrow(-1)\left(\begin{array}{ccc|ccc}1&0&2&1&0&0\\0&1&-4&-2&1&0\\0&1&-7&-3&0&1\end{array}\right)$$

$$\begin{array}{l}(3)\\(-3)\\(2)(2)\end{array}\left(\begin{array}{ccc|ccc}1&0&2&1&0&0\\0&1&-4&-2&1&0\\0&0&-3&-1&-1&1\end{array}\right)\Rightarrow\left(\begin{array}{ccc|ccc}3&0&0&1&-2&2\\0&-3&0&2&-7&4\\0&0&-3&-1&-1&1\end{array}\right)$$

a)

$$\Rightarrow\left(\begin{array}{ccc|ccc}1&0&0&1/3&-2/3&2/3\\0&1&0&-2/3&7/3&-4/3\\0&0&1&1/3&1/3&-1/3\end{array}\right)$$

$$A^t=\begin{pmatrix}1&2&3\\0&1&1\\2&0&-1\end{pmatrix}$$

$$(-2)\left(\begin{array}{ccc|ccc}1&2&3&1&0&0\\0&1&1&0&1&0\\2&0&-1&0&0&1\end{array}\right)\xrightarrow{(4)(-2)}\left(\begin{array}{ccc|ccc}1&2&3&1&0&0\\0&1&1&0&1&0\\0&-4&-7&-2&0&1\end{array}\right)$$

$$\text{b)}\left(\begin{array}{ccc|ccc}1&0&1&1&-2&0\\0&1&1&0&1&0\\0&0&-3&-2&4&1\end{array}\right)\Rightarrow\left(\begin{array}{ccc|ccc}3&0&0&1&-2&1\\0&3&0&-2&7&1\\0&0&-3&-2&4&1\end{array}\right)$$

b) $$\Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/3 & -2/3 & 1/3 \\ 0 & 1 & 0 & -2/3 & 7/3 & 1/3 \\ 0 & 0 & 1 & 2/3 & -4/3 & -1/3 \end{array}\right)$$

c) $$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 2 & 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{aligned} x+2y+3z &= 4 \\ y+z &= 0 \\ 2x+0-z &= 1 \end{aligned} \quad \Rightarrow \quad (-2)\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 0 \\ 2 & 0 & -1 & 1 \end{array}\right)$$

$$\begin{array}{c} \\ (4)(-2) \\ \\ \end{array}\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 0 \\ 0 & -4 & -7 & -7 \end{array}\right) \Rightarrow \begin{array}{c} (3) \\ (3) \\ \\ \end{array}\left(\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -3 & -7 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccc|c} 3 & 0 & 0 & 5 \\ 0 & 3 & 0 & -7 \\ 0 & 0 & -3 & -7 \end{array}\right) \quad 3x=5 \quad 3y=-7 \quad -3z=-7$$

$$x = 5/3 \qquad y = -7/3 \qquad z = 7/3$$

e.1) $$\begin{pmatrix} 1/3 & -2/3 & 2/3 \\ -2/3 & 7/3 & -4/3 \\ 1/3 & 1/3 & -1/3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$$

$$\begin{aligned} \frac{1}{3}x - \frac{2}{3}y + \frac{2}{3}z &= 2 \\ -\frac{2x}{3} + \frac{7}{3}y - \frac{4}{3}z &= 3 \\ \frac{x}{3} + \frac{y}{3} - \frac{z}{3} &= -4 \end{aligned} \qquad \begin{cases} x-2y+2z=6 \\ -2x+7y-4z=9 \\ x+y-z=-12 \end{cases}$$

$$(-1)(2)\left(\begin{array}{ccc|c} 1 & -2 & 2 & 6 \\ -2 & 7 & -4 & 9 \\ 1 & 1 & -1 & -12 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & -2 & 2 & 6 \\ 0 & 3 & 0 & 21 \\ 0 & 3 & -3 & -18 \end{array}\right)$$

$$(-1)(2)\left(\begin{array}{ccc|c} 1 & -2 & 2 & 6 \\ 0 & 1 & 0 & 7 \\ 0 & 1 & -1 & -6 \end{array}\right) \Rightarrow (2)\left(\begin{array}{ccc|c} 1 & 0 & 2 & 20 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & -1 & -13 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & -1 & -13 \end{array}\right) \quad x=-6 \quad y=7 \quad z=13$$

49.-

$$A=\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \quad B=\begin{pmatrix} m & n & o \\ p & q & r \\ s & t & u \end{pmatrix}$$

$$3A+2B=\begin{pmatrix} 3 & 8 & -3 \\ -2 & 2 & -3 \\ 7 & 2 & 4 \end{pmatrix} \quad A-B=\begin{pmatrix} 0 & 1 & 0 \\ -1 & -2 & -3 \\ 2 & 1 & 2 \end{pmatrix}$$

$$\begin{cases} 3a+2m=3 \\ a-m=0 \ (2) \end{cases}$$

$$\begin{array}{r} 3a+2m=3 \\ 2a-2m=0 \\ \hline 5a=3 \end{array}$$

$a=3/5$

$a-m=0$

$m=a$

$m=3/5$

$$\begin{cases} 3b+2n=8 \\ b-n=1 \ (2) \end{cases}$$

$$\begin{array}{r} 3b+2n=8 \\ 2b-2n=2 \\ \hline 5b=10 \end{array}$$

$b=2$

$b-n=1$

$b-1=n$

$n=2-1$

$n=1$

$$\begin{cases} 3c+2o=-3 \\ c-o=0 \ (2) \end{cases}$$

$$3c+2o=-3$$
$$2c-2o=0$$
$$5c=-3$$
$$c=-3/5$$

$$c-o=0$$
$$o=c$$
$$o=-3/5$$

$$3f+2r=-3$$
$$2f-2r=-6$$
$$5f=-9$$
$$f=-9/5$$
$$r=f+3$$
$$r=6/5$$

$$\begin{cases} 3d+2p=-2 \\ d-p=-1\ (2) \end{cases}$$

$$\begin{cases} 3e+2q=2 \\ e-q=-2\ (2) \end{cases}$$

$$\begin{cases} 3f+2r=-3 \\ f-r=-3\ (2) \end{cases}$$

$$3d+2p=-2$$
$$2d-2p=-2$$
$$5d=-4$$
$$d=-4/5$$
$$d-p=-2$$
$$p=d+2$$
$$p=6/5$$

$$3e+2q=2$$
$$2e-2q=-4$$
$$5e=-2$$
$$e=-2/5$$
$$e-q=-2$$
$$q=e+2$$
$$q=8/5$$

$$(2)\begin{cases} 3g+2s=7 \\ g-s=2 \end{cases}$$

$$(2)\begin{cases} 3h+2t=2 \\ h-t=1 \end{cases}$$

$$(2)\begin{cases} 3i+2u=4 \\ i-u=2 \end{cases}$$

$$3g+2s=7$$
$$2g-2s=4$$
$$5g=11$$
$$g=11/5$$
$$g-s=2$$
$$s=g-2$$
$$s=1/5$$

$$3h+2t=2$$
$$2h-2t=2$$
$$5h=4$$
$$h=4/5$$
$$h-t=1$$
$$t=h-1$$
$$t=-1/5$$

$$3i+2u=4$$
$$2i-2u=4$$
$$5i=8$$
$$i=8/5$$
$$i-u=2$$
$$u=i-2$$
$$u=-2/5$$

$$A=\begin{pmatrix} 3/5 & 2 & -3/5 \\ -4/5 & -2/5 & -9/5 \\ 11/5 & 4/5 & 8/5 \end{pmatrix}$$

$$B=\begin{pmatrix} 3/5 & 1 & -3/5 \\ 6/5 & 8/5 & 6/5 \\ 1/5 & -1/5 & -2/5 \end{pmatrix}$$

50 $\begin{pmatrix} 1 & \alpha \\ \beta & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1-\beta \\ \alpha \end{pmatrix}$

$$\begin{cases} x_1 + \alpha x_2 = 1 - \beta & (1) \\ \beta x_1 + x_2 = \alpha & (2) \end{cases}$$

El punto $P(2,-1)$ satisface a (1)
El punto $Q(2,0)$ satisface a (2)

Reemplazando $\underline{\underline{P}}$ en (1) y $\underline{\underline{Q}}$ en (2):

a) $$\begin{cases} 2 - \alpha = 1 - \beta & (1) \\ 2\beta = \alpha & (2) \end{cases}$$

Reemplazo (2) en (1)

$$2 - 2\beta = 1 - \beta$$
$$-2\beta + \beta = 1 - 2$$
$$-\beta = -1$$
$$\boxed{\beta = 1}$$

Reemplazo en (2)

$\alpha = 2\beta$ $\qquad \alpha = 2(1)$ $\qquad \boxed{\alpha = 2}$

b) $\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

$$\begin{cases} x_1 + 2x_2 = 0 \\ x_1 + x_2 = 2 \end{cases}$$

El sistema tiene solución única ya que la determinante $\underline{\underline{\text{no}}}$ es cero.
$= 1(1) - 2(1) \quad = 1 - 2 \quad = -1$

(51) $\begin{pmatrix} 1 & 2 & 1 \\ 0 & t & t \\ -1 & t & -1 \end{pmatrix}$

a) Por el método del pivoteo:

$$1\begin{vmatrix} t & t \\ t & -1 \end{vmatrix} - 2\begin{vmatrix} 0 & t \\ -1 & -1 \end{vmatrix} + 1\begin{vmatrix} 0 & t \\ -1 & t \end{vmatrix}$$

$$1(-t-t^2) - 2(0+t) + 1(0+t) = 0$$

$$\cancel{-t} - t^2 - 2t + \cancel{t} = 0$$

$$-t^2 - 2t = 0 \quad *$$

$$t^2 + 2t = 0$$

$$t(t+2) = 0$$

$$\boxed{t = 0} \qquad t+2 = 0 \qquad \boxed{t = -2}$$

b) Igualando $*$ a 1

$$-t^2 - 2t = 1$$

$$-t^2 - 2t - 1 = 0$$

$$t^2 + 2t + 1 = 0$$

$$(t+1)^2 = 0 \qquad t+1 = 0 \qquad \boxed{t = -1}$$

Reemplazo en la matriz inicial el valor de $t = -1$

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & -1 \\ -1 & -1 & -1 \end{pmatrix}$$

Ahora, se halla la inversa de esta matriz, utilizando estos 5 pasos siguientes:

① det = 1

② Menores

$$\begin{pmatrix} 0 & -1 & -1 \\ -1 & 0 & 1 \\ -1 & -1 & -1 \end{pmatrix}$$

③ Cofactores

$$\begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ -1 & 1 & -1 \end{pmatrix}$$

④ Transpuesta

$$\begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ -1 & -1 & -1 \end{pmatrix}$$

⑤ $\frac{1}{det}$(Transpuesta)

$$= \frac{1}{1}\begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ -1 & -1 & -1 \end{pmatrix} \boxed{= \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ -1 & -1 & -1 \end{pmatrix}}$$

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$$A = \begin{pmatrix} 1 & 2 & 1 \\ \lambda & 1 & 0 \\ -1 & 1 & \lambda \end{pmatrix}$$

a) Por el método del pivoteo

$$1\begin{vmatrix} 1 & 0 \\ 1 & \lambda \end{vmatrix} - 2\begin{vmatrix} \lambda & 0 \\ -1 & \lambda \end{vmatrix} + 1\begin{vmatrix} \lambda & 1 \\ -1 & 1 \end{vmatrix} = 0$$

$$1(\lambda - 0) - 2(\lambda^2 + 0) + 1(\lambda + 1) = 0$$

$$\lambda - 2\lambda^2 + \lambda + 1 = 0$$

$$-2\lambda^2 + 2\lambda + 1 = 0$$

$$2\lambda^2 - 2\lambda - 1 = 0$$

$a = 2 \qquad b = -2 \qquad c = -1$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\lambda = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-1)}}{2(2)}$$

$$\lambda = \frac{2 \pm \sqrt{4+8}}{4}$$

$$\lambda = \frac{2 \pm \sqrt{12}}{4} \qquad \lambda = \frac{2 \pm 2\sqrt{3}}{4}$$

$$\lambda = \frac{\cancel{2}^{1}(1 \pm \sqrt{3})}{\cancel{4}_{2}} \qquad \boxed{\lambda = \frac{1 \pm \sqrt{3}}{2}}$$

b) $A\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$$\begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \\ -1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} x_1 + 2x_2 + x_3 = 0 \\ x_1 + x_2 = 0 \\ -x_1 + x_2 + x_3 = 0 \end{cases}$$

(53) $3X - 2Y = \begin{pmatrix} 7 & 3 \\ 16 & 4 \end{pmatrix}$ (a') $\quad X + 3Y = \begin{pmatrix} 6 & 12 \\ -2 & 27 \end{pmatrix}$ (b')

Designando incógnitas cualesquiera para las matrices X i Y

$$X = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad Y = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

(a') $3X - 2Y = \begin{pmatrix} 7 & 3 \\ 16 & 4 \end{pmatrix}$

$$3\begin{pmatrix} a & b \\ c & d \end{pmatrix} - 2\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} 7 & 3 \\ 16 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 3a & 3b \\ 3c & 3d \end{pmatrix} - \begin{pmatrix} 2e & 2f \\ 2g & 2h \end{pmatrix} = \begin{pmatrix} 7 & 3 \\ 16 & 4 \end{pmatrix}$$

$3a - 2e = 7$ (1)
$3b - 2f = 3$ (2)
$3c - 2g = 16$ (3)
$3d - 2h = 4$ (4)

(b') $X + 3Y = \begin{pmatrix} 6 & 12 \\ -2 & 27 \end{pmatrix}$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + 3\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} 6 & 12 \\ -2 & 27 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} 3e & 3f \\ 3g & 3h \end{pmatrix} = \begin{pmatrix} 6 & 12 \\ -2 & 27 \end{pmatrix}$$

Resolviendo este sistema homogéneo

$$\left|\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 \end{array}\right| \begin{array}{l} F_1 - F_2 \\ \\ F_1 + F_3 \end{array}$$

$$\left|\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 3 & 2 & 0 \end{array}\right| -3F_2 + F_3$$

$$\left|\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right| -2F_2 + F_1$$

$$\left|\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right| -F_3$$

$$\left|\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right| \begin{array}{l} F_3 + F_1 \\ -F_3 + F_2 \end{array}$$

$$\left|\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right|$$

Luego, este sistema tiene la solución trivial $X_1 = 0 \quad X_2 = 0 \quad X_3 = 0$

$$S: \{X_1 = 0 \quad X_2 = 0 \quad X_3 = 0$$

Reemplazo en (2) ó (6)

En (2): $3b - 2f = 3$

$3b - 2(3) = 3$ $\qquad$ $3b - 6 = 3$

$3b = 3 + 6$ $\qquad$ $3b = 9$

$b = \frac{\cancel{9}^{3}}{\cancel{3}_{1}}$ $\qquad$ $\boxed{b = 3}$

*** Reuniendo (3) y (7)

$$\begin{array}{r} \\ (-3) \end{array}\begin{cases} 3c - 2g = 16 & (3) \\ c + 3g = -2 & (7) \end{cases}$$

$$\begin{array}{r} 3c - 2g = 16 \\ -3c - 9g = 6 \\ \hline // \quad -11g = 22 \end{array}$$

$11g = -22$

$g = -\frac{\cancel{22}^{2}}{\cancel{11}_{1}}$ $\qquad$ $\boxed{g = -2}$

Reemplazo en (3) ó (7)

En (7): $c + 3g = -2$

$c + 3(-2) = -2$ $\qquad$ $c - 6 = -2$

$c = -2 + 6$ $\qquad$ $\boxed{c = 4}$

**** Reuniendo (4) y (8)

$$\begin{array}{r} \\ (-3) \end{array}\begin{cases} 3d - 2h = 4 & (4) \\ d + 3h = 27 & (8) \end{cases}$$

$a + 3e = 6$ (5)

$b + 3f = 12$ (6)

$c + 3g = -2$ (7)

$d + 3h = 27$ (8)

*Reuniendo (1) y (5)

$$(-3)\begin{cases} 3a - 2e = 7 & (1) \\ a + 3e = 6 & (5) \end{cases}$$

$$\begin{array}{r} 3a - 2e = 7 \\ -3a - 9e = -18 \\ \hline // \quad -11e = -11 \end{array}$$

$11e = 11$

$e = \frac{\cancel{11}^{1}}{\cancel{11}_{1}}$ $\boxed{e = 1}$

Reemplazo en (1) ó (5)

En (5): $a + 3e = 6$

$a + 3(1) = 6$ $a + 3 = 6$

$a = 6 - 3$ $\boxed{a = 3}$

** Reuniendo (2) y (6)

$$(-3)\begin{cases} 3b - 2f = 3 & (2) \\ b + 3f = 12 & (6) \end{cases}$$

$$\begin{array}{r} 3b - 2f = 3 \\ -3b - 9f = -36 \\ \hline // \quad -11f = -33 \end{array}$$

$11f = 33$

$f = \frac{\cancel{33}^{3}}{\cancel{11}_{1}}$ $\boxed{f = 3}$

$$3d - 2h = 4$$
$$-3d - 9h = -81$$

$$// \quad -11h = -77 \qquad 11h = 77$$

$$h = \frac{\cancel{77}^{7}}{\cancel{11}_{1}} \qquad \boxed{h = 7}$$

Reemplazo en 4 ó 8

En 4 : $3d - 2h = 4$

$$3d - 2(7) = 4 \qquad 3d - 14 = 4$$

$$3d = 4 + 14 \qquad 3d = 18$$

$$d = \frac{\cancel{18}^{6}}{\cancel{3}_{1}} \qquad \boxed{d = 6}$$

$$\boxed{X = \begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}} \qquad \boxed{Y = \begin{pmatrix} 1 & 3 \\ -2 & 7 \end{pmatrix}}$$

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$$\begin{cases} x \quad -3z = 1 \\ 2x \quad -6z = 3 \\ \quad y + z = 2 \end{cases}$$

$$\begin{pmatrix} 1 & 0 & -3 \\ 2 & 0 & -6 \\ 0 & 1 & 1 \end{pmatrix}$$

Por el método del pivoteo:

$$1 \begin{vmatrix} 0 & -6 \\ 1 & 1 \end{vmatrix} - 0 \cancel{\begin{vmatrix} 2 & -6 \\ 0 & 1 \end{vmatrix}}\,0 - 3 \begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix}$$

$$1(0+6) - 3(2-0)$$

$$1(6) - 3(2) \qquad 6 - 6 \qquad \boxed{\det = 0}$$

(55)

$$* \begin{cases} x + ay + z = -1 \\ y + 2z = 1 \\ x + y - z = -a \end{cases}$$

$$\begin{pmatrix} 1 & a & 1 \\ 0 & 1 & 2 \\ 1 & 1 & -1 \end{pmatrix}$$

Por el método del pivoteo:

$$1\begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix} - a\begin{vmatrix} 0 & 2 \\ 1 & -1 \end{vmatrix} + 1\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} \neq 0$$

$$1(-1-2) - a(0-2) + 1(0-1) \neq 0$$

$$1(-3) - a(-2) + 1(-1) \neq 0$$

$$-3 + 2a - 1 \neq 0$$

$$2a - 4 \neq 0 \qquad 2a \neq 4$$

$$a \neq \frac{\cancel{4}^{2}}{\cancel{2}_{1}} \qquad \boxed{a \neq 2}$$

$$** \begin{cases} x - ay + bz = 0 \\ 2x - y + z = 0 \\ ax - by - z = 0 \end{cases}$$

$$\begin{pmatrix} 1 & -a & b \\ 2 & -1 & 1 \\ a & -b & -1 \end{pmatrix}$$

Por el método del pivoteo:

$$1\begin{vmatrix} -1 & 1 \\ -b & -1 \end{vmatrix} + a\begin{vmatrix} 2 & 1 \\ a & -1 \end{vmatrix} + b\begin{vmatrix} 2 & -1 \\ a & -b \end{vmatrix} = 0$$

$$1(1+b) + a(-2-a) + b(-2b+a) = 0$$

$$1 + b - 2a - a^2 - 2b^2 + ab = 0$$

Tomando $a=2$

$$1+b-2(2)-(2)^2-2b^2+(2)b=0$$
$$1+b-4-4-2b^2+2b=0$$
$$-2b^2+3b-7=0$$
$$2b^2-3b+7=0$$

$a=2 \qquad b=-3 \qquad c=7$

Como b^2-4ac es una cantidad negativa, las raices son complejas

$$b^2-4ac=(-3)^2-4(2)(7)$$
$$=9-56$$
$$=-47$$

56)
$$\begin{cases}(m+2)x+y+2z=0\\ x+my+z=0\\ 2x+2y+2z=0\end{cases}$$

$$\begin{pmatrix} m+2 & 1 & 2\\ 1 & m & 1\\ 2 & 2 & 2\end{pmatrix}$$

Por el método del pivoteo:

$$(m+2)\begin{vmatrix} m & 1\\ 2 & 2\end{vmatrix}-1\begin{vmatrix} 1 & 1\\ 2 & 2\end{vmatrix}+2\begin{vmatrix} 1 & m\\ 2 & 2\end{vmatrix}$$

$$(m+2)(2m-2)-1(2-2)+2(2-2m)$$
$$2m^2-2m+4\cancel{m}-\cancel{4}-\cancel{2}+\cancel{2}+\cancel{4}-4\cancel{m}$$
$$2m^2-2m=0$$
$$2m(m-1)=0$$
$$m(m-1)=0$$

$\boxed{m=0} \qquad m-1=0 \qquad \boxed{m=1}$

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$$(m)\begin{cases} x - my = m^2 n \\ \dfrac{x}{m} + y = 2mn \end{cases}$$

$m, n \in \mathbb{R}$
$m \neq 0$

$$\begin{array}{r} x - my = m^2 n \\ x + my = 2m^2 n \\ \hline 2x \quad // \quad = 3m^2 n \end{array}$$

$$\boxed{x = \frac{3m^2 n}{2}}$$

$$(-m)\begin{cases} x - my = m^2 n \\ \dfrac{x}{m} + y = 2mn \end{cases}$$

$$\begin{array}{r} x - my = m^2 n \\ -x - my = -2m^2 n \\ \hline \cancel{-x} \quad -2my = -m^2 n \end{array}$$

$$2my = m^2 n$$

$$y = \frac{m^{\cancel{2}} n}{2\cancel{m}}$$

$$\boxed{y = \frac{mn}{2}}$$

El sistema tiene solución única

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$$\begin{pmatrix} 0 & -2 & \alpha \\ 1 & -\alpha & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\begin{cases} -2y + \alpha z = 1 \\ x - \alpha y + z = 2 \\ x + y = 3 \end{cases}$$

Por el método del pivoteo:

$$0\begin{vmatrix} -\alpha & 1 \\ 1 & 0 \end{vmatrix} + 2\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} + \alpha\begin{vmatrix} 1 & -\alpha \\ 1 & 1 \end{vmatrix} = 0$$

$$2(0-1) + \alpha(1+\alpha) = 0$$

$$2(-1) + \alpha + \alpha^2 = 0$$

$$\alpha^2 + \alpha - 2 = 0$$

$$(\alpha+2)(\alpha-1) = 0$$

$$\alpha + 2 = 0 \qquad \alpha - 1 = 0$$

$$\boxed{\alpha = -2} \qquad \boxed{\alpha = 1}$$

59 $$A\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & a \\ 0 & 1 & b \\ 3 & 2 & c \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$\begin{cases} x_1 + 2x_2 + ax_3 = 1 & (1) \\ x_2 + bx_3 = 1 & (2) \\ 3x_1 + 2x_2 + cx_3 = -1 & (3) \end{cases}$$

Multiplicando (1) por -3 y sumando con (3)

$$-3x_1 - 6x_2 - 3ax_3 = -3$$

$$3x_1 + 2x_2 + cx_3 = -1$$

$$// \quad -4x_2 - 3ax_3 + cx_3 = -4 \qquad (4)$$

$$-4x_2 + x_3(c-3a) = -4$$

Multiplicando ② por 4 y sumando con ②

$$4X_2 + 4bX_3 = 4$$
$$-4X_2 + X_3(c-3a) = -4$$

$$// \quad 4bX_3 + cX_3 - 3aX_3 = 0$$
$$X_3(4b + c - 3a) = 0$$
$$\boxed{X_3 = 0}$$

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$$A = \begin{pmatrix} 5 \\ 1 \end{pmatrix} + \lambda\begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$$B = \begin{pmatrix} -2 \\ 2 \end{pmatrix} + t\begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 5 \\ 1 \end{pmatrix} + \begin{pmatrix} 3\lambda \\ -2\lambda \end{pmatrix} \qquad B = \begin{pmatrix} -2 \\ 2 \end{pmatrix} + \begin{pmatrix} 4t \\ t \end{pmatrix}$$

$$A = \begin{pmatrix} 5 + 3\lambda \\ 1 - 2\lambda \end{pmatrix} \qquad B = \begin{pmatrix} -2 + 4t \\ 2 + t \end{pmatrix}$$

Como $A = B$, se debe cumplir:

$$\begin{cases} 5 + 3\lambda = -2 + 4t & ① \\ 1 - 2\lambda = 2 + t & ② \end{cases}$$

$$\begin{cases} 3\lambda - 4t = -7 & ① \\ -2\lambda - t = 1 & ② \end{cases} \quad (-4)$$

$$3\lambda - 4t = -7$$
$$8\lambda + 4t = -4$$

$$11\lambda \quad // = -11$$

$$\lambda = -\frac{\cancel{11}^{1}}{\cancel{11}_{1}} \qquad \boxed{\lambda = \ldots}$$

Reemplazo en 1 ó 2:

En 2: $-2\lambda - t = 1$

$$-2(-1) - t = 1$$
$$2 - t = 1$$
$$-t = 1 - 2$$
$$-t = -1$$

$$\boxed{t = 1}$$

$$S: \begin{cases} \lambda = -1 \\ t = 1 \end{cases}$$

1) a)
$$\begin{cases} X_1 + 2X_2 + X_3 = 8 & (1) \\ X_2 + 3X_3 + X_4 = 15 & (2) \\ 4X_1 + X_3 + X_4 = 11 & (3) \\ X_1 + X_2 + 5X_4 = 23 & (4) \end{cases}$$

$$\left|\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 8 \\ 0 & 1 & 3 & 1 & 15 \\ 4 & 0 & 1 & 1 & 11 \\ 1 & 1 & 0 & 5 & 23 \end{array}\right| \begin{array}{l} -4F_1 + F_3 \\ -F_1 + F_4 \end{array}$$

$$\left|\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 8 \\ 0 & 1 & 3 & 1 & 15 \\ 0 & -8 & -3 & 1 & -21 \\ 0 & -1 & -1 & 5 & 15 \end{array}\right| \begin{array}{l} -2F_2 + F_1 \\ 8F_2 + F_3 \\ F_2 + F_4 \end{array}$$

$$\left|\begin{array}{cccc|c} 1 & 0 & -5 & -2 & -22 \\ 0 & 1 & 3 & 1 & 15 \\ 0 & 0 & 21 & 9 & 99 \\ 0 & 0 & 2 & 6 & 30 \end{array}\right| \begin{array}{l} \\ \\ F_3/3 \\ F_4/2 \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & -5 & -2 & -22 \\ 0 & 1 & 3 & 1 & 15 \\ 0 & 0 & 7 & 3 & 33 \\ 0 & 0 & 1 & 3 & 15 \end{array}\right) \begin{array}{l} \\ \\ F_3/7 \\ \\ \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & -5 & -2 & -22 \\ 0 & 1 & 3 & 1 & 15 \\ 0 & 0 & 1 & 3/7 & 33/7 \\ 0 & 0 & 1 & 3 & 15 \end{array}\right) \begin{array}{l} 5F_3+F_1 \\ -3F_3+F_2 \\ -F_3+F_4 \\ \\ \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 1/7 & 11/7 \\ 0 & 1 & 0 & -2/7 & 6/7 \\ 0 & 0 & 1 & 3/7 & 33/7 \\ 0 & 0 & 0 & 18/7 & 72/7 \end{array}\right) \begin{array}{l} \\ \\ \\ 7F_4/18 \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 1/7 & 11/7 \\ 0 & 1 & 0 & -2/7 & 6/7 \\ 0 & 0 & 1 & 3/7 & 33/7 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right) \begin{array}{l} -\frac{F_4}{7}+F_1 \\ \frac{2F_4}{7}+F_2 \\ -\frac{3F_4}{7}+F_3 \\ \\ \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right)$$

$$S: \begin{cases} x_1 = 1 \quad x_2 = 2 \quad x_3 = 3 \quad x_4 = 4 \end{cases}$$

b) $\begin{cases} X_1 - X_2 + X_3 - X_4 = -2 & ① \\ X_1 + 2X_2 - 2X_3 - X_4 = -5 & ② \\ 2X_1 + X_2 - 3X_3 + 2X_4 = -1 & ③ \\ X_1 + 2X_2 + 3X_3 - 6X_4 = -10 & ④ \end{cases}$

$$\left(\begin{array}{cccc|c} 1 & -1 & 1 & -1 & -2 \\ 1 & 2 & -2 & -1 & -5 \\ 2 & 1 & -3 & 2 & -1 \\ 1 & 2 & 3 & -6 & -10 \end{array}\right) \begin{array}{l} -F_1 + F_2 \\ -2F_1 + F_3 \\ -F_1 + F_4 \\ \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & -1 & 1 & -1 & -2 \\ 0 & 3 & -3 & 0 & -3 \\ 0 & 3 & -5 & 4 & 3 \\ 0 & 3 & 2 & -5 & -8 \end{array}\right) \begin{array}{l} \\ F_2/3 \\ \\ \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & -1 & 1 & -1 & -2 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 3 & -5 & 4 & 3 \\ 0 & 3 & 2 & -5 & -8 \end{array}\right) \begin{array}{l} F_2 + F_1 \\ -3F_2 + F_3 \\ -3F_2 + F_4 \\ \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -3 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & -2 & 4 & 6 \\ 0 & 0 & 5 & -5 & -5 \end{array}\right) \begin{array}{l} \\ \\ -F_3/2 \\ F_4/5 \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -3 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & -1 & -1 \end{array}\right) \begin{array}{l} \\ F_3 + F_2 \\ -F_3 + F_4 \\ \end{array}$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -3 \\ 0 & 1 & 0 & -2 & -4 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right| \begin{array}{l} F_4+F_1 \\ 2F_4+F_2 \\ 2F_4+F_3 \\ \end{array}$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right|$$

$$S: \{ X_1 = -1 \quad X_2 = 0 \quad X_3 = 1 \quad X_4 = 2$$

c) $$\begin{cases} 6X_1 - 5X_2 + 7X_3 + 8X_4 = 3 \\ 3X_1 + 11X_2 + 2X_3 + 4X_4 = 6 \\ 3X_1 + 2X_2 + 3X_3 + 4X_4 = 1 \\ X_1 + X_2 + X_3 = 0 \end{cases}$$

$$\left|\begin{array}{cccc|c} 6 & -5 & 7 & 8 & 3 \\ 3 & 11 & 2 & 4 & 6 \\ 3 & 2 & 3 & 4 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{array}\right|$$

$$\left|\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 0 \\ 3 & 11 & 2 & 4 & 6 \\ 3 & 2 & 3 & 4 & 1 \\ 6 & -5 & 7 & 8 & 3 \end{array}\right| \begin{array}{l} -3F_1+F_2 \\ -3F_1+F_3 \\ -6F_1+F_4 \\ \end{array}$$

d) $\begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} z = \begin{pmatrix} 3 \\ -3 \\ -2 \end{pmatrix}$

$$\begin{pmatrix} x + 2y \\ 3x - y \\ 2x - y \end{pmatrix} + \begin{pmatrix} -z \\ 2z \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} x + 2y - z \\ 3x - y + 2z \\ 2x - y + z \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ -2 \end{pmatrix}$$

$$\begin{cases} x + 2y - z = 3 \\ 3x - y + 2z = -3 \\ 2x - y + z = -2 \end{cases}$$

$$\left|\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 3 & -1 & 2 & -3 \\ 2 & -1 & 1 & -2 \end{array}\right| \begin{array}{l} -3F_1 + F_2 \\ -2F_1 + F_3 \\ \end{array}$$

$$\left|\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -12 \\ 0 & -5 & 3 & -8 \end{array}\right| \; -F_2/7$$

$$\left|\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & -5/7 & 12/7 \\ 0 & -5 & 3 & -8 \end{array}\right| \begin{array}{l} -2F_2 + F_1 \\ 5F_2 + F_3 \\ \end{array}$$

$$\left|\begin{array}{ccc|c} 1 & 0 & 3/7 & -3/7 \\ 0 & 1 & -5/7 & 12/7 \\ 0 & 0 & -4/7 & 4/7 \end{array}\right| \; -7F_3/4$$

$$\left|\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 0 \\ 0 & 8 & -1 & 4 & 6 \\ 0 & -1 & 0 & 4 & 1 \\ 0 & -11 & 1 & 8 & 3 \end{array}\right|$$

$$= \left|\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 4 & 1 \\ 0 & 8 & -1 & 4 & 6 \\ 0 & -11 & 1 & 8 & 3 \end{array}\right| \begin{array}{l} \\ -F_2 \\ \\ \\ \end{array}$$

$$\left|\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & -1 \\ 0 & 8 & -1 & 4 & 6 \\ 0 & -11 & 1 & 8 & 3 \end{array}\right| \begin{array}{l} -F_2 + F_1 \\ -8F_2 + F_3 \\ 11F_2 + F_4 \\ \\ \end{array}$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 1 & 4 & 1 \\ 0 & 1 & 0 & -4 & -1 \\ 0 & 0 & -1 & 36 & 14 \\ 0 & 0 & 1 & -36 & -8 \end{array}\right| \begin{array}{l} \\ \\ -F_3 \\ \\ \end{array}$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 1 & 4 & 1 \\ 0 & 1 & 0 & -4 & -1 \\ 0 & 0 & 1 & -36 & -14 \\ 0 & 0 & 1 & -36 & -8 \end{array}\right| \begin{array}{l} -F_3 + F_1 \\ -F_3 + F_4 \\ \\ \\ \end{array}$$

$$= \left|\begin{array}{cccc|c} 1 & 0 & 0 & 40 & 15 \\ 0 & 1 & 0 & -4 & -1 \\ 0 & 0 & 1 & -36 & -14 \\ 0 & 0 & 0 & 0 & 6 \end{array}\right|$$

El sistema es inconsistente (no tiene solución

$$\left|\begin{array}{ccc|c} 1 & 0 & 3/7 & -3/7 \\ 0 & 1 & -5/7 & 12/7 \\ 0 & 0 & 1 & -1 \end{array}\right| \quad \begin{array}{l} \dfrac{5F_3}{7}+F_2 \\ -\dfrac{3F_3}{7}+F_1 \end{array}$$

$$\left|\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right|$$

S: $\begin{cases} x = 0 \quad y = 1 \quad z = -1 \end{cases}$

62.- $\begin{cases} x + y + z = 2 \\ x + 3y + 2z = 5 \\ 2x + 3y + (a^2-1)z = a+1 \end{cases}$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 2 & 3 & a^2-1 \end{pmatrix}$$

Por el método del pivoteo:

$$1\begin{vmatrix} 3 & 2 \\ 3 & a^2-1 \end{vmatrix} - 1\begin{vmatrix} 1 & 2 \\ 2 & a^2-1 \end{vmatrix} + 1\begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix} = 0$$

$$1(3a^2-3-6) - 1(a^2-1-4) + 1(3-6) = 0$$

$$3a^2-9-a^2+5+3-6 = 0$$

$$2a^2-7=0$$

$$2a^2=7$$

$$a^2=\frac{7}{2} \qquad \boxed{a = \pm\sqrt{\frac{7}{2}}}$$

(63) a) Verdadero *
b) Verdadero **
c) Verdadero ***
d) Falso ****
e) Falso *****

$$\begin{cases} X_1 + 2X_2 - 3X_3 + X_4 - X_5 = 0 \\ 3X_1 - X_2 - 2X_3 - X_4 + X_5 = 0 \\ -2X_1 + X_2 - X_3 + X_4 + X_5 = 0 \\ X_1 + X_2 + X_3 - 2X_4 - X_5 = 0 \\ -3X_1 - 4X_2 + 4X_3 - X_4 + 4X_5 = 0 \end{cases}$$

* Como se observa, el sistema es homogéneo.

** $(2, 2, 2, 2, 2)$ es una solución

*** $m = 5$

**** La información es suficiente

***** El sistema propuesto satisface las condiciones iniciales.

El sistema tiene infinitas soluciones

(64)
$$\begin{cases} aX_1 - X_2 + X_3 = 0 \\ -X_1 + X_2 + 2X_3 = 0 \\ 3X_1 + 2X_2 - X_3 = 0 \end{cases}$$

$$\begin{pmatrix} a & -1 & 1 \\ -1 & 1 & 2 \\ 3 & 2 & -1 \end{pmatrix}$$

Veamos cuando la determinante es igual a cero y hallemos $\underline{a}$.

Por el método del pivoteo:

$$a\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} + 1\begin{vmatrix} -1 & 2 \\ 3 & -1 \end{vmatrix} + 1\begin{vmatrix} -1 & 1 \\ 3 & 2 \end{vmatrix} = 0$$

$$a(-1-4) + 1(1-6) + 1(-2-3) = 0$$

$$a(-5) + 1(-5) + 1(-5) = 0$$

$$-5a - 5 - 5 = 0$$

$$-5a - 10 = 0$$

$$-5a = 10$$

$$a = \frac{\cancel{10}^{2}}{\cancel{-5}_{1}} \qquad \boxed{a = -2}$$

La alternativa e) es falsa, ya que los sistemas homogéneos no pueden ser inconsistentes

65 a) $\begin{cases} (m-2)x + y + 2z = 0 \\ x + my + z = 0 \\ 2x + 2y + 2z = 0 \end{cases}$

El sistema tiene siempre la solución trivial

m puede tomar cualquier valor.

b) $\begin{cases} -mx + y - z = 1 & (1) \\ 2mx + y - z = 0 & (2) \\ -my + z = 0 & (3) \\ mx = 1 & (4) \end{cases}$

De la ecuación (4) se deduce que $x = \dfrac{1}{m}$, $\boxed{m \neq 0}$

Tomando solo hasta (3)

$$\begin{pmatrix} -m & 1 & -1 \\ 2m & 1 & -1 \\ 0 & -m & 1 \end{pmatrix}$$

Por el método del pivoteo:

$$-m \begin{vmatrix} 1 & -1 \\ -m & 1 \end{vmatrix} - 2m \begin{vmatrix} 1 & -1 \\ -m & 1 \end{vmatrix} \neq 0$$

$$-m(1 - m) - 2m(1 + m) \neq 0$$

$$-m + m^2 - 2m + 2m^2 \neq 0$$

$$3m^2 - 3m \neq 0$$

$$3m(m - 1) \neq 0$$

$$m(m - 1) \neq 0$$

$\boxed{m \neq 0}$ $\qquad m - 1 \neq 0$ $\boxed{m \neq 1}$

$S: \{ m \neq 0 \wedge m \neq 1$

c) $\begin{cases} (m-1)x - 2y = 2 \\ -x + y + mz = 0 \\ -y + 2z = m \end{cases}$

$$\begin{pmatrix} m-1 & -2 & 0 \\ -1 & 1 & m \\ 0 & -1 & 2 \end{pmatrix}$$

Por el método del pivoteo

$$(m-1)\begin{vmatrix} 1 & m \\ -1 & 2 \end{vmatrix} + 1\begin{vmatrix} -2 & 0 \\ -1 & 2 \end{vmatrix} \neq 0$$

$$(m-1)(2+m) + 1(-4+0) \neq 0$$

$$2m + m^2 - 2 - m - 4 \neq 0$$

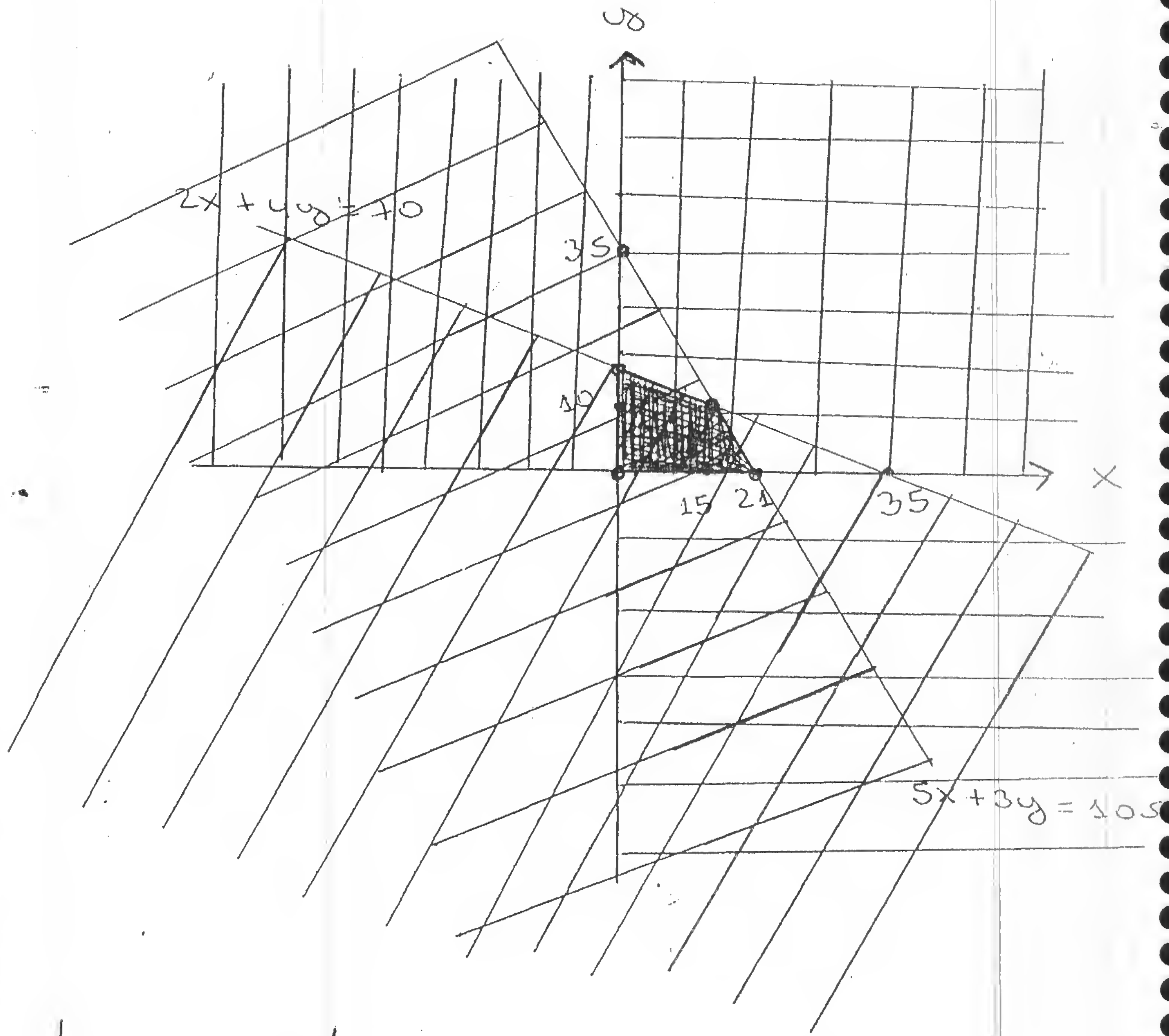
$$m^2 + m - 6 \neq 0$$

$$(m+3)(m-2) \neq 0$$

$m + 3 \neq 0$ $\quad$ $m - 2 \neq 0$

$\boxed{m \neq -3}$ $\quad$ $\boxed{m \neq 2}$

$S: \{ m \neq -3 \ \wedge \ m \neq 2$



La región más pintada es la solución del sistema y está representada por el trapecio formado.

Los vértices de dicho trapecio son: $P_1(0,0)$ $P_3(21,0)$

$P_2(0,10)$ $P_4(15,10)$

y también son soluciones del sistema de desigualdades.

66 a) $\begin{cases} 3^{x+y} = 81 & (1) \\ 3^{y-x} = 9 & (2) \end{cases}$

1) $3^{x+y} = 81 \qquad 3^{x+y} = 3^4$

$x + y = 4 \quad *$

2) $3^{y-x} = 9 \qquad 3^{y-x} = 3^2$

$-x + y = 2 \quad **$

$$\begin{array}{r} x + y = 4 \\ -x + y = 2 \\ \hline 2y = 6 \end{array}$$

$y = \frac{\cancel{6}^{3}}{\cancel{2}_{1}} \qquad \boxed{y = 3}$

Reemplazo en (1) ó (2)

En (1): $x + y = 4$

$x + 3 = 4$

$x = 4 - 3 \qquad \boxed{x = 1}$

b) $\begin{cases} 3^x + 3^y = 36 & (1) \\ 3^{y-x} = 3 & (2) \end{cases}$

Igualando y en función de x en (2)

$3^{y-x} = 3^1 \qquad y - x = 1$

$\boxed{y = 1 + x}$ (3)

Reemplazando (3) en (1), y resolviendo:

$$3^x + 3^y = 36$$

$$3^x + 3^{x+1} = 36$$

$$3^x + 3(3^x) = 36$$

$$4(3^x) = 36$$

$$3^x = \frac{\cancel{36}^{9}}{\cancel{4}_{1}} \qquad 3^x = 9$$

$$3^x = 3^2 \qquad \boxed{x = 2}$$

Reemplazando en (3):

$$y = x + 1$$

$$y = 2 + 1 \qquad \boxed{y = 3}$$

c) $\begin{cases} 2^x + 2^y = 20 & (1) \\ 2^{x+y} = 64 & (2) \end{cases}$

Resolviendo en (2):

$$2^{x+y} = 64$$

$$2^{x+y} = 2^6$$

$$x + y = 6 \qquad \boxed{x = 6 - y} \ (3)$$

Reemplazo (3) en (1)

$$2^x + 2^y = 20$$

$$2^{6-y} + 2^y = 20$$

$$\frac{2^6}{2^y} + 2^y = 20.$$

$$2^6 + 2^y(2^y) = 20(2^y)$$

$$2^6 + 2^{2y} = 20(2^y)$$

$$(2)^{2y} + 20(2^y) + 64 = 0$$

$$(2^y)^2 - 20(2^y) + 64 = 0$$

$$(2^y - 16)(2^y - 4) = 0$$

$$2^y - 16 = 0 \qquad 2^y - 4 = 0$$

$$2^y = 16 \qquad 2^y = 4$$

$$2^y = 2^4 \qquad 2^y = 2^2$$

$$\boxed{y_1 = 4} \qquad \boxed{y_2 = 2}$$

Reemplazando en 3

$$x = 6 - y$$

Con $y_1 = 4$

$$x_1 = 6 - 4$$

$$\boxed{x_1 = 2}$$

Con $y_2 = 2$

$$x_2 = 6 - 2$$

$$\boxed{x_1 = 4}$$

Las soluciones para el problema serian:

$$\begin{cases} x = 2 \\ y = 4 \end{cases} \qquad \begin{matrix} x = 4 \\ y = 2 \end{matrix}$$

d) $\begin{cases} 2^x + 3^y = 7 & (1) \\ 2^{2x+1} - 3^{2y} = 23 & (2) \end{cases}$

Tomando (1)

$$2^x + 3^y = 7$$

$$\boxed{2^x = 7 - 3^y} \quad (3)$$

Reemplazo (3) en (2):

$$2^{2x+1} - 3^{2y} = 23$$

$$2(2^{2x}) - 3^{2y} = 23$$

$$2(2^x)^2 - 3^{2y} = 23$$

$$2(7 - 3^y)^2 - 3^{2y} = 23$$

$$2\left[49 - 14(3^y) + 3^{2y}\right] - 3^{2y} = 23$$

$$98 - 28(3^y) + 2(3^{2y}) - 3^{2y} = 23$$

$$3^{2y} - 28(3^y) + 98 - 23 = 0$$

$$(3^y)^2 - 28(3^y) + 75 = 0$$

$$(3^y - 25)(3^y - 3) = 0$$

$$3^y - 25 = 0$$

$$3^y = 25$$

$$\log_3 (3)^y = \log_3 25$$

$$y \log_3 3 = \log_3 25$$

$$\boxed{y_1 = \log_3 25}$$

$$3^y - 3 = 0$$

$$3^y = 3$$

$$\boxed{y_2 = 1}$$

Reemplazando en (3)

$$2^x = 7 - 3^y$$

Con $y_1 = \log_3 25$

$$2^x = 7 - 3^{\log_3 25}$$

$$2^x = 7 - 25$$

$$2^x = -18$$

Imposible

Con $y_2 = 1$

$$2^x = 7 - 3^1$$

$$2^x = 7 - 3$$

$$2^x = 4$$

$$2^x = 2^2$$

$$\boxed{x = 2}$$

$$S: \begin{cases} x = 2 \\ y = 1 \end{cases}$$

e) $\begin{cases} 2^{2x-y} = 32 & (1) \\ 3^{x-2y} = 3 & (2) \end{cases}$

Tomando (1)

$$2^{2x-y} = 32$$

$$2^{2x-y} = 2^5$$

$$2x - y = 5 \qquad -y = 5 - 2x$$

$$\boxed{y = 2x - 5} \quad (3)$$

Tomando (2)

$$3^{x-2y} = 3$$

$$x - 2y = 1$$

$$-2y = 1 - x$$

$$2y = x - 1$$

$$\boxed{y = \frac{x-1}{2}} \quad (4)$$

Igualando 3 y 4:

$y = 2x - 5$ $\qquad$ $y = \frac{x-1}{2}$

$2x - 5 = \frac{x-1}{2}$

$2(2x-5) = x - 1$

$4x - 10 = x - 1$ $\qquad$ $4x - x = -1 + 10$

$3x = 9$ $\qquad$ $x = \frac{\cancel{9}^{3}}{\cancel{3}_{1}}$ $\qquad$ $\boxed{x = 3}$

Reemplazando en 3 ó 4:

En 3: $y = 2x - 5$

$y = 2(3) - 5$

$y = 6 - 5$ $\qquad$ $\boxed{y = 1}$

$S: \begin{cases} x = 3 \\ y = 1 \end{cases}$

f) $\begin{cases} 3^x \cdot 4^y = 3^8 & (1) \\ 2^{x-1} \cdot 2^{y+1} = 2^6 & (2) \end{cases}$

Tomando 2

$2^{x-1} \cdot 2^{y+1} = 2^6$

$\frac{2^x}{2} \cdot 2(2^y) = 64$

$2^x \cdot \cancel{2}(2^y) = 128$

$2^y = \frac{128}{2(2^x)}$ $\qquad$ $2^y = \frac{64}{2^x}$

$2^y = \frac{2^6}{2^x}$ $\qquad$ $2^y = (2)^{6-x}$ $\qquad$ $\boxed{y = 6 - x}$

Reemplazando (3) en (1):

$$3^x \cdot 4^y = 3^8$$

$$3^x \cdot 4^{6-x} = 3^8$$

$$3^x \cdot \frac{4^6}{4^x} = 3^8 \qquad \frac{3^x}{4^x} = \frac{3^8}{4^6}$$

* y **: Soluciones definitivas

$$\left(\frac{3}{4}\right)^x = \frac{6561}{4096}$$

$$\log_{\frac{3}{4}} \left(\frac{3}{4}\right)^x = \log_{\frac{3}{4}} \left(\frac{6561}{4096}\right)$$

$$x \log_{\frac{3}{4}} \frac{3}{4} = \log_{\frac{3}{4}} \left(\frac{6561}{4096}\right)$$

$$* \quad \boxed{x = \log_{\frac{3}{4}} \left(\frac{6561}{4096}\right)} \quad *$$

Reemplazo en (3)

$$y = 6 - x$$

$$\boxed{y = 6 - \log_{\frac{3}{4}} \left(\frac{6561}{4096}\right)}$$

De aquí se deduce

$$y = \log_{\frac{3}{4}} \left(\frac{3}{4}\right)^6 - \log_{\frac{3}{4}} \frac{6561}{4096}$$

$$y = \log_{\frac{3}{4}} \left(\frac{729}{4096}\right) - \log_{\frac{3}{4}} \left(\frac{6561}{4096}\right)$$

$$y = \log_{\frac{3}{4}} \left[\left(\frac{729}{4096}\right) \div \left(\frac{6561}{4096}\right)\right]$$

$$y = \log_{\frac{3}{4}} \left(\frac{\cancel{729}^{1}}{\cancel{4096}} \cdot \frac{\cancel{4096}}{\cancel{6561}_{9}}\right) \quad ** \quad \boxed{y = \log_{\frac{3}{4}} \left(\frac{1}{9}\right)} \; **$$

g) $\begin{cases} x+y=110 & (1) \\ \log x+\log y=3 & (2) \end{cases}$

Tomando (2):

$$\log x+\log y=3$$

$$\log(xy)=3 \qquad xy=10^3$$

$$xy=1000 \qquad \boxed{x=\frac{1000}{y}} \quad (3)$$

Reemplazo (3) en (1)

$$x+y=110$$

$$\frac{1000}{y}+y=110$$

$$1000+y^2=110y \qquad y^2-110y+1000=0$$

$$(y-100)(y-10)=0$$

$$y-100=0 \qquad y-10=0$$

$$\boxed{y_1=100} \qquad \boxed{y_2=10}$$

Reemplazando en (3):

$$x=\frac{1000}{y}$$

$$x_1=\frac{10\cancel{00}}{1\cancel{00}} \qquad x_2=\frac{100\cancel{0}}{1\cancel{0}}$$

$$\boxed{x_1=10} \qquad \boxed{x_2=100}$$

$$S: \begin{cases} x=10 \\ y=100 \end{cases} \qquad \begin{cases} x=100 \\ y=10 \end{cases}$$

h) $\begin{cases} \log_2(x-y) = 2 & (1) \\ \log_2 x - \log_2 y = 1 & (2) \end{cases}$

De la ecuación (2):

$$\log_2 x - \log_2 y = 1$$

$$\log_2 \frac{x}{y} = 1$$

$$\frac{x}{y} = 2^1 \qquad \frac{x}{y} = 2 \qquad \boxed{x = 2y} \; (3)$$

Reemplazo (3) en (1):

$$\log_2(x-y) = 2$$

$$\log_2(2y-y) = 2$$

$$\log_2(y) = 2$$

$$\cancel{2^{\log_2}} y = 2^2 \qquad y = 2^2 \qquad \boxed{y = 4}$$

Reemplazo en (3):

$$x = 2y \qquad x = 2(4) \qquad \boxed{x = 8}$$

$$S: \begin{cases} x = 8 \\ y = 4 \end{cases}$$

i) $\begin{cases} \log x + \log y = 4 & (1) \\ y - 4x = 0 & (2) \end{cases}$

De (2) tenemos:

$$y - 4x = 0$$

$$\boxed{y = 4x} \; (3)$$

Reemplazo 3 en 1:

$\log x + \log y = 4$

$\log xy = 4$

$\cancel{\log_{10}} xy = 10^4$ $\qquad xy = 10000$

* Como $y = 4x$

$x(4x) = 10000$

$4x^2 = 10000$ $\qquad x^2 = \frac{10000}{4}$

$x^2 = 2500$ $\qquad \cancel{\sqrt{x^2}} = \sqrt{2500}$

$\boxed{x = 50}$

Reemplazando en 3:

$y = 4x$

$y = 4(50)$ $\qquad \boxed{y = 200}$

$S: \begin{cases} x = 50 \\ y = 200 \end{cases}$

67. a) $\begin{cases} \operatorname{Sen} x + \operatorname{Sen} y = 1 & (1) \\ \operatorname{Cos}(x - y) = 1 & (2) \end{cases}$

$x, y \in (0, 2\pi)$

De 2 tenemos:

$\operatorname{Cos}(x - y) = 1$

$x - y = \operatorname{Cos}^{-1}(1)$

$x - y = 0$ $\qquad \boxed{x = y}$ (3)

Reemplazando 3 en 1

$$\operatorname{Sen} x + \operatorname{Sen} y = 1$$

$$\operatorname{Sen} x + \operatorname{Sen} x = 1$$

$$2 \operatorname{Sen} x = 1$$

$$\operatorname{Sen} x = \frac{1}{2}$$

$$x = \operatorname{Sen}^{-1}\left(\frac{1}{2}\right)$$

$$\boxed{x_1 = \frac{\pi}{6}} \qquad \boxed{x_2 = \frac{5\pi}{6}}$$

Reemplazo en 3

$$y = x$$

$$\boxed{y_1 = \frac{\pi}{6}} \qquad \boxed{y_2 = \frac{5\pi}{6}}$$

$$S: \begin{cases} x = \frac{\pi}{6} \\ y = \frac{\pi}{6} \end{cases} \qquad \begin{matrix} x = \frac{5\pi}{6} \\ y = \frac{5\pi}{6} \end{matrix}$$

b) $\begin{cases} \operatorname{Cos} x \operatorname{Tan} x = \frac{\sqrt{3}}{2} & (1) \\ \operatorname{Sen}(x+y) = 1 & (2) \end{cases}$

De 1, tenemos:

$$\operatorname{Cos} x \operatorname{Tan} x = \frac{\sqrt{3}}{2}$$

$$\cancel{\operatorname{Cos} x} \cdot \frac{\operatorname{Sen} x}{\cancel{\operatorname{Cos} x}} = \frac{\sqrt{3}}{2} \qquad x \neq 0 \quad x \neq 2\pi \quad x \neq \frac{\pi}{2}$$

$$\operatorname{Sen} x = \frac{\sqrt{3}}{2}$$

$$x = \operatorname{Sen}^{-1}\left(\frac{\sqrt{3}}{2}\right).$$

$$\boxed{x_1 = \frac{\pi}{3}}$$

$$\boxed{x_2 = \frac{2\pi}{3}}$$

Reemplazando en (2):

$$\operatorname{Sen}(x+y) = 1$$

$$x + y = \operatorname{Sen}^{-1}(1)$$

$$x + y = \frac{\pi}{2}$$

Con $x_1 = \frac{\pi}{3}$

$$\frac{\pi}{3} + y = \frac{\pi}{2} \qquad y = \frac{\pi}{2} - \frac{\pi}{3} \qquad \boxed{y_1 = \frac{\pi}{6}}$$

Con $x_2 = \frac{2\pi}{3}$

$$\frac{2\pi}{3} + y = \frac{\pi}{2} \qquad y = \frac{\pi}{2} - \frac{2\pi}{3}$$

$$y = -\frac{\pi}{6} \qquad y = \frac{-\pi}{6} + 2\pi \qquad \boxed{y_2 = \frac{11\pi}{12}}$$

$$S: \begin{cases} x = \frac{\pi}{3} & x = \frac{2\pi}{3} \\ y = \frac{\pi}{6} & y = \frac{11\pi}{12} \end{cases}$$

c) $\begin{cases} \operatorname{Sen} x \operatorname{Sen} y = \frac{1}{4} & (1) \\ \operatorname{Cos} x \operatorname{Cos} y = \frac{3}{4} & (2) \end{cases}$ $\quad x \neq 0$, $\quad x \neq 2\pi$

Sumando (1) y (2), obtenemos:

$$\operatorname{Sen} x \operatorname{Sen} y + \operatorname{Cos} x \operatorname{Cos} y = 1$$

$$\operatorname{Cos}(x - y) = 1$$

$$x - y = \operatorname{Cos}^{-1}(1)$$

$$x - y = 0 \qquad\qquad x - y = 2\pi$$

$$\boxed{x = y} \; (3) \qquad\qquad \boxed{x = 2\pi + y}$$

Descarto por *

Reemplazo 3 en 1

$$\operatorname{Sen} x \operatorname{Sen} y = \frac{1}{4}$$

$$\operatorname{Sen} x \operatorname{Sen} x = \frac{1}{4}$$

$$\operatorname{Sen}^2 x = \frac{1}{4} \qquad \cancel{\sqrt{\ }}\operatorname{Sen}^{\cancel{2}} x = \pm\sqrt{\frac{1}{4}}$$

$$\operatorname{Sen} x = \pm\frac{1}{2} \qquad x = \operatorname{Sen}^{-1}\left(\pm\frac{1}{2}\right)$$

$$x = \operatorname{Sen}^{-1}\left(\frac{1}{2}\right)$$

$$\boxed{x_1 = \frac{\pi}{6}} \qquad \boxed{x_2 = \frac{5\pi}{6}}$$

$$x = \operatorname{Sen}^{-1}\left(-\frac{1}{2}\right)$$

$$\boxed{x_3 = \frac{7\pi}{6}} \qquad \boxed{x_4 = \frac{11\pi}{6}}$$

Como $x = y$, obtenemos

$$S: \begin{cases} x = \frac{\pi}{6} \\ y = \frac{\pi}{6} \end{cases} \qquad \begin{matrix} x = \frac{5\pi}{6} \\ y = \frac{5\pi}{6} \end{matrix} \qquad \begin{matrix} x = \frac{7\pi}{6} \\ x = \frac{7\pi}{6} \end{matrix} \qquad \begin{matrix} x = \frac{11\pi}{6} \\ y = \frac{11\pi}{6} \end{matrix}$$

d) $\begin{cases} x + y = \dfrac{2\pi}{3} & (1) \\ \operatorname{Sen} x + \operatorname{Sen} y = \dfrac{3}{2} & (2) \end{cases}$

De (1): $x + y = \dfrac{2\pi}{3}$

$$\boxed{x = \frac{2\pi}{3} - y} \quad (3)$$

Reemplazo en (2)

$$\operatorname{Sen} x + \operatorname{Sen} y = \frac{3}{2}$$

$$\operatorname{Sen}\left(\frac{2\pi}{3} - y\right) + \operatorname{Sen} y = \frac{3}{2}$$

$$\operatorname{Sen}\frac{2\pi}{3}\operatorname{Cos} y - \operatorname{Sen} y \operatorname{Cos}\frac{2\pi}{3} + \operatorname{Sen} y = \frac{3}{2}$$

$$\frac{\sqrt{3}}{2}\operatorname{Cos} y + \frac{1}{2}\operatorname{Sen} y + \operatorname{Sen} y = \frac{3}{2}$$

$$\sqrt{3}\operatorname{Cos} y + 3\operatorname{Sen} y = 3$$

$$\sqrt{3}\sqrt{1 - \operatorname{Sen}^2 y} = 3 - 3\operatorname{Sen} y$$

$$3(1 - \operatorname{Sen}^2 y) = 9 - 18\operatorname{Sen} y + 9\operatorname{Sen}^2 y$$

$$3 - 3\operatorname{Sen}^2 y - 9 + 18\operatorname{Sen} y - 9\operatorname{Sen}^2 y = 0$$

$$-3\operatorname{Sen}^2 y - 9\operatorname{Sen}^2 y + 18\operatorname{Sen} y - 6 = 0$$

$$-12\operatorname{Sen}^2 y + 18\operatorname{Sen} y - 6 = 0$$

$$2\operatorname{Sen}^2 y - 3\operatorname{Sen} y + 1 = 0$$

$$(2\operatorname{Sen} y - 1)(\operatorname{Sen} y + 1) = 0$$

$2\,\text{Sen}\,y - 1 = 0$

$2\,\text{Sen}\,y = 1$

$\text{Sen}\,y = \frac{1}{2}$

$y = \text{Sen}^{-1}\left(\frac{1}{2}\right)$

$\boxed{y_1 = \frac{\pi}{6}}$

$\text{Sen}\,y - 1 = 0$

$\text{Sen}\,y = 1$

$y = \text{Sen}^{-1}(1)$

$\boxed{y_2 = \frac{\pi}{2}}$

Reemplazo en (3):

$x = \frac{2\pi}{3} - y$

Con $y_1 = \frac{\pi}{6}$

$x = \frac{2\pi}{3} - \frac{\pi}{6}$

$\boxed{x_1 = \frac{\pi}{2}}$

Con $y_2 = \frac{\pi}{2}$

$x_2 = \frac{2\pi}{3} - \frac{\pi}{2}$

$\boxed{x_2 = \frac{\pi}{6}}$

$S: \begin{cases} x = \frac{\pi}{2} \\ y = \frac{\pi}{6} \end{cases} \qquad \begin{matrix} x = \frac{\pi}{6} \\ y = \frac{\pi}{2} \end{matrix}$

e) $\begin{cases} \text{Sen}\, x\ \text{Sen}\, y = \text{Cos}\, x\ \text{Cos}\, y & (1) \\ x - y = \dfrac{\pi}{6} & (2) \end{cases}$

$x \neq 0$ $\qquad$ $x \neq 2\pi$

De (1):

$$\text{Sen}\, x\ \text{Sen}\, y - \text{Cos}\, x\ \text{Cos}\, y = 0$$

$$\text{Cos}\, x\ \text{Cos}\, y - \text{Sen}\, x\ \text{Sen}\, y = 0$$

$$\text{Cos}\,(x+y) = 0$$

$$x + y = \text{Cos}^{-1}(0)$$

$x + y = 0$ $\qquad$ $x + y = 2\pi$

$\boxed{x = -y}$ (3) $\qquad$ $\boxed{x = 2\pi - y}$ (4)

De (2):

$x - y = \dfrac{\pi}{6}$ $\qquad$ $\boxed{x = \dfrac{\pi}{6} + y}$ (5)

$\dfrac{180}{12}$

Igualando (3) y (5):

$-y = \dfrac{\pi}{6} + y$ $\qquad$ $-2y = \dfrac{\pi}{6}$

$y = \dfrac{\frac{\pi}{6}}{-2}$ $\qquad$ $y = -\dfrac{\pi}{12} + 2\pi$

$$\boxed{y = \frac{23\pi}{12}}$$

Igualando (4) y (5):

$$2\pi - y = \frac{\pi}{6} + y$$

$2\pi - \dfrac{\pi}{6} = y + y$ $\qquad$ $\dfrac{11\pi}{6} = 2y$

$2y = \dfrac{11\pi}{6}$ $\qquad$ $\boxed{y = \dfrac{11\pi}{12}}$

S: $\begin{cases} x = \dfrac{13\pi}{12} \\ y = 11\pi \end{cases}$ $\qquad$ $x = \dfrac{\pi}{12}$, $y = \dfrac{23\pi}{12}$

$$\frac{\sqrt{3}}{2}\cos y + \frac{1}{2}\operatorname{Sen} y + \operatorname{Sen} y = \frac{2}{3}$$

$$\frac{\sqrt{3}}{2}\cos y + \frac{3}{2}\operatorname{Sen} y = \frac{2}{3}$$

$$3\sqrt{3}\cos y + 9\operatorname{Sen} y = 4$$

$$9\operatorname{Sen} y = 4 - 3\sqrt{3}\cos y$$

$$9\sqrt{1-\cos^2 y} = 4 - 3\sqrt{3}\cos y$$

$$\left(9\sqrt{1-\cos^2 y}\right)^2 = \left(4 - 3\sqrt{3}\cos y\right)^2$$

$$81(1-\cos^2 y) = 16 - 24\sqrt{3}\cos y + 27\cos^2 y$$

$$81 - 81\cos^2 y = 16 - 24\sqrt{3}\cos y + 27\cos^2 y$$

$$0 = 16 - 24\sqrt{3}\cos y + 27\cos^2 y - 81 + 81\cos^2 y = 0$$

$$81\cos^2 y - 24\sqrt{3}\cos y - 65 = 0$$

$a = -2$ $\quad a = 81 \quad b = -24\sqrt{3} \quad c = -65.$

$$\cos y = \frac{24\sqrt{3} \pm \sqrt{(-24\sqrt{3})^2 - 4(81)(-65)}}{2(81)}$$

$$\cos y = \frac{24\sqrt{3} \pm \sqrt{1728 + 21060}}{162}$$

$$\cos y = \frac{24\sqrt{3} \pm \sqrt{22788}}{162}$$

$$\cos y = \frac{24\sqrt{3} \pm 3\sqrt{2532}}{162}$$

Las soluciones son inconmensurables.

f) $\begin{cases} \operatorname{Sen} x + \operatorname{Cos} y = \frac{1}{2} & (1) \\ \operatorname{Sen} x + \operatorname{Sen} y = \frac{3}{2} & (2) \end{cases}$

Restando (1) de (2)

$$\operatorname{Sen} x + \operatorname{Sen} y = \frac{3}{2}$$
$$-\operatorname{Sen} x - \operatorname{Cos} y = -\frac{1}{2}$$

$$\operatorname{Sen} y - \operatorname{Cos} y = 1$$

$$\operatorname{Sen} y = 1 + \operatorname{Cos} y$$

$$\operatorname{Sen}^2 y = 1 + 2\operatorname{Cos} y + \operatorname{Cos}^2 y$$

$$1 - \operatorname{Cos}^2 y = 1 + 2\operatorname{Cos} y + \operatorname{Cos}^2 y$$

$$\cancel{1} + 2\operatorname{Cos} y + \operatorname{Cos}^2 y - \cancel{1} + \operatorname{Cos}^2 y = 0$$

$$2\operatorname{Cos}^2 y + 2\operatorname{Cos} y = 0$$

$$2\operatorname{Cos} y(\operatorname{Cos} y + 1) = 0$$

$$\operatorname{Cos} y(\operatorname{Cos} y + 1) = 0$$

$*$ $\operatorname{Cos} y = 0$

$y = \operatorname{Cos}^{-1}(0)$

$\boxed{y_1 = \frac{\pi}{2}}$ $\qquad$ $\boxed{y_2 = \frac{3\pi}{2}}$

$**$ $\operatorname{Cos} y + 1 = 0$

$\operatorname{Cos} y = -1$ $\qquad$ $y = \operatorname{Cos}^{-1}(-1)$

$\boxed{y = \pi}$

Reemplazando en (1) ó (2):

En 1: $\operatorname{Sen} x + \operatorname{Cos} y = \frac{1}{2}$

$y_1 = \frac{\pi}{2}$ $\quad \operatorname{Sen} x + \operatorname{Cos} \frac{\pi}{2} = \frac{1}{2}$

$\operatorname{Sen} x = \frac{1}{2}$ $\qquad$ $x = \operatorname{Sen}^{-1}\left(\frac{1}{2}\right)$

$$\boxed{x_1 = \frac{\pi}{6}} \qquad \boxed{x_3 = \frac{5\pi}{6}}$$

Con $y_2 = \frac{3\pi}{2}$

$$\operatorname{Sen} x + \operatorname{Cos} y = \frac{1}{2}$$

$$\operatorname{Sen} x = \frac{1}{2}$$

$$\boxed{x_1 = \frac{\pi}{6}} \qquad \boxed{x = \frac{5\pi}{6}}$$

Con $y_3 = \pi$

$$\operatorname{Sen} x + \operatorname{Cos} y = \frac{1}{2}$$

$$\operatorname{Sen} x + \operatorname{Cos} \pi = \frac{1}{2}$$

$$\operatorname{Sen} x = \frac{1}{2} - \operatorname{Cos} \pi$$

$$\operatorname{Sen} x = \frac{1}{2} - (-1)$$

$$\operatorname{Sen} x = \frac{3}{2} \qquad x = \operatorname{Sen}^{-1}\left(\frac{3}{2}\right)$$

Esto es imposible ya que el seno está entre $[-1, 1]$

Por lo tanto, la solución del problema, nos queda:

$$S: \begin{cases} x = \frac{\pi}{6} \\ y = \frac{3\pi}{2} \end{cases} \qquad \begin{matrix} x = \frac{5\pi}{6} \\ y = \frac{3\pi}{2} \end{matrix}$$

$$\begin{cases} x = \frac{\pi}{6} \\ y = \frac{\pi}{2} \end{cases} \qquad \begin{matrix} x = \frac{5\pi}{6} \\ y = \frac{\pi}{2} \end{matrix}$$

(68) $\begin{cases} p^2 + x^2 = 169 & (1) \\ p = x + 7 & (2) \end{cases}$

De (2); reemplazo en (1)

$$p^2 + x^2 = 169$$
$$(x+7)^2 + x^2 = 169$$
$$x^2 + 14x + 49 + x^2 = 169$$
$$2x^2 + 14x + 49 - 169 = 0$$
$$2x^2 + 14x - 120 = 0$$
$$x^2 + 7x - 60 = 0$$
$$(x+12)(x-5) = 0$$

$x + 12 = 0$ $x - 5 = 0$

$\boxed{x = -12}$ $\boxed{x = 5}$

Imposible

(69)

x = tiempo de trabajo del padre
y = tiempo de trabajo del hijo

$$\begin{cases} \frac{1}{x} + \frac{1}{y} = \frac{1}{12} & (1) \\ x = y - 7 & (2) \end{cases}$$

Reemplazo (2) en (1)

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{12}$$
$$\frac{1}{y-7} + \frac{1}{y} = \frac{1}{12}$$
$$12y + 12(y-7) = y(y-7)$$
$$12y + 12y - 84 = y^2 - 7y$$
$$0 = y^2 - 7y - 24y + 84$$ $y^2 - 31y + 84 = 0$

$(y-28)(y-3)=0$

$y-28=0$ $\quad$ $y-3=0$

$\boxed{y_1=28}$ $\quad$ $\boxed{y_2=3}$

...ero descarto por naturaleza $y_2=3$

Reemplazo en (2):

$x=y-7$

$x=28-7$

$\boxed{x=21}$

10)

$$\begin{cases} R_1+R_2=25 & (1) \\ \dfrac{1}{R_1}+\dfrac{1}{R_2}=\dfrac{1}{6} & (2) \end{cases}$$

De (1), tenemos:

$R_1+R_2=25$ $\quad$ $\boxed{R_1=25-R_2}$ (3)

Reemplazo en (2), el valor (3):

$$\frac{1}{R_1}+\frac{1}{R_2}=\frac{1}{6}$$

$$\frac{1}{25-R_2}+\frac{1}{R_2}=\frac{1}{6}$$

$$6R_2+6(25-R_2)=\cancel{6}R_2(25-R_2)$$

$$(25=150R_2-6R_2^2)$$

$$(6R_2^2-150R_2+25=0)$$

$$6R_2+150-6R_2=25R_2-R_2^2$$

$$R_2^2-25R_2+\cancel{6R_2}+150-\cancel{6R_2}=0$$

$$R_2^2-25R_2+150=0$$

$$(R_2 - 15)(R_2 - 10) = 0$$

$$R_2 - 15 = 0 \qquad R_2 - 10 = 0$$

$$\boxed{R_2 = 15} \qquad \boxed{R_2 = 10}$$

Reemplazo en (3):

$$R_1 = 25 - R_2$$

Con $R_2 = 15$

$$R_1 = 25 - 15$$

$$\boxed{R_1 = 10}$$

Con $R_2 = 10$

$$R_1 = 25 - 10$$

$$\boxed{R_2 = 15}$$

$$\begin{cases} R_1 = 10 \\ R_2 = 15 \end{cases} \qquad \begin{matrix} R_1 = 15 \\ R_2 = 10 \end{matrix}$$

(71)
$$\begin{cases} 2x^2 - 3y^2 = 15 & (1) \\ x^2 + 2y^2 = 11 & (2) \end{cases}$$

$$\begin{array}{r} 2x^2 - 3y^2 = +15 \\ -2x^2 - 4y^2 = -22 \\ \hline // \quad -7y^2 = -7 \end{array}$$

$$7y^2 = 7$$

$$y^2 = \frac{\cancel{7}^{1}}{\cancel{7}_{1}} \qquad y^2 = 1 \qquad y = \pm 1$$

Reemplazo en (1) ó (2):

En (2): $x^2 + 2y^2 = 11$

$$x^2 + 2(\pm 1)^2 = 11$$

$$x^2 + 2 = 11 \qquad x^2 = 11 - 2$$

$$x^2 = 9 \qquad x = \pm 3$$

$$S: \begin{cases} x = 3 \\ y = 1 \end{cases} \qquad \begin{matrix} x = 3 \\ y = -1 \end{matrix} \qquad \begin{matrix} x = -3 \\ y = 1 \end{matrix} \qquad \begin{matrix} x = -3 \\ y = -1 \end{matrix}$$

b) $\begin{cases} x^2 + y^2 = 25 & (1) \\ x + y = 1 & (2) \end{cases}$

De (2):

$x + y = 1$ $\quad$ $\boxed{x = 1 - y}$ (3)

Reemplazo (3) en (1)

$x^2 + y^2 = 25$

$(1 - y)^2 + y^2 = 25$

$1 - 2y + y^2 + y^2 - 25 = 0$

$2y^2 - 2y + 24 = 0$

$y^2 - y + 12 = 0$

$(y - 4)(y + 3) = 0$

$y - 4 = 0$ $\qquad$ $y + 3 = 0$

$\boxed{y_1 = 4}$ $\qquad$ $\boxed{y_2 = -3}$

Reemplazo en (3)

$x = 1 - y$

Con $y_1 = 4$ $\qquad$ Con $y_2 = -3$

$x = 1 - 4$ $\qquad$ $x = 1 + (-3)$

$\boxed{x_1 = -3}$ $\qquad$ $\boxed{x_2 = 4}$

S: $\begin{cases} x = -3 \\ y = 4 \end{cases}$ $\qquad$ $\begin{matrix} x = 4 \\ y = -3 \end{matrix}$

c) $\begin{cases} x^2 + 3xy = -8 & (1) \\ xy + y^2 = -4 & (2) \end{cases}$

De (2):

$$xy + y^2 = -4$$
$$xy = -4 - y^2 \qquad \boxed{x = \frac{-4-y^2}{y}} \quad (3)$$

Reemplazo (3) en (1):

$$x^2 + 3xy = -8$$
$$\left(\frac{-4-y^2}{y}\right)^2 + 3xy = -8$$
$$\frac{16 + 8y^2 + y^4}{y^2} + 3\cancel{y}\left(\frac{-4-y^2}{\cancel{y}}\right) = -8$$
$$16 + 8y^2 + y^4 + y^2(-12 - 3y^2) = -8y^2$$
$$16 + 8y^2 + y^4 - 12y^2 - 3y^4 + 8y^2 = 0$$
$$-2y^4 + 4y^2 + 16 = 0$$
$$y^4 - 2y^2 - 8 = 0$$
$$(y^2 - 4)(y^2 + 2) = 0$$

$y^2 - 4 = 0$ $\qquad$ $y^2 + 2 = 0$

$y^2 = 4$ $\qquad$ $y^2 = -2$

$\boxed{y = \pm 2}$ $\qquad$ Imposible

Reemplazo en (3)

$$x = \frac{-4-y^2}{y}$$

Con $y_1 = 2$

$$x_1 = \frac{-4-(2)^2}{2} \qquad x_1 = \frac{-4-4}{2}$$

$$x_1 = -\frac{8}{2} \qquad \boxed{x_1 = -4}$$

on $y = -2$

$$x_2 = \frac{-4-(-2)^2}{-2} \qquad x_2 = \frac{-4-4}{-2}$$

$$x_2 = \frac{-\cancel{8}^{\,4}}{\cancel{-2}_{\,1}} \qquad \boxed{x_2 = 4}$$

S: $\begin{cases} x = -4 \\ y = 2 \end{cases} \qquad \begin{matrix} x = 4 \\ y = -2 \end{matrix}$

d) $\begin{cases} x^2 + y^2 = 29 & (1) \\ xy = 10 & (2) \end{cases}$

De (2):

$$xy = 10 \qquad \boxed{x = \frac{10}{y}}$$

Reemplazo en (1):

$$x^2 + y^2 = 29$$

$$\left(\frac{10}{y}\right)^2 + y^2 = 29$$

$$\frac{100}{y^2} + y^2 = 29 \qquad 100 + y^4 = 29y^2$$

$$y^4 - 29y^2 + 100 = 0$$

$$(y^2 - 25)(y^2 - 4) = 0$$

$$y^2 - 25 = 0 \qquad y^2 - 4 = 0$$

$$y^2 = 25 \qquad y^2 = 4$$

$$\boxed{y = \pm 5} \qquad \boxed{y = \pm 2}$$

S: $\begin{cases} x = 2 \\ y = 5 \end{cases} \qquad \begin{matrix} x = 5 \\ y = 2 \end{matrix}$

e) $\begin{cases} xy - 6 = \dfrac{y^3}{x} & (1) \\ xy + 24 = \dfrac{x^3}{y} & (2) \end{cases}$

Multiplicando miembro a miembro (1) y (2) nos queda

$$(xy-6)(xy+24) = x^2y^2$$

$$x^2y^2 - 6xy + 24xy - 144 = x^2y^2$$

$$\cancel{x^2y^2} - \cancel{x^2y^2} + 18xy = 144$$

$$18xy = 144$$

$$x = \frac{\overset{8}{\cancel{144}}}{\underset{1}{\cancel{18}}y} \qquad \boxed{x = \frac{8}{y}} \quad (3)$$

Reemplazo en (1) ó (2)

En (1): $xy - 6 = \dfrac{y^3}{x}$

$\frac{8}{y}\Big)\ y - 6 = \dfrac{y^3}{\frac{8}{y}}$

$8 - 6 = \dfrac{y^4}{8}$ $\qquad 8(2) = y^4$

$y^4 = 16$ $\qquad \sqrt[4]{y^4} = \sqrt[4]{16}$

$\boxed{y = \pm 2}$

Reemplazando en (3):

$x = \dfrac{8}{y}$

$y_1 = 2$ $\qquad x_1 = \dfrac{\cancel{8}^{4}}{\cancel{2}_1}$ $\qquad \boxed{x_1 = 4}$

$y_2 = -2$ $\qquad x_2 = \dfrac{\cancel{8}^{4}}{\cancel{-2}_1}$ $\qquad \boxed{x_2 = -4}$

S: $\begin{cases} x = 4 \\ y = 2 \end{cases}$ $\qquad \begin{matrix} x = -4 \\ y = -2 \end{matrix}$

f) $\begin{cases} 3x^2 - 8xy + 4y^2 = 0 & (1) \\ 5x^2 - 7xy - 6y^2 = 0 & (2) \end{cases}$

Factorizando (1):

$(3x - 2y)(x - 2y) = 0$

$3x - 2y = 0$ $\qquad x - 2y = 0$

$3x = 2y$ $\qquad \boxed{x = 2y}$ (4)

$\boxed{x = \dfrac{2y}{3}}$ (3)

Reemplazo en (2), el valor de (3):

$5x^2 - 7xy - 6y^2 = 0$

$5\left(\dfrac{2y}{3}\right)^2 - 7\left(\dfrac{2y}{3}\right)y - 6y^2 = 0$

$$5\left(\frac{4y^2}{9}\right)-\frac{14y^2}{3}-6y^2=0$$

$$20y^2-42y^2-54y^2=0$$

$$-76y^2=0$$

$$y^2=0 \qquad \boxed{y=0}$$

Reemplazo en (3):

$$x=\frac{2y}{3} \qquad x=\frac{2(0)}{3} \qquad \boxed{x=0}$$

- Lo mismo ocurre al reemplazar el valor de (4) en (2)

$$S: \begin{cases} x=0 \\ y=0 \end{cases}$$

* g)

$$\begin{cases} 3x^2-2xy=160 & (1) \\ x^2-3xy-2y^2=8 & (2) \end{cases}$$

Despejando y en función de x en (1):

$$\boxed{y=\frac{3x^2-160}{2x}} \quad (3)$$

Reemplazo (3) en (2):

$$x^2-3x\left(\frac{3x^2-160}{2x}\right)-2\left(\frac{3x^2-160}{2x}\right)^4=8$$

$$x^2-\frac{3(3x^2-160)}{2}-\frac{9x^4-960x^2+25600}{2x^2}$$

$$2x^4-9x^4+480x^2-9x^4+960x^2+25600=16x^2$$

Reduciendo, nos queda:

$$-16x^4+1424x^2-25600=0$$

$$x^4-89x^2+1600=0$$

actorizando:

$$(x^2-64)(x^2-25)=0$$

$x^2-64=0$ $\quad$ $x^2-25=0$

$x^2=64$ $\quad$ $x^2=25$

$\boxed{x=\pm 8}$ $\quad$ $\boxed{x=\pm 5}$

Reemplazando en (3)

$$y=\frac{3x^2-160}{2x}$$

Con $x=8$ $\quad$ $y=2$

Con $x=-8$ $\quad$ $y=-2$

Con $x=5$ $\quad$ $y=-\frac{17}{2}$

Con $x=-5$ $\quad$ $y=\frac{17}{2}$

$S: \begin{cases} x=8 \\ y=2 \end{cases}$ $\quad$ $\begin{matrix} x=-8 \\ y=-2 \end{matrix}$ $\quad$ $\begin{matrix} x=5 \\ y=-\frac{17}{2} \end{matrix}$ $\quad$ $\begin{matrix} x=-5 \\ y=\frac{17}{2} \end{matrix}$

72 $\begin{cases} 5x + 3y \leq 105 \\ 2x + 4y \leq 70 \\ x \geq 0 \\ y \geq 0 \end{cases}$

a) $\begin{matrix} (2) \\ (-5) \end{matrix} \begin{cases} 5x + 3y = 105 & (1) \\ 2x + 4y = 70 & (2) \end{cases}$

$$\begin{array}{r} 10x + 6y = 210 \\ -10x - 20y = -350 \\ \hline // \quad -14y = -140 \end{array}$$

$14y = 140 \qquad y = \dfrac{\cancel{140}^{10}}{\cancel{14}_{1}} \qquad \boxed{y = 10}$

Reemplazo en (1) ó (2)

En (2): $2x + 4y = 70$

$2x + 4(10) = 70$

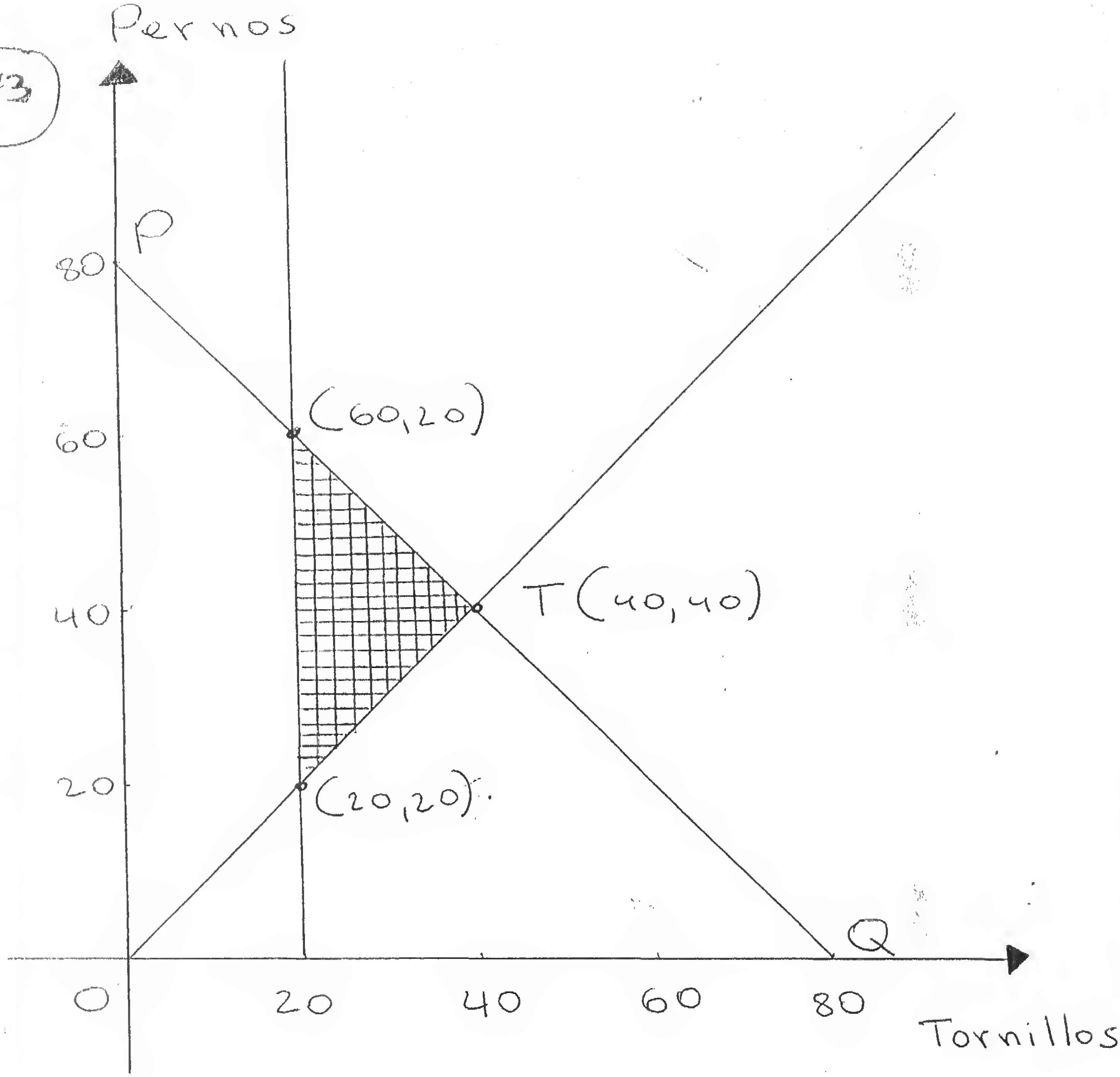
$2x + 40 = 70 \qquad 2x = 70 - 40$

$2x = 30 \qquad x = \dfrac{\cancel{30}^{15}}{\cancel{2}_{1}} \qquad \boxed{x = 15}$

$P(15, 10)$

S: $\{ X = 15 \quad Y = 10$

73

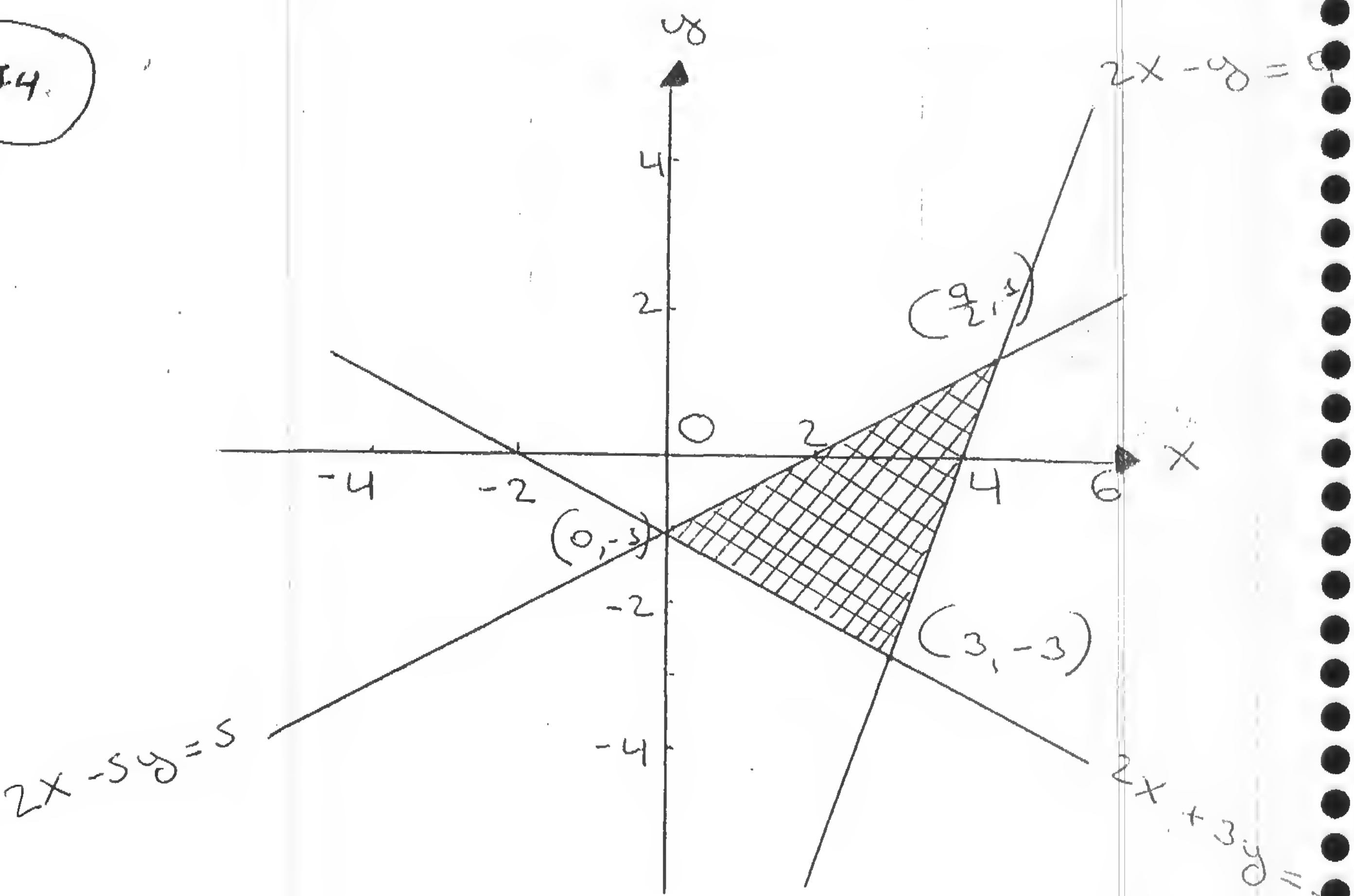


Los vértices del triángulo formado son: a) (20, 20), b) (60, 20), c) (40, 40)

Se observa claramente que $Q \geq 20$ y que $P \geq Q$ y además por b se puede establecer $P \leq 80 - Q$

$$\begin{cases} P \geq Q \\ P \leq 80 - Q \\ Q \geq 20 \end{cases}$$

74.



Los vértices del triángulo son:

a) $(0, -1)$ b) $(3, -3)$ c) $(\frac{9}{2}, 1)$

Se observa que: $2x + 3y + 3 \geq 0 \quad \wedge$

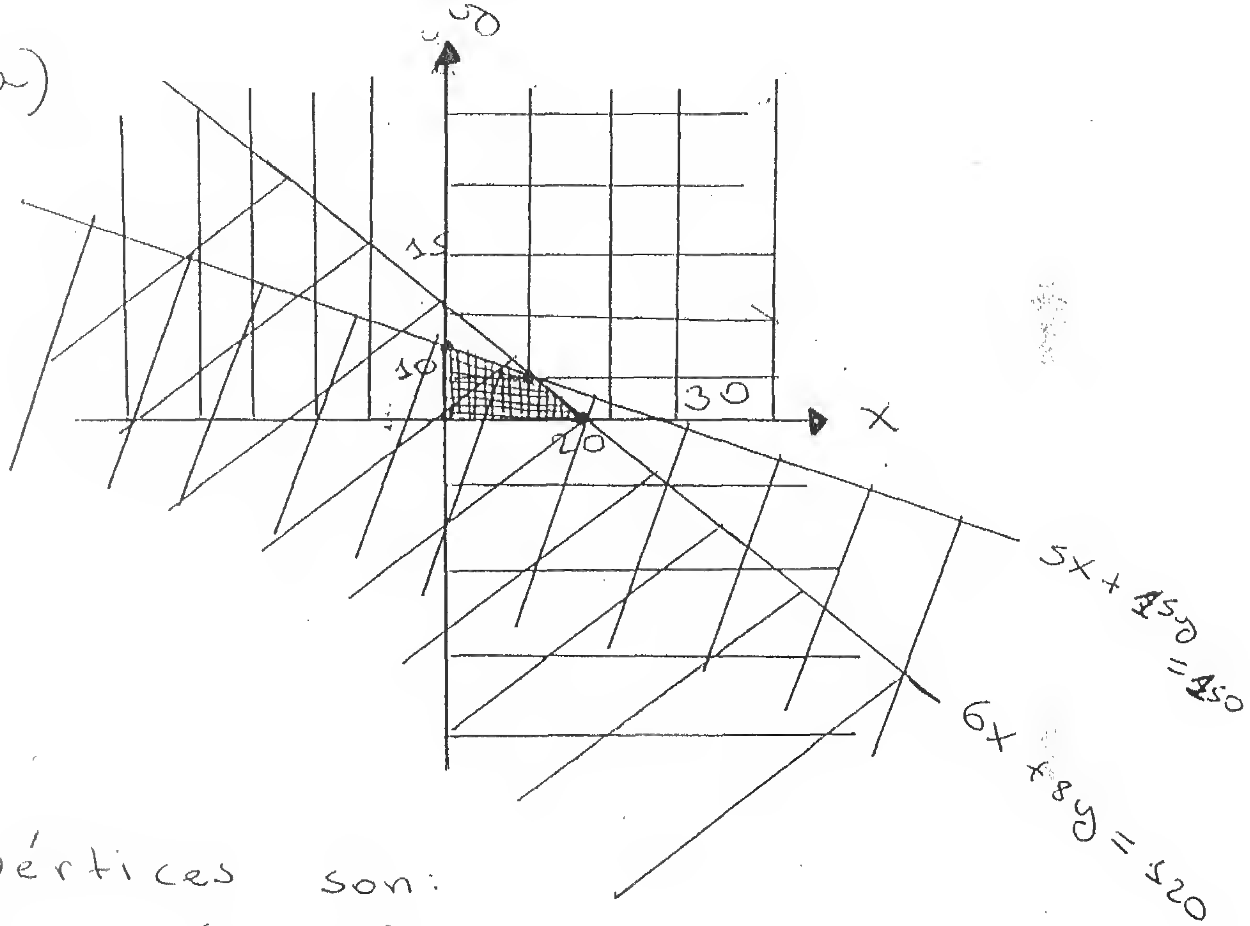
$2x - y - 9 \leq 0 \quad \wedge \quad 2x - 5y - 5 \geq 0$

Por tanto, el sistema corresponde a:

$$\begin{cases} 2x + 3y \geq -3 \\ 2x - y - 9 \leq 0 \\ 2x - 5y - 5 \geq 0 \end{cases}$$

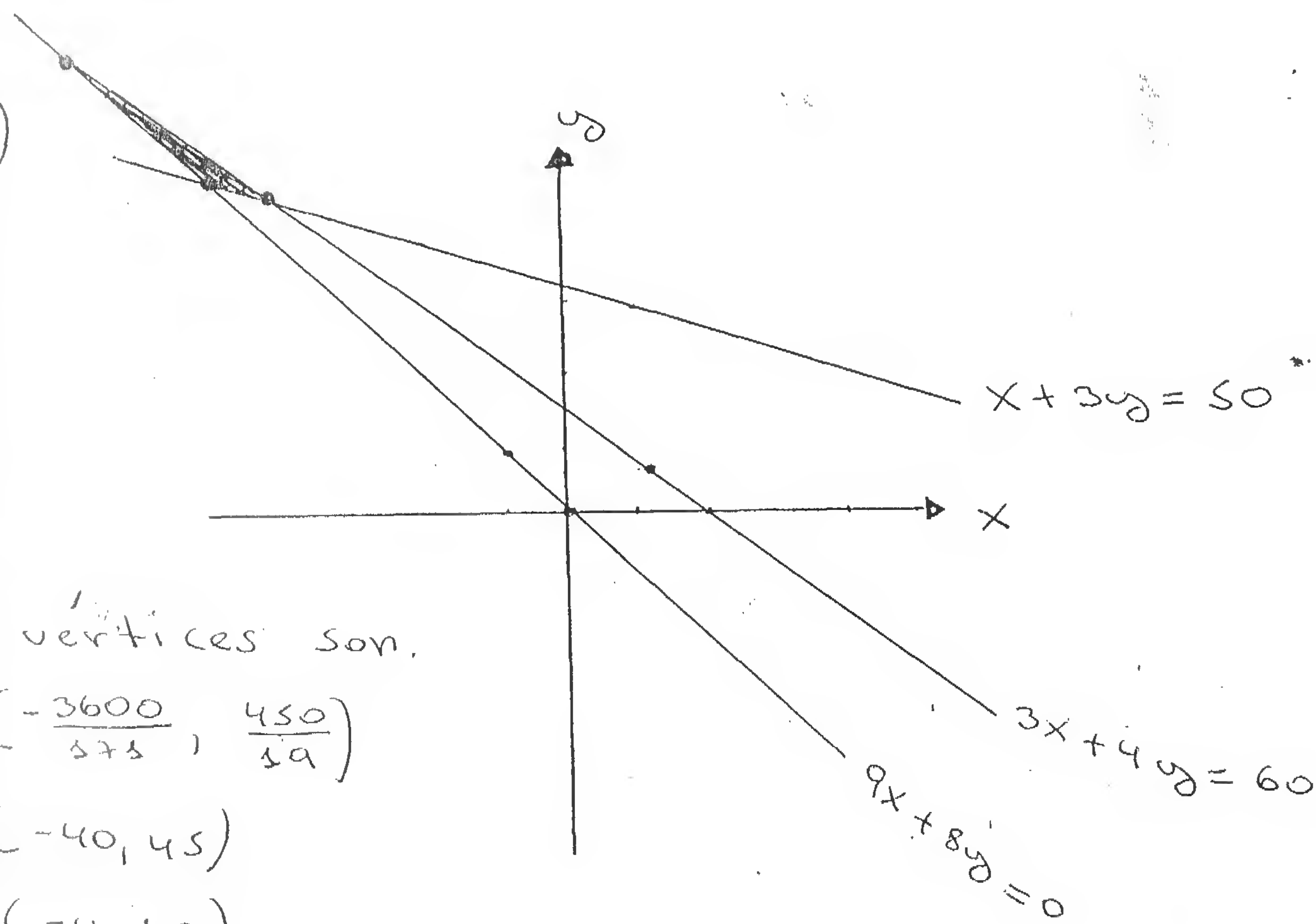
La región sombreada en el interior del triángulo corresponde al conjunto solución del sistema de desigualdades descrito.

7-5) a)



Los vértices son:
(0, 0) , (0, 10) , (20, 0) , (12, 6)

b)



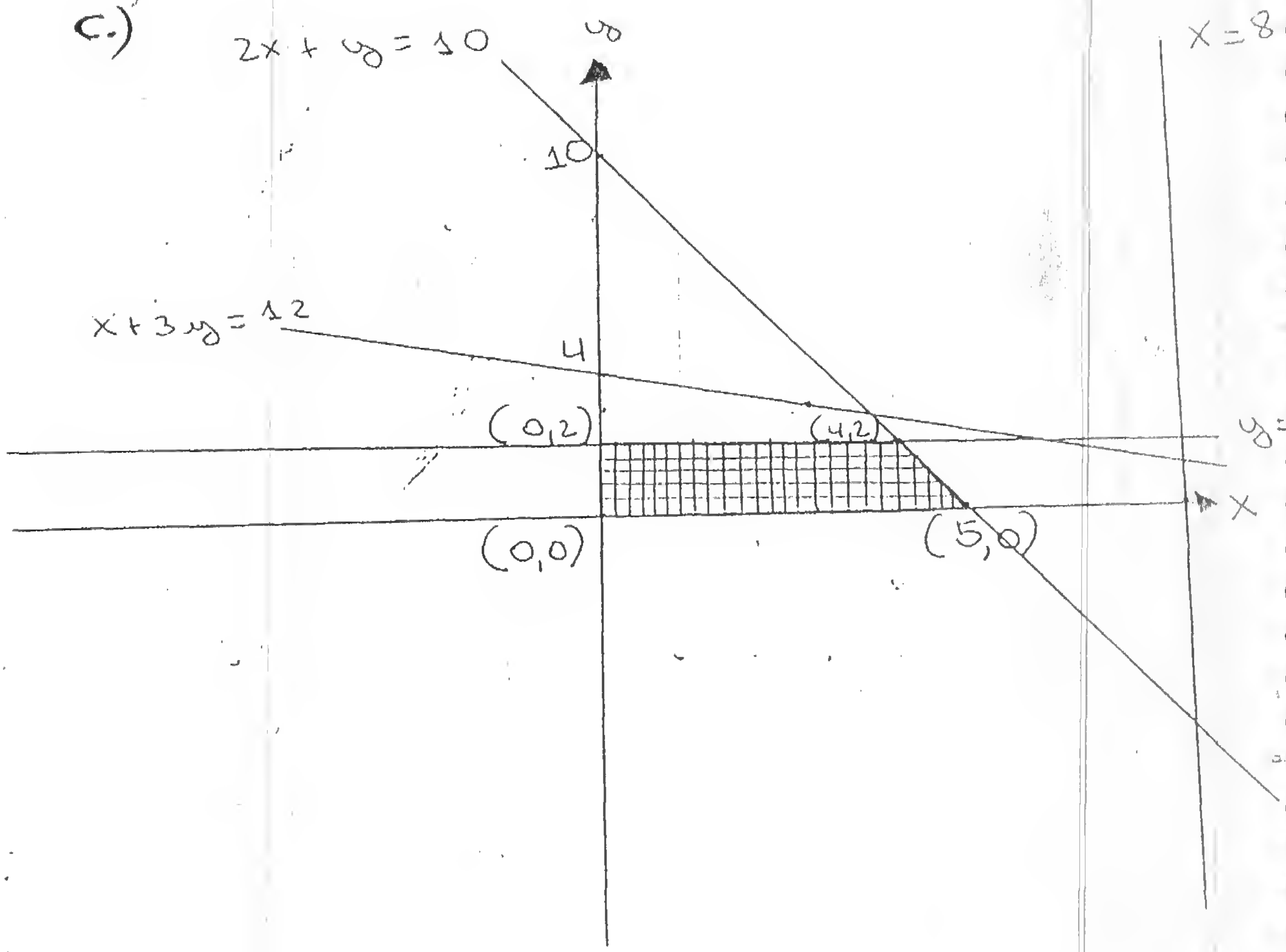
Los vértices son.

P $\left(-\frac{3600}{171}, \frac{450}{19}\right)$

Q (−40, 45)

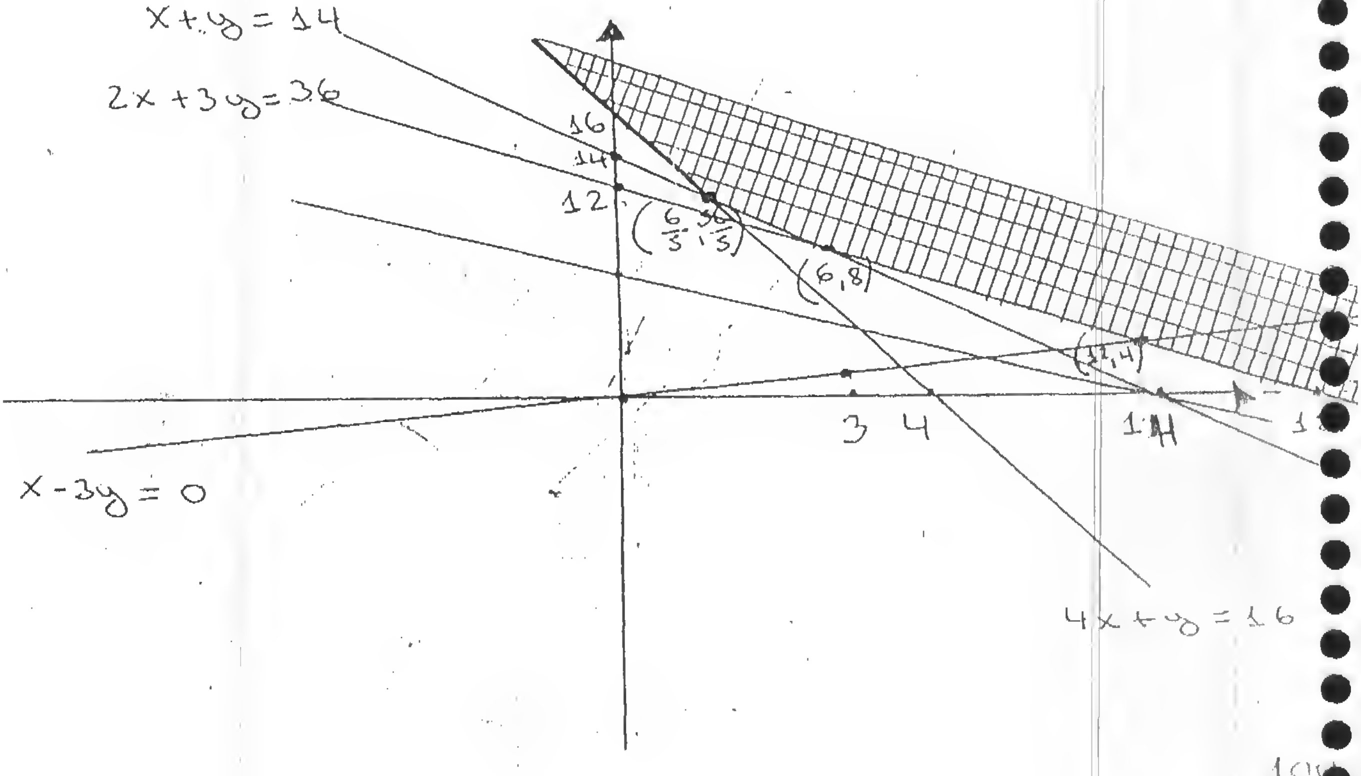
R (−4, 18)

c.)

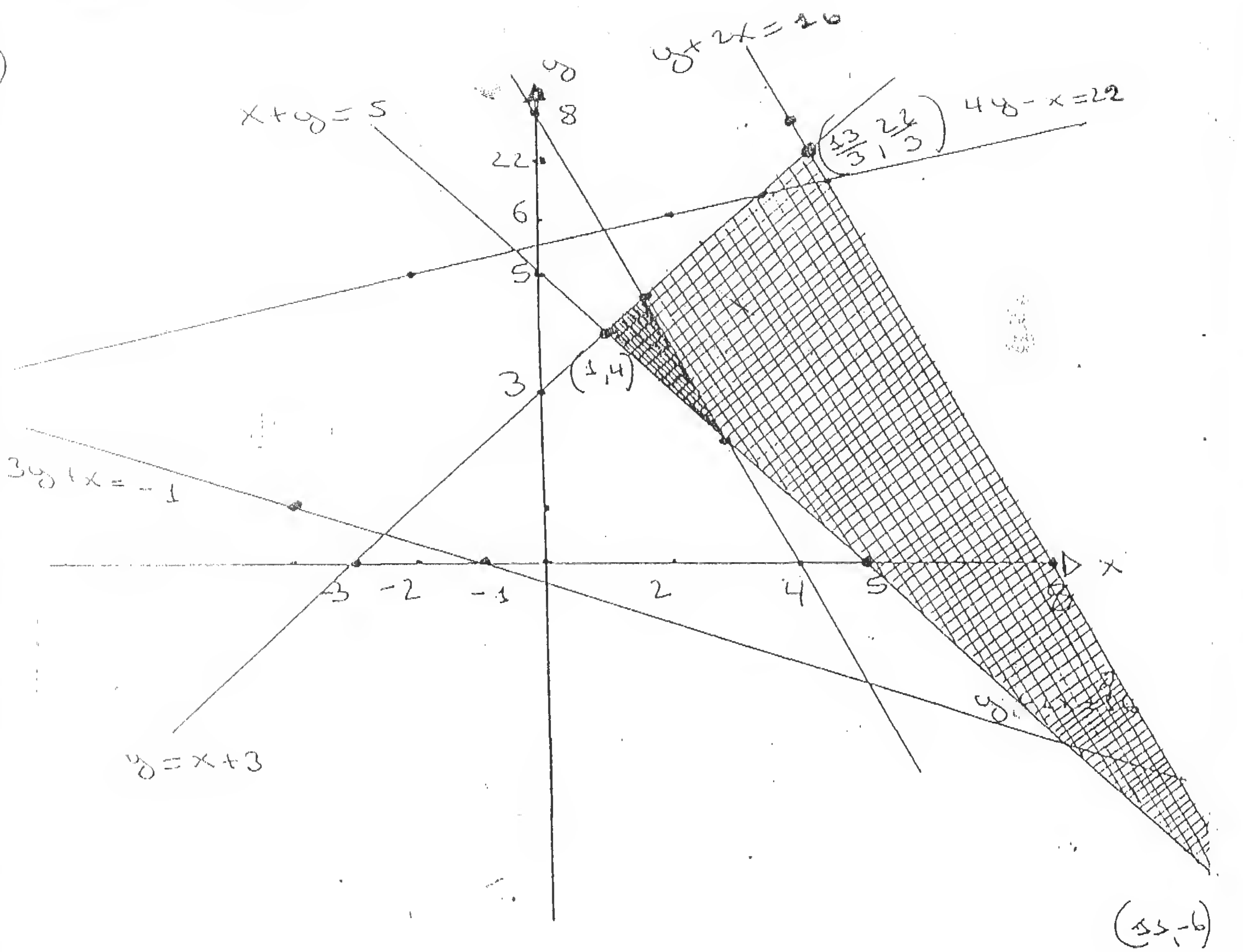


Los vértices son:
(0,2) , (0,0) , (4,2) , (5,0)

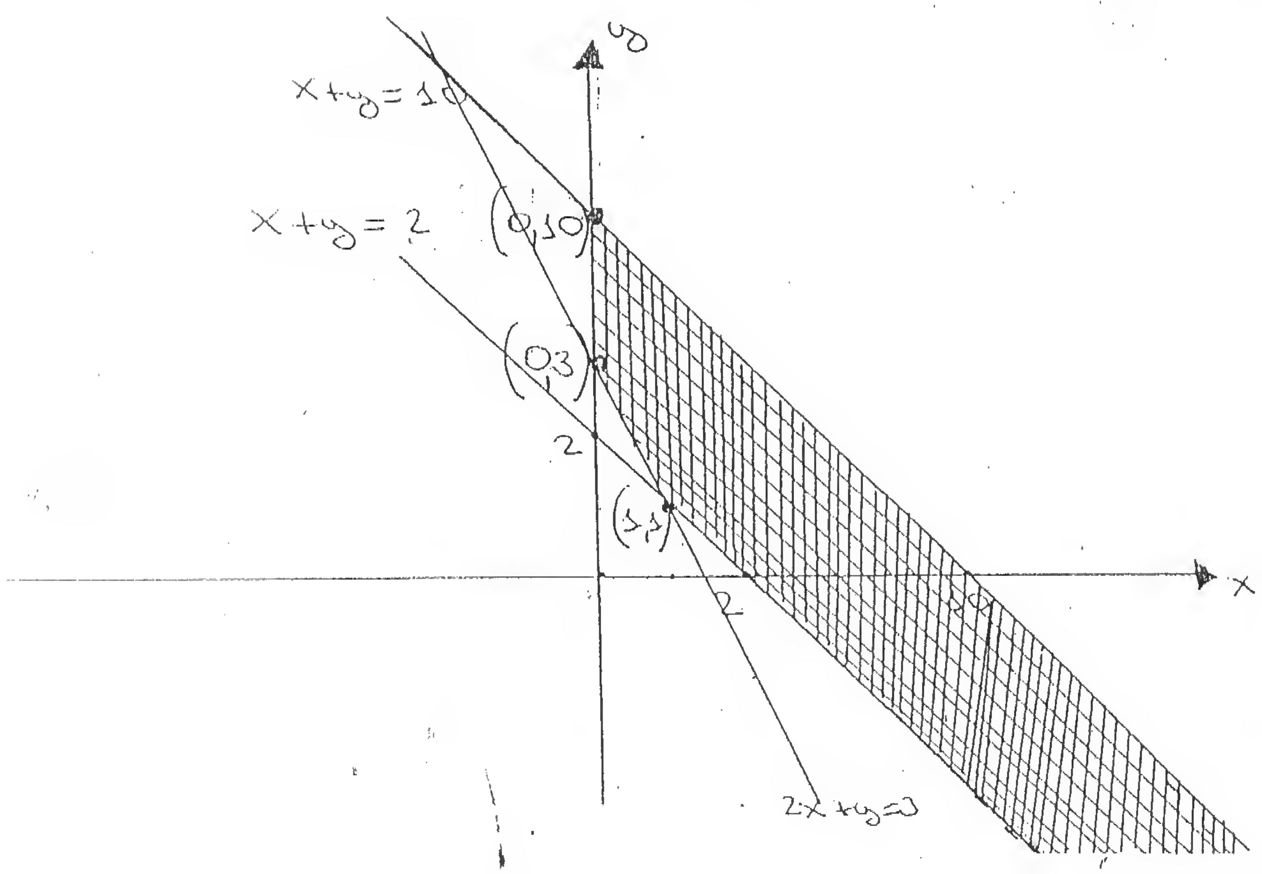
d.)



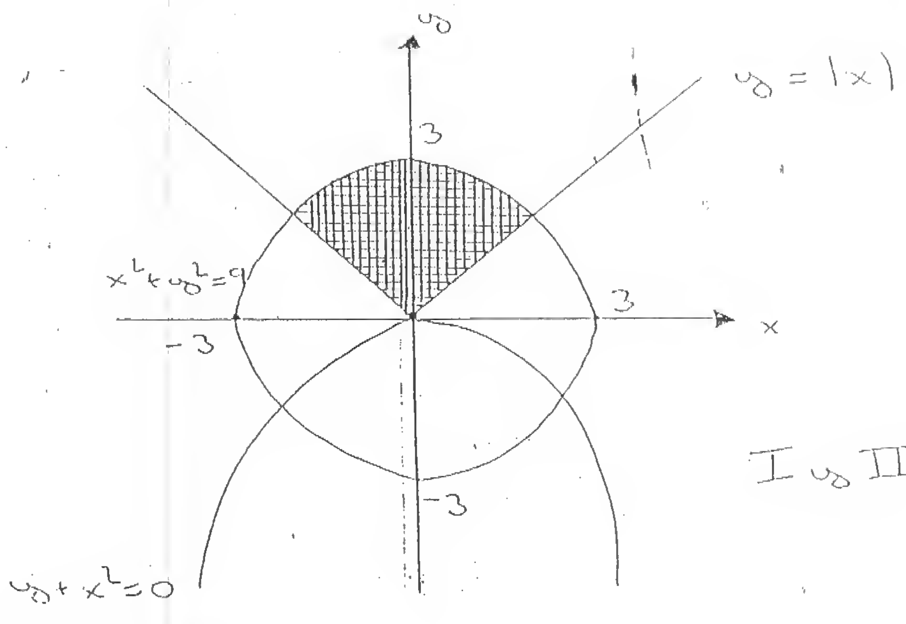
(a)


y + 2x = 16
4y - x = 22
x + y = 5
(13/3, 22/3)
(1, 4)
3y + x = -1
y = x + 3
(11, -6)
y
x
8
22
6
5
3
-3
-2
-1
2
4
5

(b)

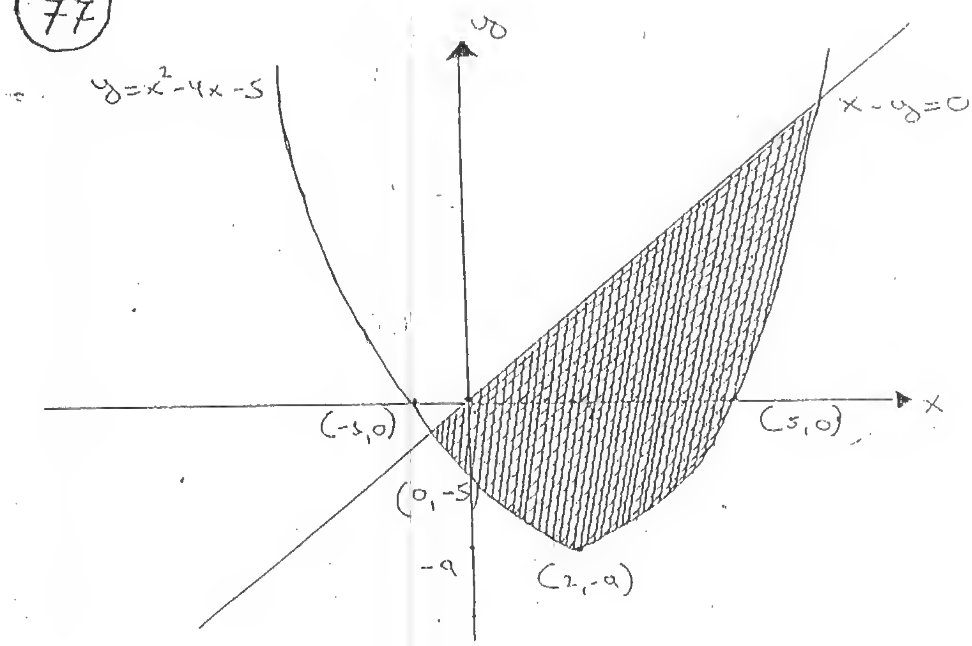

x + y = 10
x + y = 2
(0, 10)
(0, 3)
(1, 1)
2x + y = 3
y
x
2
2

76.

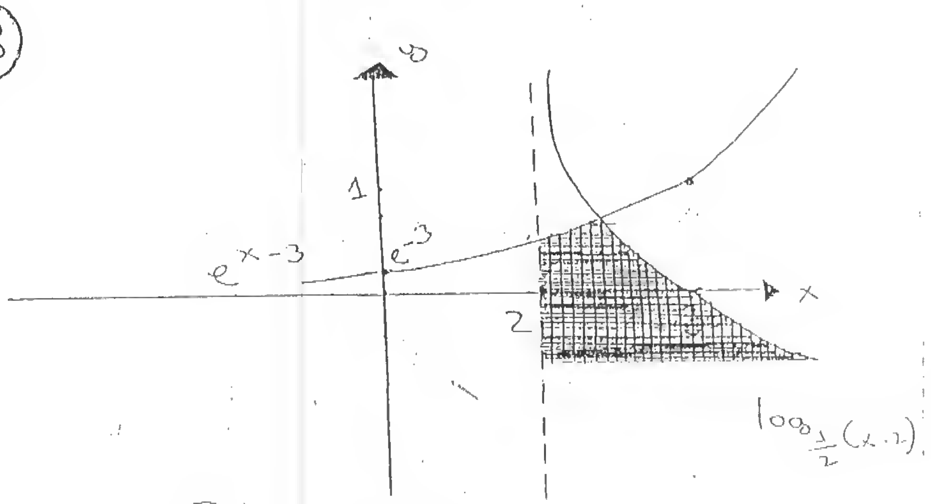


I y II

77



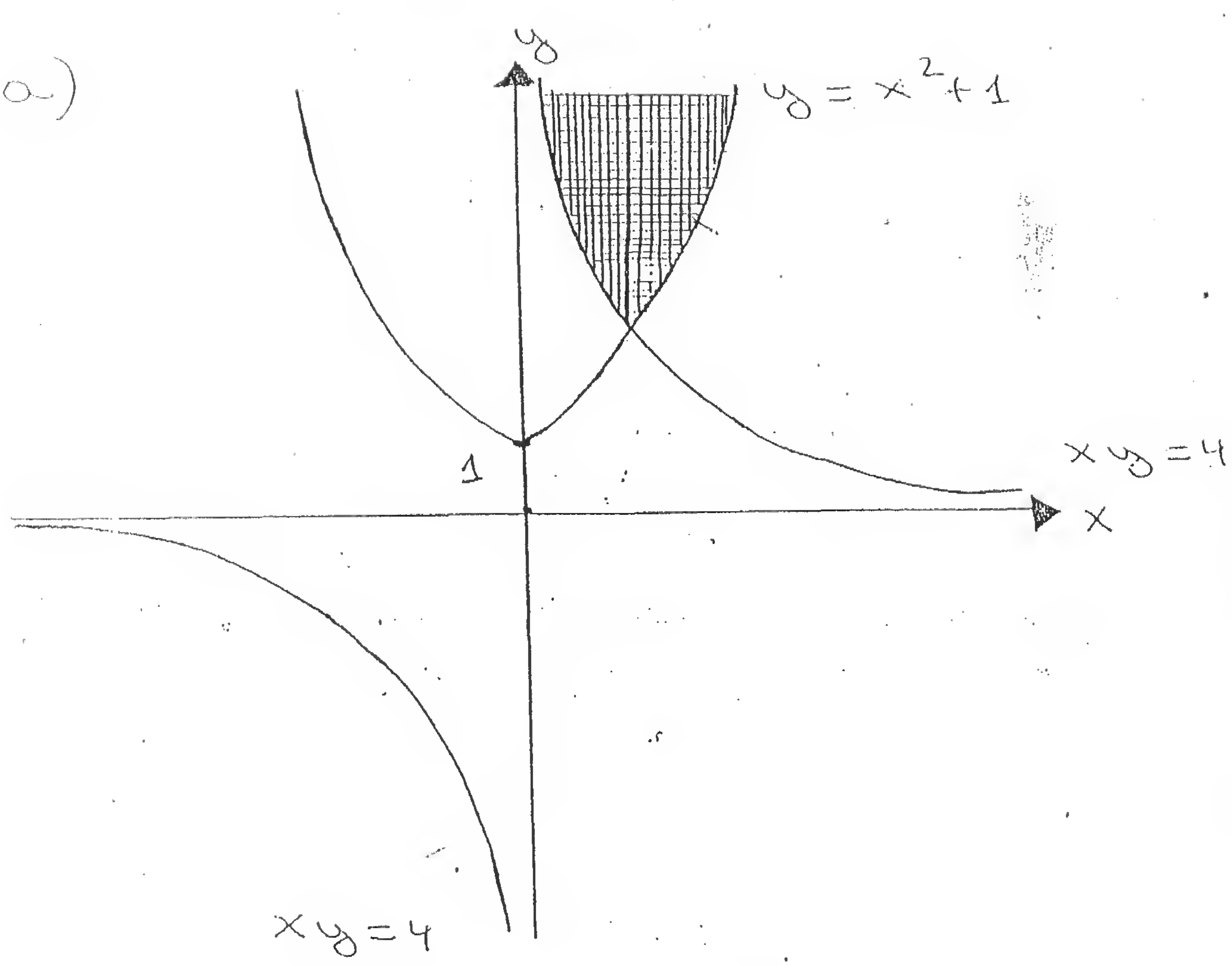
78



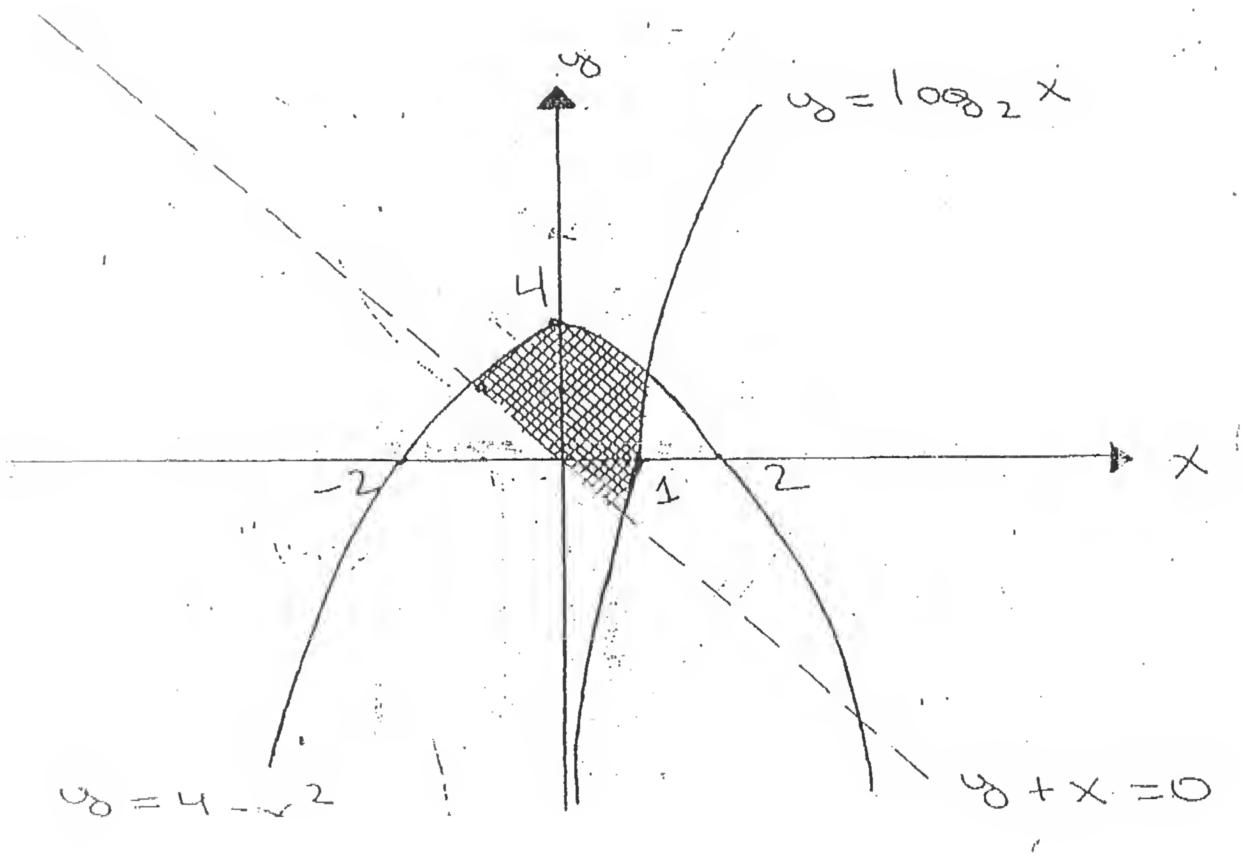
$$S: \begin{cases} \log_{\frac{1}{2}}(x-2) \geq y \\ e^{x-3} \geq y \end{cases}$$

79

a)

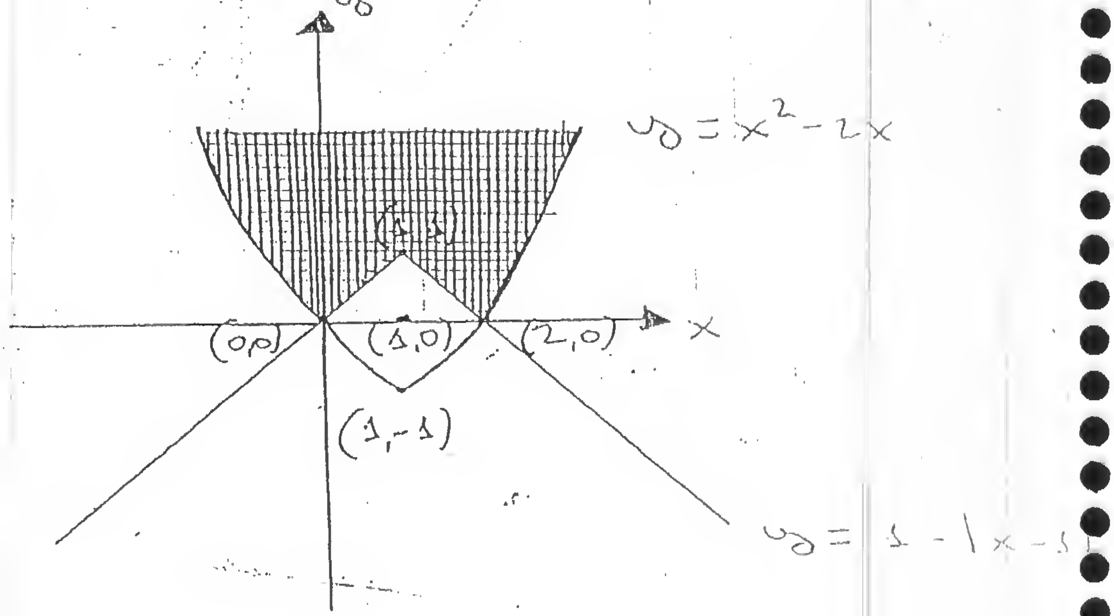


b)



(80)

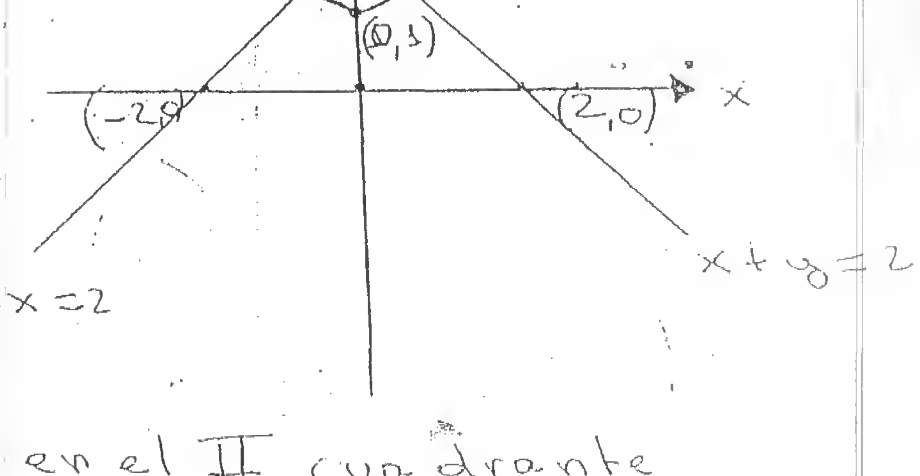
$$\begin{cases} y \geq x^2 - 2x \\ y > 1 - |x - 1| \end{cases}$$



a) F b) F c) F d) F e) ✓

C es falso porque el punto (1, 1) no pertenece a la gráfica ya que $y > 1 - |x - 1|$

(81)



c) Solo en el II cuadrante

419

Solución

EJERCICIOS PROPUESTOS

CAPÍTULO SEIS

NÚMEROS COMPLEJOS

(1) $\overline{z \cdot w} = \overline{z} \cdot \overline{w}$

Por propiedades de los # complejos esto es verdadero

$z = 1 + i \qquad w = 2 - i$

$z \cdot w = 2 + i + 1 \qquad \boxed{\overline{z \cdot w} = 3 - i}$ *

$\overline{z} = 1 - i \qquad \overline{w} = 2 + i$

$\boxed{\overline{z} \cdot \overline{w} = 3 - i}$ **

Luego * y ** son iguales (a)

(2) $\left(\sqrt{1 - \sqrt{3}}\right)^2 = \sqrt{(1 - \sqrt{3})^2}$

Si efectuamos $1 - \sqrt{3}$ nos da una cantidad negativa; no se puede sacar raiz cuadrada a esa cantidad negativa, mientras que en el otro lado se simplifican el exponente con el indice (b)

(3) $f(x) = x^4 + a_1 x^3 + a_2 x^2 + a_3 x + a_4 = 0$

$f(i) = 1 - a_1(i) + a_2(-1) + a_3(i) + a_4 = 0$

$f(1+i) = -4 + a_1(-2+2i) + a_2(2i) + a_3(1+i) + a_4 = 0$

$f(1+i) = -4 + 2i\,a_1 - 2a_1 + 2i\,a_2 + a_3 + i\,a_3 + a_4 = 0$

$\therefore a_1 + a_2 + a_3 + a_4 = 4$ (d)

(4) $p(x): \det\begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix} + \det\begin{vmatrix} 8x & 2 \\ -1 & x \end{vmatrix} - \det\begin{vmatrix} 4 & 1 \\ 2 & x \end{vmatrix} = 0$

$\det\begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix} = -2 \qquad \det = 8x^2 + 2 \qquad \det = 4x - 2$

$\cancel{-2} + 8x^2 \cancel{+2} - 4x + 2 = 0 \qquad 8x^2 - 4x + 2 = 0$

$4x^2 - 2x + 1 \qquad x_1 + x_2 = -\dfrac{-\cancel{2}^{1}}{\cancel{4}_{2}} \qquad \boxed{x_1 + x_2 = \frac{1}{2}}$ (a)

(5) $\frac{2i}{x} + iy + 2 = 3i - \frac{3}{x} + y$

$i\left(\frac{2}{x} + y - 3\right) = y - \frac{3}{x} + 2 = 0$

$y - \frac{3}{x} + 2 = 0$ $\qquad \boxed{y = \frac{3}{x} - 2}$

Reemplazo x en $\frac{2}{x} + y - 3 = 0$ $\qquad \boxed{x = 1}$

$\frac{2}{x} + \frac{3}{x} - 2 - 3 = 0$ $\qquad \frac{5}{x} = 5$

Reemplazo en x

$y = \frac{3}{x} - 2$

$y = \frac{3}{1} - 2$ $\qquad y = 3 - 2$ $\qquad \boxed{y = 1}$

$S: \{x = 1 \quad y = 1$

(6) i) $3 + 2ix + 3yi = 8i + x - 2y$

$2ix + 3iy - 8i = x - 2y - 3$

$i(2x + 3y - 8) = x - 2y - 3$

$\begin{cases} 2x + 3y = 8 \\ x - 2y = 3 \end{cases}$ $\qquad \begin{array}{r} 2x + 3y = 8 \\ -2x + 4y = -6 \\ \hline // \quad 7y = 14 \end{array}$

$\boxed{y = 2}$

$x - 2y = 3$

$x - 2(2) = 3$

$x = 3 + 4$ $\qquad \boxed{x = 7}$

ii) $(1 - i)x + (2 + i)y = 4 + 2i$

$x - ix + 2y + iy = 4 + 2i$

$x + 2y - 4 = ix - iy + 2i$

$x + 2y - 4 = i(x - y + 2)$

$\begin{cases} x + 2y = 4 \\ x - y = -2 \end{cases}$ $\qquad \begin{array}{r} x + 2y = 4 \\ -x + y = 2 \\ \hline // \quad 3y = 6 \end{array}$

$y = \frac{6}{3}$ $\qquad \boxed{y = 2}$

$x - y = -2$

$x - 2 = -2$

$x = 2 - 2$ $\qquad \boxed{x = 0}$

iii) $\frac{8i}{x} + iy - 2 = 7i - \frac{10}{x} + y$

$\frac{8i}{x} + iy - 7i = y + 2 - \frac{10}{x}$

$i\left(\frac{8}{x} + y - 7\right) = y + 2 - \frac{10}{x}$

$$\begin{cases} \frac{8}{x} + y = 7 \\ -\frac{10}{x} + y = -2 \end{cases}$$

$-\frac{8}{x} - y = -7$

$-\frac{10}{x} + y = -2$

$-\frac{18}{x} \quad // = -9$

$\frac{18}{x} = 9$

$9x = 18$

$x = \frac{18}{9}$ $\boxed{x=2}$

$\frac{8}{x} + y = 7$

$\frac{8}{2} + y = 7$

$4 + y = 7$ $\quad y = 7 - 4$ $\boxed{y=3}$

*** Del ejercicio 3

Las raíces son: $i, -i, 1+i, 1-i$

$(x-i)(x+i)(x-1-i)(x-1+i) = x^4 - 2x^3 + 3x^2 - 2x + 2$

Por lo tanto: $a_1 + a_2 + a_3 + a_4 = 1$ ***

7)

$$A = \begin{pmatrix} 1 & 0 & 2i \\ 0 & 1-i & 1 \\ -i & 0 & 1 \end{pmatrix} \qquad \bar{A} = \begin{pmatrix} 1 & 0 & -2i \\ 0 & 1+i & 1 \\ i & 0 & 1 \end{pmatrix}$$

$$A - \bar{A} = \begin{pmatrix} 0 & 0 & 4i \\ 0 & -2i & 0 \\ -2i & 0 & 0 \end{pmatrix}$$

$$\det(A - \bar{A}) = 4i \begin{vmatrix} 0 & -2i \\ -2i & 0 \end{vmatrix}$$

$4i(-4i^2) = -16i^3$ $\quad -16(-i)$ $\quad \boxed{= +16i}$

8)

$\frac{-4 + mi}{2 - 3i} = n - 2i$

$-4 + mi = (2-3i)(n-2i)$

$-4 + mi = 2n - 4i - 3in - 6$

$-4 + mi = (2n - 6) - i(4 + 3n)$

$-4 + mi$

$-4 = 2n - 6$

$-4 + 6 = 2n$

$2 = 2n$

$\boxed{n = 1}$

$mi = -i(4+3n)$

$m = -4 - 3n$

$m = -4 - 3(1)$

$\boxed{m = -7}$

(9) $z_1 = 1 + \sqrt{3}i \qquad z_2 = -1 + \sqrt{3}i$

$$\frac{z_2}{z_1} = \frac{-1+\sqrt{3}i}{1+\sqrt{3}i} \cdot \frac{1-\sqrt{3}i}{1-\sqrt{3}i} = \frac{2+2\sqrt{3}i}{4}$$

$$\frac{2+2\sqrt{3}i}{4} = \frac{\cancel{2}^{1}(1+\sqrt{3}i)}{\cancel{4}_{2}} = \frac{1+\sqrt{3}i}{2}$$

$$\left(\frac{z_2}{z_1}\right)^2 = \left(\frac{1+\sqrt{3}i}{2}\right)^2 = \frac{-2+2\sqrt{3}i}{4}$$

(c)

$$2\left(\frac{z_2}{z_1}\right)^2 \cdot \cancel{2}\left[\cancel{2}\left(\frac{-1+\sqrt{3}i}{\cancel{4}}\right)\right] \boxed{= -1+\sqrt{3}i}$$

(10) $z = \dfrac{4+Ki}{2+i}$

$$z = \frac{4+Ki}{2+i} \cdot \frac{2-i}{2-i} = \frac{8+K-4i+2Ki}{5}$$

$$z = \frac{8+K+i(2K-4)}{5}$$

* Para que z sea real puro es necesario que

$2K-4 = 0 \qquad 2K = 4 \qquad K = \frac{\cancel{4}^{2}}{\cancel{2}_{1}} \qquad \boxed{K=2}$

* Para que z sea imaginario puro:

$8+K=0 \qquad \boxed{K=-8}$

(11) $x^3 - 5x^2 + 7x + 13 = 0$

Los divisores de 13 son: $\pm 1, \pm 13$

El polinomio se anula cuando $x = -1$

$$\frac{x^3-5x^2+7x+13}{x+1} = x^2 - 6x + 13$$

Ahora, hay que resolver $x^2-6x+13=0$

$a=1 \qquad b=-6 \qquad c=13$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \qquad x=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(13)}}{2(1)}$$

$$x=\frac{6\pm\sqrt{36-52}}{2} \qquad x=\frac{6\pm\sqrt{-16}}{2}$$

$$x=\frac{6\pm 4i}{2} \qquad x=\cancel{2}\left(\frac{3\pm 2i}{\cancel{2}}\right) \qquad \boxed{x=3\pm 2i}$$

2)

$$\sqrt{a+ib}=u+iv$$

$$u=\sqrt{\tfrac{1}{2}\left(a+\sqrt{a^2+b^2}\right)} \qquad v=\sqrt{\tfrac{1}{2}\left(-a+\sqrt{a^2+b^2}\right)}$$

$$\sqrt{a+ib}=u+iv$$

$$\left(\cancel{\sqrt{a+ib}}\right)^{\cancel{2}}=(u+iv)^2$$

$$a+ib=u^2+2iuv+i^2v^2$$

$$*\quad a+ib=u^2+2iuv-v^2$$

Resolviendo $u^2+2iuv-v^2$

$$\left[\cancel{\sqrt{\tfrac{1}{2}\left(a+\sqrt{a^2+b^2}\right)}}\right]^{\cancel{2}}+2i\sqrt{\tfrac{1}{4}\left[\left(\cancel{\sqrt{a^2+b^2}}\right)^{\cancel{2}}-a^2\right]}-\left[\cancel{\sqrt{\tfrac{1}{2}\left(-a+\sqrt{a^2+b^2}\right)}}\right]^{\cancel{2}}$$

$$\tfrac{1}{2}\left(a+\sqrt{a^2+b^2}\right)+\cancel{2}i\left(\tfrac{1}{\cancel{2}}\right)\sqrt{\cancel{a^2}+b^2-\cancel{a^2}}-\tfrac{1}{2}\left(-a+\sqrt{a^2+b^2}\right)$$

$$\frac{a}{2}+\cancel{\frac{\sqrt{a^2+b^2}}{2}}+i\sqrt{b^2}+\frac{a}{2}-\cancel{\frac{\sqrt{a^2+b^2}}{2}}$$

$$\frac{a}{2}+bi+\frac{a}{2}=a+bi \quad *$$

Luego: $a+bi=a+bi$

(13) a) $(1-4i)(3+11i)-(1+i)^{-1}$

$3+11i-12i+44 - \frac{1}{1+i}$

$47-i-\frac{1}{1+i} = \frac{(1+i)(47-i)-1}{1+i}$

$\frac{47-i+47i+\cancel{1}-\cancel{1}}{1+i} = \frac{47+46i}{1+i}$

$\frac{47+46i}{1+i}\cdot\frac{1-i}{1-i} = \frac{47-47i+46i+46}{(1+i)(1-i)}$

$= \frac{93-i}{1+1}$ $\boxed{=\frac{93-i}{2}}$

b) $(1-i)^3(1+i)$

$(1-i)(1-i)^2(1+i)$

$(1-i)^2(1-i^2)$

$-2i(2)$ $\boxed{=-4i}$

c) $\left[\frac{2i}{1+i}\right]^4$

$\frac{2i}{1+i}\cdot\frac{1-i}{1-i} = \frac{2+2i}{2} = 2\left(\frac{1+i}{2}\right) = 1+i$

$(1+i)^4 = 1+4i+6i^2+4i^3+i^4$

$= 1+\cancel{4i}-6-\cancel{4i}+1$ $\boxed{=-4}$

d) $\frac{i}{1+i}+\frac{1+i}{i}$

$= \frac{i^2+(1+i)^2}{i(1+i)} = \frac{-1+2i}{-1+i}$

$$\frac{1+2i}{-1+i} \cdot \frac{-1-i}{-1-i} = \frac{1+i-2i+2}{1+1} \boxed{= \frac{3-i}{2}}$$

$$\left[1 + i\sqrt{3} + (1-\sqrt{3})i\right]^3$$

$$\left[1 + \cancel{\sqrt{3}i} + i - \cancel{\sqrt{3}i}\right]^3$$

$$(1+i)^3 = 1 + 3i + 3i^2 + i^3$$

$$= 1 + 3i - 3 - i \qquad \boxed{= -2 + 2i}$$

7) $\begin{array}{l} -(1-3i) \\ (2+3i) \end{array} \begin{cases} (2+3i)x - (1+i)y = 3+4i & (1) \\ (1-3i)x - (1-2i)y = -2-6i & (2) \end{cases}$

$$-(1-3i)(2+3i)x + (1+i)(1-3i)y = -(3+4i)(1-3i)$$

$$(1-3i)(2+3i)x - (1-2i)(2+3i)y = (-2-6i)(2+3i)$$

$$// \quad y\left[(1+i)(1-3i) - (1-2i)(2+3i)\right] = -15 + 5i + 14 - 18i$$

$$y\left[4 - 2i - 8 + i\right] = -15 + 5i + 14 - 18i$$

$$y(-4 - 1i) = -1 - 13i$$

$$y(4 + 1i) = 1 + 13i \qquad y = \frac{1+13i}{4+1i}$$

$$y = \frac{1+13i}{4+1i} \cdot \frac{4-1i}{4-1i} = \frac{4 - 1i + 52i - 13i^2}{16+1}$$

$$y = \frac{17+51i}{17} \qquad y = \frac{\cancel{17}(1+3i)}{\cancel{17}} \qquad \boxed{y = 1+3i}$$

Reemplazo en (1) ó (2)

En (1): $(2+3i)x - (1+i)y = 3+4i$

$$(2+3i)x - (1+i)(1+3i) = 3+4i$$

$$(2+3i)x - (-2+4i) = 3+4i$$

(15) $A = \begin{pmatrix} 1+i & 2+i & 4+i \\ 0 & 1 & 2 \\ -i & 0 & i \end{pmatrix}$, hallar A^{-1}

1) $\det |A| = (1+i)\begin{vmatrix} 1 & 2 \\ 0 & i \end{vmatrix} - (2+i)\begin{vmatrix} 0 & 2 \\ -i & i \end{vmatrix} + (4+i)\begin{vmatrix} 0 & 1 \\ -i & 0 \end{vmatrix}$

$\det |A| = (1+i)(i) - (2+i)(2i) + (4+i)(i)$

$\det |A| = i + \cancel{i^2} - \cancel{4i} - 2\cancel{i^2} + \cancel{4i} + \cancel{i^2}$

$\det |A| = i$

2) Menores

$\begin{pmatrix} i & 2i & i \\ -1+2i & -2+5i & -1+2i \\ i & 2+2i & 1+i \end{pmatrix}$

3) Cofactores

$\begin{pmatrix} i & -2i & i \\ 1-2i & -2+5i & 1-2i \\ i & -2-2i & 1+i \end{pmatrix}$

4) Transpuesta

$\begin{pmatrix} i & 1-2i & i \\ -2i & -2+5i & -2-2i \\ i & 1-2i & 1+i \end{pmatrix}$

5) Fórmula

$A^{-1} = \dfrac{1}{\det A}(A^T)$

5) $\dfrac{1}{i}\begin{pmatrix} i & 1-2i & i \\ -2i & -2+5i & -2-2i \\ i & 1-2i & 1+i \end{pmatrix}$

$A^{-1} = \begin{pmatrix} 1 & -2-i & 1 \\ -2 & 5+2i & -2+2i \\ 1 & -2-i & 1-i \end{pmatrix}$

(16) a) $\begin{vmatrix} 1+i & 2+i & 4+i \\ 0 & 1 & 2 \\ -i & 0 & i \end{vmatrix}$

$\det = (1+i)\begin{vmatrix} 1 & 2 \\ 0 & i \end{vmatrix} - (2+i)\begin{vmatrix} 0 & 2 \\ -i & i \end{vmatrix} + (4+i)\begin{vmatrix} 0 & 1 \\ -i & 0 \end{vmatrix}$

$\det = (1+i)(i) - (2+i)(2i) + (4+i)(i)$

$\det = i + \cancel{i^2} - \cancel{4i} - 2\cancel{i^2} + \cancel{4i} + \cancel{i^2}$

$\boxed{\det = i}$

$(2+3i)x = 3+4i-2+4i$

$(2+3i)x = 1+8i$

$$x = \frac{1+8i}{2+3i}$$

$$x = \frac{1+8i}{2+3i} \cdot \frac{2-3i}{2-3i} = \frac{2-3i+16i+24}{4+9}$$

$$x = \frac{26+13i}{13} \qquad \frac{\cancel{13}(2+i)}{\cancel{13}} \qquad \boxed{x = 2+i}$$

b) $\begin{cases} (2+i)x + 2y = 1+7i & (1) \\ (1-i) + yi = 0 & (2) \end{cases}$

De la ecuación (2)

$1 - i + yi = 0$

$yi = -1+i \qquad y = \dfrac{-1+i}{i}$

$$\frac{-1+i}{i} \cdot \frac{i}{i} = \frac{i(-1+i)}{i^2}$$

$$-1(-i+i^2) = i - i^2 = 1+i \qquad \boxed{y = 1+i}$$

Reemplazo en (1)

$(2+i)x + 2y = 1+7i$

$(2+i)x + 2(1+i) = 1+7i$

$(2+i)x + 2+2i = 1+7i$

$(2+i)x = 1+7i-2-2i \qquad x = \dfrac{-1+5i}{2+i}$

$(2+i)x = -1+5i$

$$x = \frac{-1+5i}{2+i} \cdot \frac{2-i}{2-i}$$

$$x = \frac{-2+i+10i+5}{4+1} \qquad \boxed{x = \frac{3+11i}{5}}$$

b) $\begin{vmatrix} 1-i & 0 & i \\ 2-i & 1 & 0 \\ 4-i & 2 & -i \end{vmatrix}$

$$\det = (1-i)\begin{vmatrix} 1 & 0 \\ 2 & -i \end{vmatrix} + i\begin{vmatrix} 2-i & 1 \\ 4-i & 2 \end{vmatrix}$$

$$\det = (1-i)(-i) + i(4-2i-4+i)$$

$$\det = -i + i^2 + i(-i)$$

$$\det = -i + \cancel{i^2} - \cancel{i^2} \qquad \boxed{\det = -i}$$

(17) a) $\dfrac{(3+5i)(2-i)^3}{-1+4i}$

$$\frac{(3+5i)(2-11i)}{-1+4i} = \frac{61-23i}{-1+4i}$$

$$\frac{61-23i}{-1+4i} \cdot \frac{-1-4i}{-1-4i} = \frac{-153-221}{17}$$

$$\cancel{17}\left(\frac{-9-13i}{\cancel{17}}\right) = -9-13i$$

b) $(2+3i)(3-4i)^2$

$(2+3i)(-7-24i)$

$= -14-48i-21i+72 \qquad \boxed{= 58-69i}$

c) $\frac{1}{2}(1+i)(1+i^{-8})$

$$\frac{1}{2}(1+i)\left(1+\frac{1}{i^8}\right) = \frac{1}{2}(1+i)(1+1)$$

$$= \frac{1}{\cancel{2}}(1+i)(\cancel{2}) \qquad \boxed{= 1+i}$$

d) $\dfrac{(1+i)(2+i)(3+i)}{1-i}$

$$\frac{(1+i)^2(2+i)(3+i)}{(1+i)(1-i)} = \frac{2i(2+i)(3+i)}{1+1}$$

$$= \frac{\cancel{2}i(6+2i+3i-1)}{\cancel{2}} = i(5+5i)$$

$$= 5i+5i^2 \qquad \boxed{= -5+5i}$$

e) $(2+5i)^3$

$$= 8+3(2)^2(5i)+3(2)(5i)^2+125i^3$$

$$= 8+60i+150i^2+125i^3$$

$$= 8+60i-150-125i$$

$$\boxed{= -142-65i}$$

18) $$\left(\frac{z_1}{z_1+z_2}\right) + \left(\frac{z_2}{z_1+z_2}\right) = 1$$

$$\frac{z_1+z_2}{z_1+z_2} = 1$$

19) $$\sqrt{z} = \frac{2}{1-i} + 1-4i$$

$$\sqrt{z} = \frac{2+(1-i)(1-4i)}{1-i}$$

$$\sqrt{z} = \frac{2-3-5i}{1-i} \qquad \sqrt{z} = \frac{-1-5i}{1-i}$$

$$\sqrt{z} = \frac{-1-5i}{1-i}\cdot\frac{1+i}{1+i} \qquad \sqrt{z} = \frac{4-6i}{2}$$

$$\sqrt{z} = \frac{\cancel{2}(2-3i)}{\cancel{2}} \qquad \sqrt{z} = 2-3i$$

$$(\sqrt{z})^{\cancel{2}} = (2-3i)^2 \qquad z = 4 - 12i + 9i^2$$

$$z = 4 - 12i - 9 \qquad \boxed{z = -5 - 12i}$$

(20) $(x+iy)(2+3i)$

$$2x + 3ix + 2iy + 3i^2y$$

$$2x + i(3x+2y) - 3y = 0$$

Para que esto sea un número real:

$$3x + 2y = 0$$

$$3x = -2y \qquad \boxed{\frac{x}{y} = -\frac{2}{3}}$$

(21) a) $-\sqrt{3} + i = z$

Hay que hallar: módulo y argumento

módulo $= \sqrt{a^2+b^2}$ $\quad |z|$

argumento $=$ $\quad \Theta = \text{Tan}^{-1}\left(\frac{b}{a}\right)$

siendo $z = a + bi$

a: parte real $\quad$ b: parte imaginaria

$$|z| = \sqrt{(-\sqrt{3})^2 + (1)^2} = \sqrt{3+1} = \sqrt{4} = 2$$

$$\Theta = \text{Tan}^{-1}\left(\frac{1}{-\sqrt{3}}\right) \qquad \Theta = \text{Tan}^{-1}\left(-\frac{\sqrt{3}}{3}\right)$$

Θ está en el II cuadrante ya que: $x(-)$ $y(+)$

$$\Theta = \text{Tan}^{-1}\left(-\frac{\sqrt{3}}{3}\right) \qquad \Theta = \frac{5\pi}{6}$$

Forma Polar: $|z| \text{ Cis } \Theta$ $\quad \boxed{= 2 \text{ Cis } \frac{5\pi}{6}}$

b) $z = 3 - 3i$

$|z| = \sqrt{(3)^2 + (-3)^2}$

$|z| = \sqrt{9+9}$ $\qquad |z| = \sqrt{18}$

$|z| = 3\sqrt{2}$

$\theta = Tan^{-1}\left(\frac{-3}{3}\right)$ $\qquad \theta = Tan^{-1}(-1)$ $\qquad x = +$, $y = -$

$\theta = 7\frac{\pi}{4}$

$\boxed{3\sqrt{2}\ Cis\ \frac{7\pi}{4}}$

c) $z = 1 + \sqrt{3}\, i$

$|z| = \sqrt{(1)^2 + (\cancel{\sqrt{3}})^{\cancel{2}}}$

$|z| = \sqrt{1+3}$

$|z| = \sqrt{4}$

$|z| = 2$

$\theta = Tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$ $\qquad \theta = Tan^{-1}(\sqrt{3})$

$\theta = \frac{\pi}{3}$

$\boxed{= 2\ Cis\ \frac{\pi}{3}}$

θ cae en el I cuadrante ya que:
$x\,(+)$, $y\,(+)$

(22) $$\frac{1+\cos\alpha+i\,\text{Sen}\,\beta}{1+\cos\beta+i\,\text{Sen}\,\beta}$$

$$\frac{1+\cos\alpha+i\,\text{Sen}\,\beta}{1+\cos\beta+i\,\text{Sen}\,\beta}\cdot\frac{1+\cos\beta-i\,\text{Sen}\,\beta}{1+\cos\beta-i\,\text{Sen}\,\beta}$$

Resolviendo y reduciendo

$$\frac{1+\cos\beta+\cos\alpha(1+\cos\beta)+\text{Sen}^2\beta+i\,\text{Sen}\,\beta(\cos\beta-\cos\alpha)}{2(1+\cos\beta)}$$

Parte real:

$$\frac{1+\cos\beta+\cos\alpha+\cos\alpha\cos\beta+\text{Sen}^2\beta}{2(1+\cos\beta)}$$

Parte imaginaria:

$$\frac{i\,\text{Sen}\,\beta(\cos\beta-\cos\alpha)}{2(1+\cos\beta)}$$

(23) $$\left[\frac{1+\text{Sen}\,\alpha+i\cos\alpha}{1+\text{Sen}\,\alpha-i\cos\alpha}\right]^n$$

$$\frac{(1+\text{Sen}\,\alpha+i\cos\alpha)(1+\text{Sen}\,\alpha+i\cos\alpha)}{(1+\text{Sen}\,\alpha-i\cos\alpha)(1+\text{Sen}\,\alpha+i\cos\alpha)}$$

$$=\frac{1+\text{Sen}^2\alpha-\cos^2\alpha+2\,\text{Sen}\,\alpha+2i\cos\alpha(1+\text{Sen}\,\alpha)}{2(1+\text{Sen}\,\alpha)}$$

$$=\frac{2\,\text{Sen}\,\alpha(\text{Sen}\,\alpha+1)+2i\cos\alpha(\text{Sen}\,\alpha+1)}{2(1+\text{Sen}\,\alpha)}$$

$$=\frac{\cancel{2(\text{Sen}\,\alpha+1)}(\text{Sen}\,\alpha+i\cos\alpha)}{\cancel{2(\text{Sen}\,\alpha+1)}}$$

$$=(\text{Sen}\,\alpha+i\cos\alpha)^n$$ *

$$\cos\left[n\left(\frac{\pi}{2}-\alpha\right)\right] + i\,\mathrm{Sen}\left[n\left(\frac{\pi}{2}-\alpha\right)\right]$$

$$\left[\cos\left(\frac{\pi}{2}-\alpha\right) + i\,\mathrm{Sen}\left(\frac{\pi}{2}-\alpha\right)\right]^n$$

$$\left[\left(\cos\frac{\pi}{2}\cancel{\cos}^{0}\alpha + \mathrm{Sen}\frac{\pi}{2}^{1}\mathrm{Sen}\,\alpha\right) + i\left(\mathrm{Sen}\frac{\pi}{2}^{1}\cos\alpha - \mathrm{Sen}\,\alpha\,\cancel{\cos\frac{\pi}{2}}^{0}\right)\right]^n$$

$$\left(\mathrm{Sen}\,\alpha + i\cos\alpha\right)^n \quad * \qquad \text{L.Q.Q.D.}$$

4) a) $i^{25} = i$

$i^{25} = i^{21} \cdot i^{4} = i^{13} = i^{9} = i^{5} = i$ (V)

b) $|z_1 + z_2| \leq |z_1| + |z_2|$

Es propiedad de los números complejos (V)

c) $(x, y)(0, 1) = (x, y)$

$(x, y)(0, 1) = (0, y)$ (F)

d) $z \cdot z = z^2$

Es propiedad de los números complejos (V)

(25) $\left(\frac{1+\sqrt{3}i}{-1+\sqrt{3}i}\right)^{10}$

$$\frac{1+\sqrt{3}i}{-1+\sqrt{3}i}\cdot\frac{-1-\sqrt{3}i}{-1-\sqrt{3}i}=\frac{-1-\sqrt{3}i-\sqrt{3}i-3i^2}{1-3i^2}$$

$$=\frac{2-2\sqrt{3}i}{1+3}=\frac{\cancel{2}(1-\sqrt{3}i)}{\cancel{4}_2}=\frac{1-\sqrt{3}i}{2}$$

$$z=\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^{10}$$

$$|z|=\sqrt{\left(\frac{1}{2}\right)^2+\left(-\frac{\sqrt{3}}{2}\right)^2}\qquad |z|=\sqrt{\frac{1}{4}+\frac{3}{4}}$$

$$|z|=\sqrt{1}\qquad |z|=1 \;*$$

$$\theta=\tan^{-1}\left(\frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right)\qquad \theta=\tan^{-1}(-\sqrt{3})$$

θ está en el IV cuadrante ya que: $x(+)$, $y(-)$

$$\theta=\frac{5\pi}{3}\;**$$

$$z^N=|z|^n e^{i\theta n}$$

$$=(1)^{10}\left(\cos\frac{5\pi}{3}+i\,\text{Sen}\,\frac{5\pi}{3}\right)^{10}$$

$$=1(\cos 3000^\circ+i\,\text{Sen}\,3000^\circ)$$

$$=\cos 120^\circ+i\,\text{Sen}\,120^\circ$$

$$\boxed{=-\frac{1}{2}+\frac{\sqrt{3}}{2}i}\quad (a)$$

(26) $\left(z+z^{-1}\right)^5$

$= \left(z+\frac{1}{z}\right)^5 \qquad = \left(\frac{z^2+1}{z}\right)^5$

$= \frac{z^{10}+5z^8+10z^6+10z^4+5z^2+1}{z^5}$ *

$\cos 5\theta \quad \cos(3\theta+2\theta)$

$= \cos 3\theta \cos 2\theta - \operatorname{Sen} 3\theta \operatorname{Sen} 2\theta$

$= (4\cos^3\theta - 3\cos\theta)(2\cos^2\theta - 1) - 2\operatorname{Sen}\theta\cos\theta(3\operatorname{Sen}\theta - 4\operatorname{Se}$

$= 8\cos^5\theta - 4\cos^3\theta - 6\cos^3\theta + 3\cos\theta - 2\operatorname{Sen}^2\theta\cos\theta(3-4\operatorname{Sen}^2\theta$

$= 8\cos^5\theta - 10\cos^3\theta + 3\cos\theta - 2\cos\theta(1-\cos^2\theta)(3-4+4\cos$

$= 8\cos^5\theta - 10\cos^3\theta + 3\cos\theta - 2\cos\theta(1-\cos^2\theta)(4\cos^2\theta$

$= 8\cos^5\theta - 10\cos^3\theta + 3\cos\theta - 2\cos\theta(5\cos^2\theta - 4\cos^4\theta -$

$= 8\cos^5\theta - 10\cos^3\theta + 3\cos\theta - 10\cos^3\theta + 8\cos^5\theta + 2\cos\theta$

$= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$

* $\cos^3\theta = (3\cos\theta + \cos 3\theta)/4$

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$$

Reemplazando *

$$\cos 5\theta = 16\cos^5\theta - 20\left(\frac{3\cos\theta + \cos 3\theta}{4}\right) + 5\cos\theta$$

$$\cos 5\theta = 16\cos^5\theta - 5(3\cos\theta + \cos 3\theta) + 5\cos\theta$$

$$\cos 5\theta = 16\cos^5\theta - 15\cos\theta - 5\cos 3\theta + 5\cos\theta$$

$$\cos 5\theta = 16\cos^5\theta - 5\cos 3\theta - 10\cos\theta$$

$$\cos 5\theta + 5\cos 3\theta + 10\cos\theta = 16\cos^5\theta$$

$$\boxed{\cos^5\theta = \frac{\cos 5\theta + 5\cos 3\theta + 10\cos\theta}{16}}$$

Lo que se quería demostrar.

$n = 1 \qquad m = 5 \qquad \lambda = 10$

27. a) $\left[\sqrt{2}\left(\cos\frac{\pi}{4} + i\,\mathrm{Sen}\,\frac{\pi}{4}\right)\right]^4$

$$= (\sqrt{2})^4\left[\cos\cancel{4}\left(\frac{\pi}{\cancel{4}}\right) + i\,\mathrm{Sen}\,\cancel{4}\left(\frac{\pi}{\cancel{4}}\right)\right]$$

$$= (2)^2(\cos\pi + i\,\mathrm{Sen}\,\pi)$$

$$= 4\,\mathrm{Cis}\,\pi$$

b) $\left[3\left(\cos\frac{\pi}{5} + i\,\mathrm{Sen}\,\frac{\pi}{5}\right)\right]^{-3}$

$$= (3)^{-3}\left[\cos(-3)\frac{\pi}{5} + i\,\mathrm{Sen}\,(-3)\frac{\pi}{5}\right]$$

$$= \frac{1}{27}\left[\cos\left(-\frac{3\pi}{5}\right) + i\,\mathrm{Sen}\left(-\frac{3\pi}{5}\right)\right]$$

$$= \frac{1}{27}\,\mathrm{Cis}\left(-\frac{3\pi}{5}\right)$$

(28) $z = 2 + 2i$

$zi = i(2 + 2i)$

$zi = 2i + 2i^2$

$zi = -2 + 2i$

Cambia el signo de la parte real

(29) $z - \frac{1}{z} = \frac{z^2 - 1}{z}$

En el caso de que z sea un número complejo, haciendo operaciones al final seguirá existiendo parte imaginaria.
Por lo tanto, z debe ser real $\quad z \neq 0$

(30) i) $\left[1 + \sqrt{3}i + (1 - \sqrt{3})i\right]^3$

$= \left[1 + \cancel{\sqrt{3}i} + i - \cancel{\sqrt{3}i}\right]^3$

$= (1 + i)^3$

$= 1 + 3i + 3i^2 + i^3$

$= 1 + 3i - 3 - i$ $\qquad \boxed{= -2 + 2i}$

ii) $\left(\frac{1+i}{1-i}\right)^2 - \left(\frac{1-i}{1+i}\right)^3$

$\frac{1+i}{1-i} \cdot \frac{1+i}{1+i} = \frac{2i}{1+1} = \frac{\cancel{2}i}{\cancel{2}} = i$

$\frac{1-i}{1+i} \cdot \frac{1-i}{1-i} = \frac{-2i}{1+1} = -\frac{\cancel{2}i}{\cancel{2}} = -i$

$= (i)^2 - (-i)^3$

$= i^2 - (i) \qquad = -1 - (+i)$

$\boxed{= -1 - i}$

(31) a) $e^{-i\pi}$

$\cos \pi + i \operatorname{Sen} \pi = -1$

Parte real $= -1$ Módulo $= 1$

Parte imaginaria $= 0$

b) $e^{1 - i\frac{\pi}{6}}$

$= e \cdot e^{-i\frac{\pi}{6}}$

$= e \left[\cos \frac{11\pi}{6} + i \operatorname{Sen} \frac{11\pi}{6} \right]$

$= e \left(\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right) = \frac{\sqrt{3}e}{2} - \frac{e}{2} i$

Módulo $= e$ Parte real $= \frac{\sqrt{3}e}{2}$

Parte imaginaria $= -\frac{e}{2}$

c) e^{1+i}

$= e \cdot e^{i}$

$= e \left[\cos 1 + i \operatorname{Sen} 1 \right]$

Módulo $= e$ Parte real $= e \cos 1$

Parte imaginaria $= e \operatorname{Sen} 1$

(32) $z_1 = 7 - i$ $z_2 = 3 + i$

$\frac{z_1}{z_2} = \frac{7-i}{3+i}$ $\frac{7-i}{3+i} \cdot \frac{3-i}{3-i} = \frac{21 - 7i - 3i + i^2}{3^2 + 1^2}$

$= \frac{21 - 10i - 1}{10} = \frac{20 - 10i}{10} = \frac{\cancel{10}(2-i)}{\cancel{10}} = 2 - i$

$e^{2-i} = e^2 \cdot e^{-i}$ Módulo $= e^2$

(33) $z = \dfrac{1}{1+i} + \dfrac{1}{(1+i)^2} + \dfrac{1}{(1+i)^3} + \dfrac{1}{(1+i)^4} + \dots$

$A = \dfrac{1}{1+i}$ $\qquad R = \dfrac{1}{(1+i)^2} \cdot \dfrac{\cancel{1+i}}{1} = \dfrac{1}{1+i}$

$S = \dfrac{A}{1-R}$ $\qquad S = \dfrac{\frac{1}{1+i}}{1 - \frac{1}{1+i}} = \dfrac{\frac{1}{1+i}}{\frac{\cancel{1}+i-\cancel{1}}{1+i}}$

$S = \dfrac{1}{\cancel{1+i}} \cdot \dfrac{\cancel{1+i}}{i}$ $\qquad S = \dfrac{1}{i}$ $\qquad S = i^{-1}$

$z^2 = (i^{-1})^2$ $\qquad z^2 = (i^2)^{-1}$ $\qquad \boxed{z^2 = -1}$

(34) a) $e^{i\pi}$

$= \cos \pi + i \cancel{\sin \pi}^{\,0}$ $\qquad \boxed{= -1}$

b) $-2e^{-i\pi}$

$= -2[\cos \pi + i \operatorname{sen} \pi]$

$= -2(-1)$ $\qquad \boxed{= 2}$

c) $i + e^{i\frac{3\pi}{2}}$

$i + \left(\cancel{\cos\frac{3\pi}{2}}^{0} + i \operatorname{Sen}\frac{3\pi}{2}\right)$

$= i + i(-1)$ $\quad i - i \quad \boxed{= 0}$

d) $e^{i\frac{\pi}{4}} - e^{i\frac{7\pi}{8}}$

$\left(\cos\frac{\pi}{4} + i \operatorname{Sen}\frac{\pi}{4}\right) - \left(\cos\frac{7\pi}{8} + i \operatorname{Sen}\frac{7\pi}{8}\right)$

$\cancel{\frac{\sqrt{2}}{2}} + \frac{\sqrt{2}}{2} i - \cancel{\frac{\sqrt{2}}{2}} - \left(-\frac{\sqrt{2}}{2} i\right)$

$\frac{\sqrt{2}}{2} i + \frac{\sqrt{2}}{2} i \quad = \cancel{2}\left(\frac{\sqrt{2}}{\cancel{2}}\right) i \quad \boxed{= \sqrt{2}\, i}$

e) $e^{i\frac{\pi}{3}} + e^{i\frac{7\pi}{6}}$

$\left(\cos\frac{\pi}{3} + i \operatorname{Sen}\frac{\pi}{3}\right) + \left(\cos\frac{7\pi}{6} + i \operatorname{Sen}\frac{7\pi}{6}\right)$

$\frac{1}{2} + \frac{\sqrt{3}}{2} i + \left(-\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right)$

$\frac{1}{2} + \frac{\sqrt{3}}{2} i - \frac{\sqrt{3}}{2} - \frac{1}{2} i$

$\boxed{= \frac{1}{2}\left(1 - \sqrt{3}\right) + i\,\frac{1}{2}\left(\sqrt{3} - 1\right)}$

35 a) $2e^{\frac{\pi}{8}}$

$= 2\left(\cos\frac{\pi}{8} + i \operatorname{Sen}\frac{\pi}{8}\right) \quad \boxed{= 2\operatorname{Cis}\frac{\pi}{8}}$

b) e^{2+i}

$= e^2 \cdot e^i = e^2\left(\cos 1 + i \operatorname{Sen} 1\right) \quad \boxed{= e^2 \operatorname{Cis} 1}$

c) $2e^{1+\frac{\pi}{4}i}$

$= 2e \cdot e^{i\frac{\pi}{4}} = 2e\left(\cos\frac{\pi}{4} + i \operatorname{Sen}\frac{\pi}{4}\right) \quad \boxed{= 2e \operatorname{Cis}\frac{\pi}{4}}$

d) 2^{3i}

$= 8^{i}$ $= e^{i \ln 8}$ $\boxed{= \text{Cis}(\ln 8)}$

e) $3^{-3/2}$

$= \left(\frac{1}{3}\right)^{3/2}$ $= \sqrt{\frac{1}{3}}$ $\boxed{= \frac{\sqrt{3}}{3}}$

f) 5^{1+i}

$= 5 \cdot 5^{i}$ $= 5 \cdot e^{i \ln 5}$ $\boxed{= 5(\text{Cis}(\ln 5))}$

g) 10^{1-i}

$= 10 \cdot 10^{-i}$ $= \frac{10}{10^{i}}$

$= \frac{10}{e^{i \ln 10}}$ $\boxed{= 10/[\text{Cos}(\ln 10) - i\,\text{Sen}(\ln 10)]}$

16) h) $(\text{Cos}\,30^\circ + i\,\text{Sen}\,30^\circ)^2 = \text{Cos}\,\frac{\pi}{3} + i\,\text{Sen}\,\frac{\pi}{3}$

$(\text{Cos}\,30^\circ + i\,\text{Sen}\,30^\circ)^2 = \text{Cos}\,2(30^\circ) + i\,\text{Sen}\,2(30^\circ)$

$= \text{Cos}\,60^\circ + i\,\text{Sen}\,60^\circ$

$\text{Cos}\,60^\circ + i\,\text{Sen}\,60^\circ = \text{Cos}\,\frac{\pi}{3} + i\,\text{Sen}\,\frac{\pi}{3}$

h) i) $\left(\text{Cos}\,\frac{\pi}{6} + i\,\text{Sen}\,\frac{\pi}{6}\right)^3 = i$

$= \text{Cos}\,3\left(\frac{\pi}{6}\right) + i\,\text{Sen}\,3\left(\frac{\pi}{6}\right)$

$= \text{Cos}\,\frac{\pi}{2} + i\,\text{Sen}\,\frac{\pi}{2}$

$= 0 + i(1) = i$ $\qquad i = i$

(37) i) $2 \text{ Cis } \frac{\pi}{4}$

$= \left[2\left(\cos\frac{\pi}{4} + i\,\text{Sen}\,\frac{\pi}{4}\right)\right]^4$

$= 2^4\left(\cos \cancel{4}\left(\frac{\pi}{\cancel{4}}\right) + i\,\text{Sen}\,\cancel{4}\left(\frac{\pi}{\cancel{4}}\right)\right)$

$= 16\left(\cos\pi + i\,\text{Sen}\,\pi\right)$

$= 16(-1 + 0i)$

$= 16(-1)$ $\boxed{= -16}$

ii) $\sqrt{3} \text{ Cis } 225°$

$= \left[\sqrt{3}\left(\cos 225° + i\,\text{Sen}\,225°\right)\right]^4$

$= (\sqrt{3})^4\left(\cos 900° + i\,\text{Sen}\,900°\right)$

$= \sqrt{81}\left(\cos 180° + i\,\text{Sen}\,180°\right)$

$= 9(-1 + 0i)$ $\boxed{= -9}$

$= 9(-1)$

iii) $\frac{1}{2} e^{i\alpha}$

$= \left[\frac{1}{2}\left(\cos\alpha + i\,\text{Sen}\,\alpha\right)\right]^4$

$= \left(\frac{1}{2}\right)^4\left(\cos 4\alpha + i\,\text{Sen}\,4\alpha\right)$

$\boxed{= \frac{1}{16}\left(\cos 4\alpha + i\,\text{Sen}\,4\alpha\right)}$

(38) i) $27 \text{ Cis } \frac{3\pi}{2}$

$= \left[27\left(\cos\frac{3\pi}{2} + i\,\text{Sen}\,\frac{3\pi}{2}\right)\right]^{1/3}$

$= (27)^{1/3}\left[\cos \frac{1}{\cancel{3}}\left(\frac{\cancel{3}\pi}{2}\right) + i\,\text{Sen}\,\frac{1}{\cancel{3}}\left(\frac{\cancel{3}\pi}{2}\right)\right]$

$= 3(0 + i)$ $\boxed{= 3i}$

$|z| = 27 \qquad \Theta = \frac{3\pi}{2} \qquad \frac{2\pi}{3} = 120^\circ$

$** \quad z^{1/3} = (27)^{1/3} e^{i\theta/3}$

$= 3\left(\cos\frac{7\pi}{6} + i\,\mathrm{Sen}\,\frac{7\pi}{6}\right)$

$= 3\left(-\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right)$

$$\boxed{z_2 = -\frac{3\sqrt{3}}{2} - \frac{3}{2}i}$$

$** \quad z^{1/3} = 3\left(\cos\frac{11\pi}{6} + i\,\mathrm{Sen}\,\frac{11\pi}{6}\right)$

$= 3\left(\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right)$

$$\boxed{z_3 = \frac{3\sqrt{3}}{2} - \frac{3}{2}i}$$

ii) $z = -1 + 0i$

$* \; |z| = 1 \qquad \Theta = \mathrm{Tan}^{-1}\left(\frac{0}{-1}\right) \qquad \Theta = \pi \; *$

$\frac{2\pi}{4} = \frac{\pi}{2} \; *$

$** \quad z^{1/4} = (1)^{1/4} e^{i\theta/4}$

$= \cos\frac{\pi}{4} + i\,\mathrm{Sen}\,\frac{\pi}{4}$

$$\boxed{z_1 = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i}$$

$** \quad z^{1/4} = \cos\frac{3\pi}{4} + i\,\mathrm{Sen}\,\frac{3\pi}{4}$

$$\boxed{z_2 = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i}$$

$** \quad z^{1/4} = \cos\frac{5\pi}{4} + i\,\mathrm{Sen}\,\frac{5\pi}{4}$

$$\boxed{z_3 = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i}$$

$$** \; z^{1/4} = \cos\frac{7\pi}{4} + i\,\mathrm{Sen}\,\frac{7\pi}{4}$$

$$\boxed{z_4 = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i}$$

iii) $z = i$

$|z| = 1 \qquad \Theta = \mathrm{Tan}^{-1}\left(\frac{1}{0}\right) \qquad \Theta = \frac{\pi}{2}$

$z^{1/N} = |z|^{1/N} e^{i(\Theta + 2\pi K)/N} \qquad K = 0$

$= 1\left(\cos\frac{\pi}{8} + i\,\mathrm{Sen}\,\frac{\pi}{8}\right)$

$$\boxed{z_1 = \cos\frac{\pi}{8} + i\,\mathrm{Sen}\,\frac{\pi}{8}}$$

$$\frac{2\pi}{N} = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$\boxed{z_2 = \cos\frac{5\pi}{8} + i\,\mathrm{Sen}\,\frac{5\pi}{8}}$$

$$\boxed{z_3 = \cos\frac{9\pi}{8} + i\,\mathrm{Sen}\,\frac{9\pi}{8}}$$

$$\boxed{z_4 = \cos\frac{13\pi}{8} + i\,\mathrm{Sen}\,\frac{13\pi}{8}}$$

iv) $z = 64\cos\pi$

$z = 64(-1) \qquad z = -64$

$|z| = 64 \qquad \Theta = \mathrm{Tan}^{-1}\left(\frac{0}{-64}\right) \qquad \Theta = \pi$

$** \; z^{1/3} = (64)^{1/3} e^{i\Theta/3}$

$= 4\left(\cos\frac{\pi}{3} + i\,\mathrm{Sen}\,\frac{\pi}{3}\right)$

$= 4\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \qquad \boxed{z_1 = 2 + 2\sqrt{3}i}$

$\frac{2\pi}{1} = \frac{2\pi}{3}$

$*\ z^{1/3} = 4(\operatorname{Cos} \pi + i \operatorname{Sen} \pi)$

$= 4(-1 + 0i)$

$\boxed{z_2 = -4}$

$**\ z^{1/3} = 4\left(\operatorname{Cos} \frac{5\pi}{3} + i \operatorname{Sen} \frac{5\pi}{3}\right)$

$= 4\left(\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right)$

$\boxed{z_3 = 2 - 2\sqrt{3}\, i}$

i) $z = 27 \operatorname{cis} \frac{3\pi}{2}$

$z = 27\left(\operatorname{Cos} \frac{3\pi}{2} + i \operatorname{Sen} \frac{3\pi}{2}\right)$

$z = 27(0 - i)$

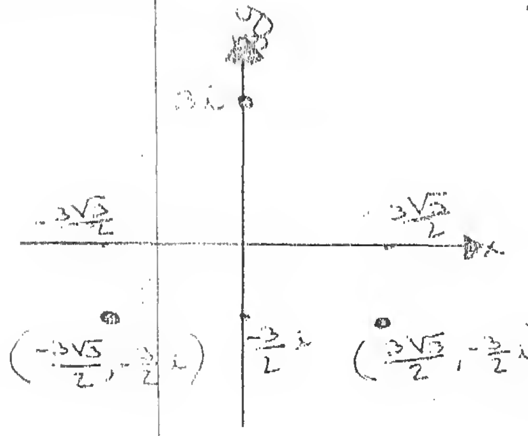
$z = -27i$

$|z| = 27 \qquad \theta = \operatorname{Tan}^{-1}\left(-\frac{27}{0}\right) \qquad \theta = \frac{3\pi}{2}$

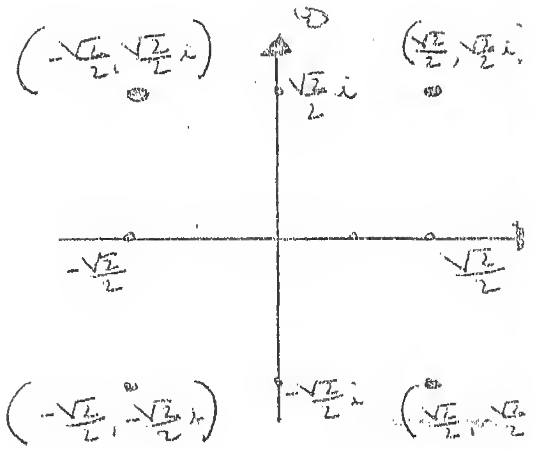
Las soluciones de i) están correctas.

Gráficos:

i)

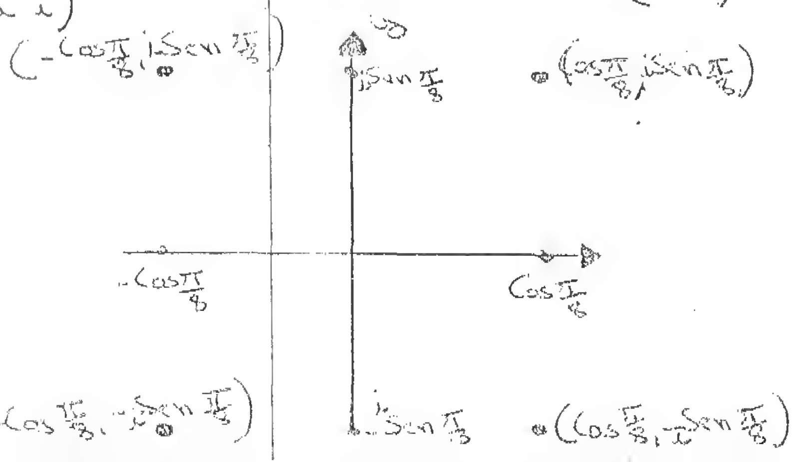


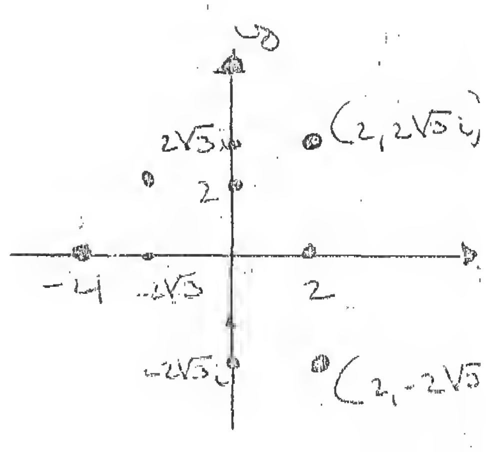
ii)



(iv)

iii)





(39) Como uno de los vértices del hexágono regular es (0, 2i), entonces:

$$(2i)^6 = 2^6 i^6 = 64(-1) = -64$$

Por lo tanto $z = -64 + 0i$

Hay que hallar $z^{1/6} = (-64 + 0i)^{1/6}$

$|z| = \sqrt{(-64)^2 + (0)^2}$ $\quad |z| = \sqrt{4096}$ $\quad * |z| = 64$

$\theta = \tan^{-1}\left(\frac{0}{-64}\right)$ $\quad ** \theta = \pi$ $\quad$ ya que $x = -$ $\quad y = 0$

Módulo $= 64$ $\quad$ Argumento $= \pi$

$$z^{1/n} = |z|^{1/n} e^{i(\theta + 2\pi k)/n} \qquad k = 0, 1, 2, 3, 4,$$

$$* \; z^{1/6} = (64)^{1/6} e^{i\theta/n} \qquad \text{con } k = 0$$

$$= 2\left(\cos\frac{\pi}{6} + i \operatorname{Sen}\frac{\pi}{6}\right)$$

$$= 2\left(\frac{\sqrt{3}}{2} + i\left(\frac{1}{2}\right)\right)$$

$$= 2\left(\frac{1}{2}\right)(\sqrt{3} + i) \qquad \boxed{z_1 = \sqrt{3} + i}$$

Luego, utilizo la fórmula $\frac{2\pi}{N}$ para sumarlo a $\frac{\pi}{6}$ y seguir hallando raíces

$$\frac{2\pi}{N} = \frac{2\pi}{6} = \frac{\pi}{3}$$

$$* \; z^{1/6} = 2\left(\cos\frac{\pi}{2} + i \operatorname{Sen}\frac{\pi}{2}\right)$$

$$= 2(0 + i) \qquad \boxed{z_2 = 2i}$$

$$= 2$$

$$* \; z^{1/6} = 2\left(\cos\frac{5\pi}{6} + i \operatorname{Sen}\frac{5\pi}{6}\right)$$

$$= 2\left(-\frac{\sqrt{3}}{2} + i\left(\frac{1}{2}\right)\right)$$

$$= 2\left(\frac{1}{2}\right)(-\sqrt{3} + i) \qquad \boxed{z_3 = -\sqrt{3} + i}$$

$$= \sqrt{3} \ldots$$

$z^{1/6} = 2\left(\cos\frac{7\pi}{6} + i\,\text{Sen}\,\frac{7\pi}{6}\right)$

$= 2\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}(i)\right)$

$= \cancel{2}\left(\frac{1}{\cancel{2}}\right)(-\sqrt{3} - i)$

$= -\sqrt{3} - i$ $\boxed{z_4 = -\sqrt{3} - i}$

$z^{1/6} = 2\left(\cos\frac{3\pi}{2} + i\,\text{Sen}\,\frac{3\pi}{2}\right)$

$= 2(0 - i)$

$= -2i$ $\boxed{z_5 = -2i}$

$z^{1/6} = 2\left(\cos\frac{11\pi}{6} + i\,\text{Sen}\,\frac{11\pi}{6}\right)$

$= 2\left(\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right)$

$= \cancel{2}\left(\frac{1}{\cancel{2}}\right)(\sqrt{3} - i)$

$\sqrt{3} - i$ $\boxed{z_6 = \sqrt{3} - i}$

*En la práctica, bastaba hallar z_1, z_2 y z_3, para después utilizando sus conjugadas hallar z_4, z_5 y z_6 *.

40 $z = \sqrt[4]{8 + 8\sqrt{3}\,i}$

$z = (8 + 8\sqrt{3}\,i)^{1/4}$

$|z| = \sqrt{(8)^2 + (8\sqrt{3})^2}$ $|z| = \sqrt{64 + 192}$

$|z| = \sqrt{256}$ $|z| = 16$

$\theta = \text{Tan}^{-1}\left(\frac{8\sqrt{3}}{8}\right)$ $\theta = \text{Tan}^{-1}(\sqrt{3})$

$\theta = \frac{\pi}{3}$

Módulo = 16 Argumento = $\frac{\pi}{3}$

$$z^{1/N} = |z|^{1/N}\, e^{i(\theta + 2\pi K)/N} \qquad * \frac{2\pi}{4} = \frac{\pi}{2}$$

$$z^{1/4} = (16)^{1/4}\, e^{i\theta/N} \qquad \text{con } K = 0$$

$$= 2\left(\cos\frac{\pi}{12} + i\,\mathrm{Sen}\,\frac{\pi}{12}\right)$$

$$= 2\left[\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) + i\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)\right]$$

$$\boxed{z_1 = \left(\frac{\sqrt{6}+\sqrt{2}}{2}\right) + \left(\frac{\sqrt{6}-\sqrt{2}}{2}\right) i}$$

$$z^{1/4} = (16)^{1/4}\left(\cos\frac{7\pi}{12} + i\,\mathrm{Sen}\,\frac{7\pi}{12}\right)$$

$$= 2\left[\left(\frac{-\sqrt{6}-\sqrt{2}}{4}\right) + i\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)\right]$$

$$\boxed{z_2 = \left(\frac{-\sqrt{6}-\sqrt{2}}{2}\right) + \left(\frac{\sqrt{6}-\sqrt{2}}{2}\right) i}$$

$$z^{1/4} = 2\left(\cos\frac{13\pi}{12} + i\,\mathrm{Sen}\,\frac{13\pi}{12}\right)$$

$$= 2\left[\left(\frac{-\sqrt{6}-\sqrt{2}}{4}\right) + i\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)\right]$$

$$\boxed{z_3 = \left(\frac{-\sqrt{6}-\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}-\sqrt{6}}{2}\right) i}$$

$$z^{1/4} = 2\left(\cos\frac{19\pi}{12} + i\,\mathrm{Sen}\,\frac{19\pi}{12}\right)$$

$$= 2\left[\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) + i\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)\right]$$

$$\boxed{z_4 = \left(\frac{\sqrt{6}+\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}-\sqrt{6}}{2}\right) i}$$

41 $z = a + bi$

Se debe cumplir: $z = (\bar{z})^2$

$= (a - bi)^2 \qquad = a^2 - 2abi + b^2 i^2$

$$\boxed{= (a^2 - b^2) - 2abi}$$

a) $x^2 - 6x + 13 = 0$

$a = 1 \qquad b = -6 \qquad c = 13$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(13)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 - 52}}{2} \qquad x = \frac{6 \pm \sqrt{-16}}{2}$$

$$x = \frac{6 \pm 4i}{2} \qquad x = \frac{\cancel{2}(3 \pm 2i)}{\cancel{2}}$$

$\boxed{x_1 = 3 + 2i} \qquad \boxed{x_2 = 3 - 2i}$

b) $2x^2 + 5x + 6$

$a = 2 \qquad b = 5 \qquad c = 6$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(2)(6)}}{2(2)}$$

$$x = \frac{-5 \pm \sqrt{25 - 48}}{4} \qquad \boxed{x = \frac{-5 \pm \sqrt{23}\, i}{4}}$$

c) $x^2 - 2(1+i)x + (2i - 1) = 0$

$a = 1 \qquad b = -2(1+i) \qquad c = 2i - 1$

$$x = \frac{-[-2(1+i)] \pm \sqrt{[-2(1+i)]^2 - 4(1)(2i-1)}}{2(1)}$$

$$x = \frac{2 + 2i \pm \sqrt{\cancel{8i} - \cancel{8i} + 4}}{2}$$

$$x = \frac{2 + 2i \pm \sqrt{4}}{2} \qquad x = \frac{2 + 2i \pm 2}{2}$$

$$x = \frac{\cancel{2}(1 + i \pm 1)}{\cancel{2}} \qquad x = 1 + i \pm 1$$

$$x_1 = 1 + i + 1 \qquad x_2 = \cancel{1} + i - \cancel{1}$$

$$\boxed{x_1 = 2 + i} \qquad \boxed{x_2 = i}$$

d) $x^3 - 2x + 4 = 0$

$$(x+2)(x^2 - 2x + 2) = 0$$

$$x + 2 = 0$$

$$\boxed{x_1 = -2}$$

$$x^2 - 2x + 2 = 0$$

$a = 1 \quad b = -2 \quad c = 2$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4-8}}{2} \qquad x = \frac{2 \pm \sqrt{-4}}{2}$$

$$x = \frac{2 \pm 2i}{2} \qquad x = \frac{\cancel{2}(1 \pm i)}{\cancel{2}}$$

$$\boxed{x_1 = 1 + i} \qquad \boxed{x_2 = 1 - i}$$

e) $x^2 - 3x + (3-i) = 0$

$a = 1 \quad b = -3 \quad c = 3 - i$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(3-i)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{4 - 32 + 4i}}{2}$$

$$\boxed{x = \frac{3 \pm \sqrt{4i - 13}}{2}}$$

F) $x^3 + 3x^2 - 3x - 14 = 0$

$(x-2)(x^2 + 5x + 7) = 0$

$x - 2 = 0 \qquad \boxed{x_1 = 2}$

$x^2 + 5x + 7 = 0$

$a = 1 \qquad b = 5 \qquad c = 7$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(7)}}{2(1)}$$

$$x = \frac{-5 \pm \sqrt{25 - 28}}{2} \qquad \boxed{x = \frac{-5 \pm \sqrt{3}\, i}{2}}$$

3) $z = 2 + 3i \qquad \bar{z} = 2 - 3i$

z y $\bar{z}$ son conjugados

$z^3 = (2 + 3i)^3 \qquad z^3 = 8 + 36i + 54i^2 + 27i^3$

$z^3 = -46 + 9i$

$(\bar{z})^3 = (2 - 3i)^3 \qquad (\bar{z})^3 = 8 - 36i + 54i^2 - 27i^3$

$(\bar{z})^3 = -46 - 9i$

Luego z^3 y $(\bar{z})^3$ también son conjugados

44) $\log_{\frac{1}{2}} |z| + \log_{\frac{1}{2}} (|z| + 1) < \log_{\frac{1}{2}} (2|z| + 5)$

$\log_2 |z| + \log_2 (|z| + 1) > \log_2 (2|z| + 5)$

$\log_2 [(|z|)(|z| + 1)] > \log_2 (2|z| + 5)$

$$|z|(|z|+1) > 2|z|+5$$

$$|z|^2+|z|-2|z|-5>0$$

$$|z|^2-|z|-5=0$$

$$a=1 \quad b=-1 \quad c=-5$$

$$|z| = \frac{-(-1) \pm \sqrt{(-1)^2-4(1)(-5)}}{2(1)}$$

$$|z| = \frac{1 \pm \sqrt{1+20}}{2} \qquad |z| = \frac{1 \pm \sqrt{21}}{2}$$

$|z| = \frac{1+\sqrt{21}}{2}$ Descarto $\frac{1-\sqrt{21}}{2}$ ya que esto es un valor negativo y $|z|$ siempre es +.

Por lo tanto: $\boxed{|z| > \frac{1+\sqrt{21}}{2}}$

(45) $-2 < 2\log_{\frac{1}{3}}|x+1-i\sqrt{5}| < -\log_3 2$

$$2\log_{\frac{1}{3}}|x+1-\sqrt{5}i| < \log_{\frac{1}{3}}2 - \log_{\frac{1}{3}}\left(\tfrac{1}{3}\right)$$

$$2\log_{\frac{1}{3}}|x+1-\sqrt{5}i| < \log_{\frac{1}{3}}6$$

$$\log_{\frac{1}{3}}|x+1-\sqrt{5}i| < \tfrac{1}{2}\log_{\frac{1}{3}}6$$

$$\log_{\frac{1}{3}}|x+1-\sqrt{5}i| < \log_{\frac{1}{3}}\sqrt{6}$$

$$|x+1-\sqrt{5}i| < \sqrt{6}$$

$$|x+1-\sqrt{5}i| < \sqrt{6} \qquad |x+1-\sqrt{5}i| > -\sqrt{6}$$

$$x+1-\sqrt{5}i < \sqrt{6} \qquad x+1-\sqrt{5}i > -$$

$$* \boxed{x < \sqrt{6}-1+\sqrt{5}i} \qquad ** \boxed{x > \sqrt{5}i-1-}$$

$2\log_{\frac{1}{3}}|x+1-i\sqrt{5}| > -2$

$-2\log_{3}|x+1-i\sqrt{5}| > -2$

$\log_{3}|x+1-i\sqrt{5}| < 1$

$\log_{3}|x+1-i\sqrt{5}| < \log_{3}3$

$|x+1-i\sqrt{5}| < 3$

$x+1-i\sqrt{5} < 3 \qquad x+1-i\sqrt{5} > -3$

$x < 3-1+i\sqrt{5} \qquad x > -3-1+i\sqrt{5}$

$***\ \boxed{x < 2+i\sqrt{5}} \qquad ****\ \boxed{x > -4+i\sqrt{5}}$

Intersectando las soluciones:

$$\boxed{\left(-4+i\sqrt{5},\ -\sqrt{6}-1+i\sqrt{5}\right) \cup \left(\sqrt{6}-1+\sqrt{5}i,\ 2+\sqrt{5}i\right)}$$

(46) $a^2+ab+b^2=0$

$a=1 \quad b=b \quad c=b^2$

$$a=\frac{-b \pm \sqrt{(b)^2-4(1)(b^2)}}{2}$$

$$a=\frac{-b \pm \sqrt{b^2-4b^2}}{2} \qquad a=\frac{-b \pm \sqrt{-3b^2}}{2}$$

$$a=\frac{-b \pm \sqrt{3}ib}{2}$$

$$\left[a-\left(\frac{-b+\sqrt{3}ib}{2}\right)\right]\left[a-\left(\frac{-b-\sqrt{3}ib}{2}\right)\right]$$

$$\left(a+\frac{b-\sqrt{3}ib}{2}\right)\left(a+\frac{b+\sqrt{3}ib}{2}\right)$$

$$\left[a+\left(\frac{1-\sqrt{3}i}{2}\right)b\right]\left[a+\left(\frac{1+\sqrt{3}i}{2}\right)b\right]$$

$$\boxed{\left[a+\left(\frac{1-i\sqrt{3}}{2}\right)b\right]\left[a+\left(\frac{1+i\sqrt{3}}{2}\right)b\right]}$$

Solución

EJERCICIOS PROPUESTOS

CAPÍTULO SIETE

MATEMÁTICAS

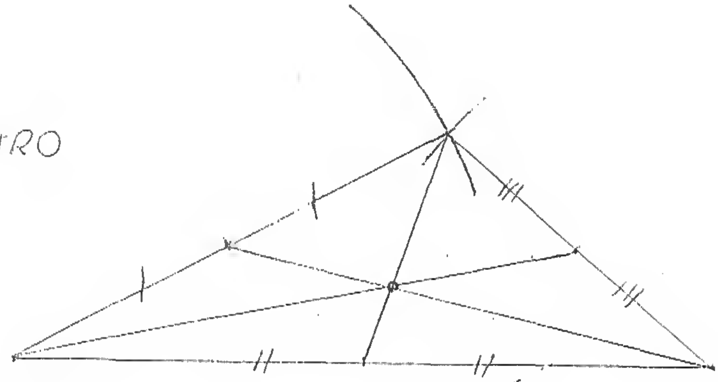
GEOMETRÍA PLANA

TRIÁNGULOS

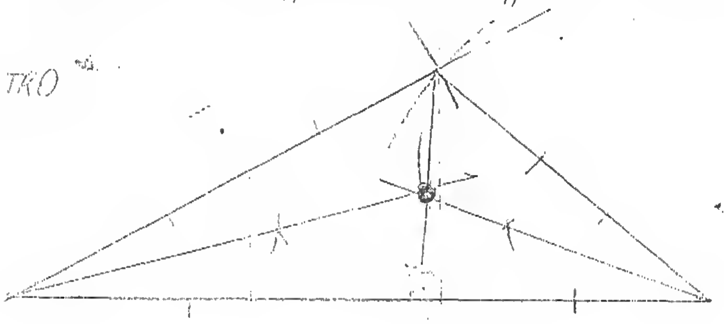
1.-

a) a=10cm
b= 5cm
c= 7cm

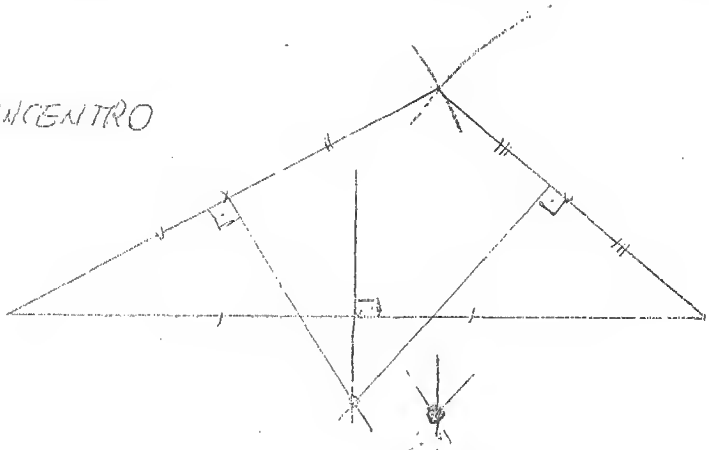
BARICENTRO



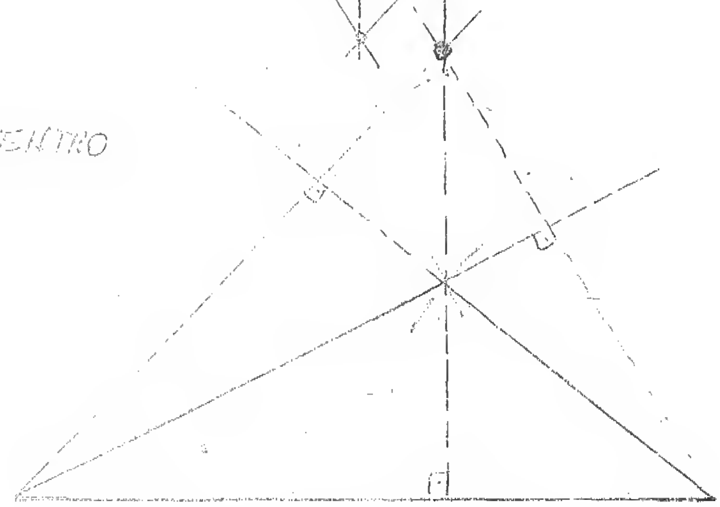
INCENTRO



CIRCUNCENTRO



ORTOCENTRO

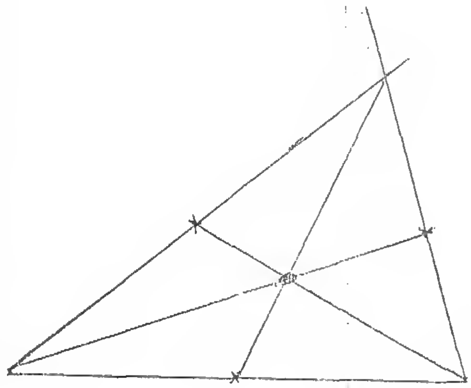


b)

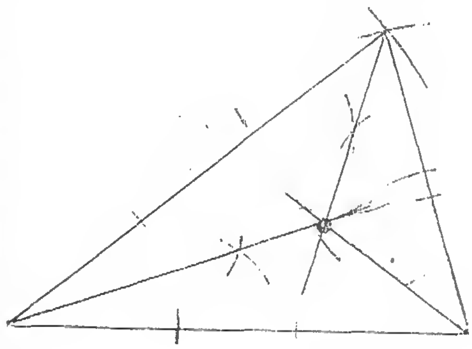
NO SE PRODUCE INTERSECCION.
NO HAY TRIANGULO

c) BARICENTRO

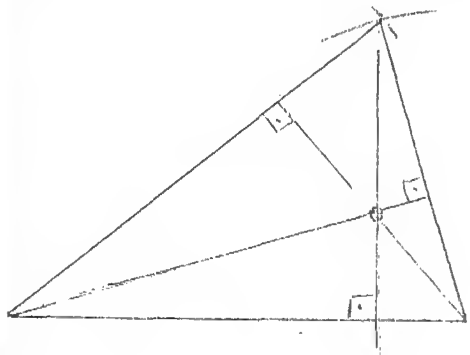
$a = 6$ cm.



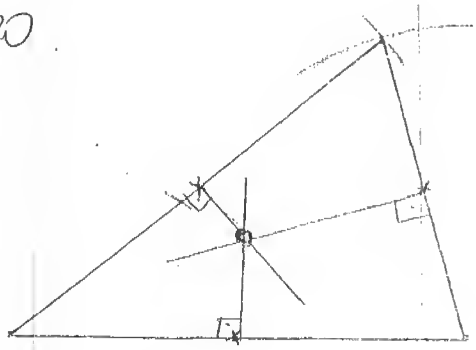
INCENTRO



ORTOCENTRO



CIRCUNCENTRO



d) $c = 8cm$

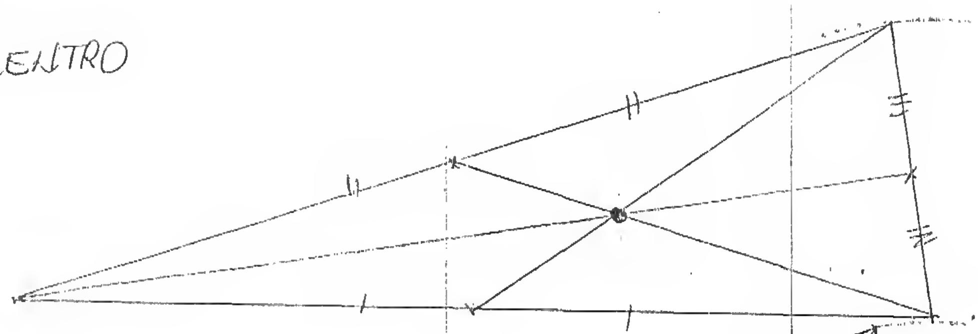
la suma de ángulos internos es 180°: $\alpha+\beta+\gamma=180°$

$\cancel{120}+\cancel{60}+\gamma=\cancel{180}$

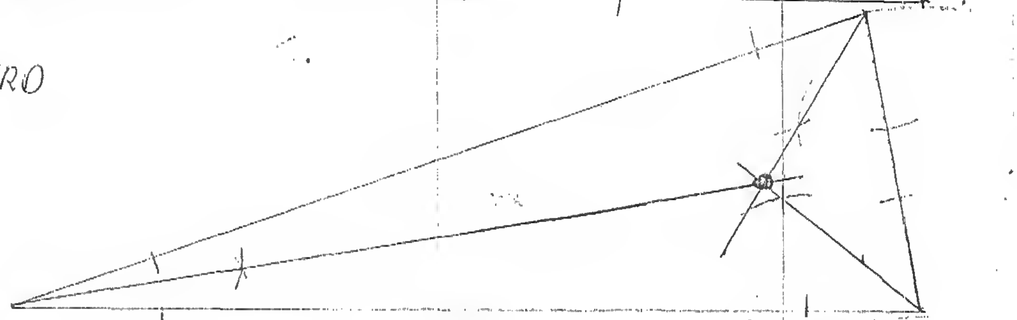
$\gamma=0°$ NO ES TRIANGULO

e) $a=12 cm$

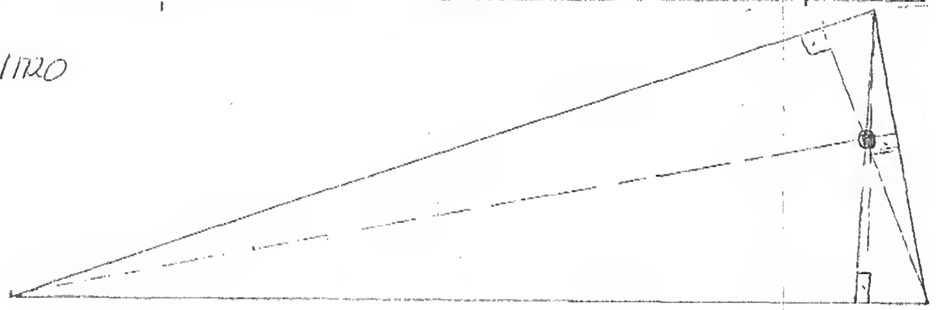
BARICENTRO



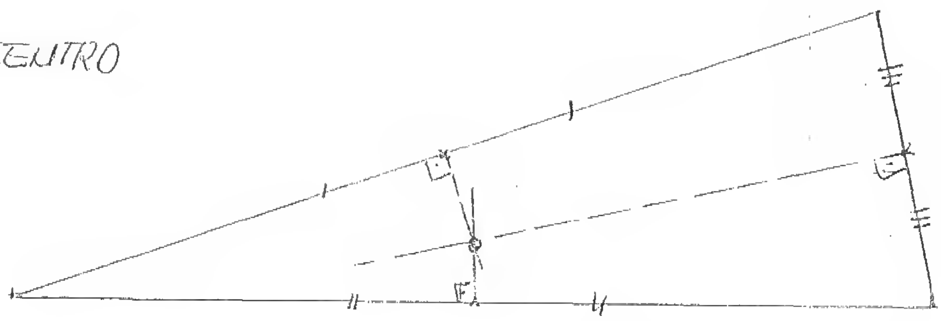
INCENTRO



ORTOCENTRO



CIRCONCENTRO



f) $b = 12\,cm$

$c = 4\,cm$

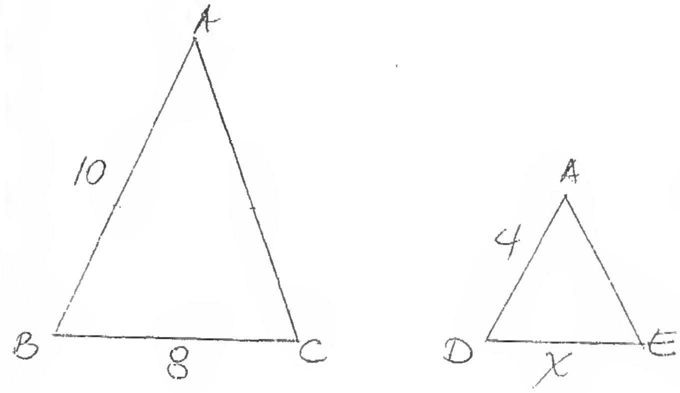
$\alpha = 180°$

$$\alpha + \beta + \gamma = 180°$$

$$\cancel{180} + \beta + \gamma = \cancel{180}$$

$$\beta + \gamma = 0°$$ NO ES TRIANGULO

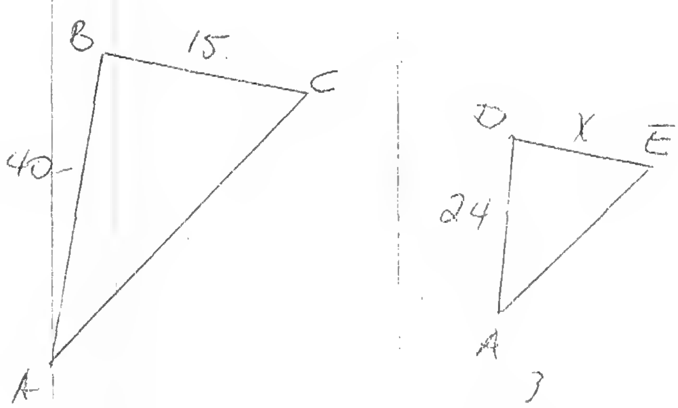
2.-



$$\frac{x}{4} = \frac{8}{10}$$

$$x = \frac{\cancel{32}^{16}}{\cancel{10}_{5}} \Rightarrow \frac{16}{5}$$

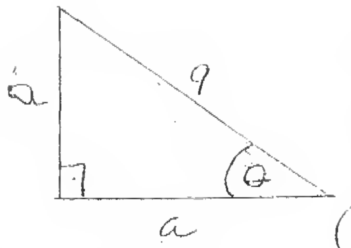
3.-



$$\frac{x}{\cancel{24}_{3}} = \frac{\cancel{15}^{3}}{\cancel{40}_{5}}$$

$$x = 9$$

4.-)



$\text{sen}\,\theta = \frac{a}{9}$ $\quad a = 9\,\text{sen}\,\theta$

$a = 9\frac{\sqrt{2}}{2}$

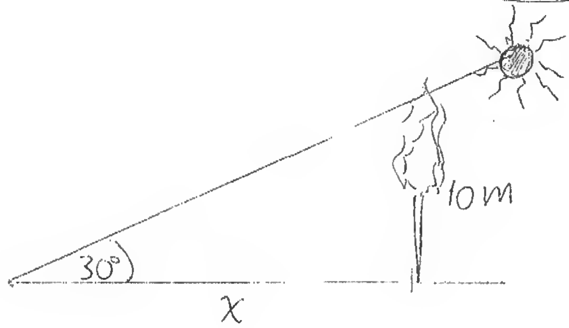
$\cos\theta = \frac{a}{9}$

$\tan\theta = \frac{a}{a}$

$\theta = 45°$

$a^2 + 6^2 = 81$

5.-)

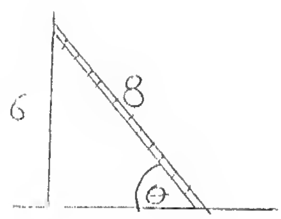


$\tan 30° = \frac{10}{x}$

$x = \frac{10}{\tan 30} \Rightarrow \frac{10}{\sqrt{3}/3} \Rightarrow \frac{30}{\sqrt{3}}$

$x = 10\sqrt{3}\text{ m.}$

6.-)

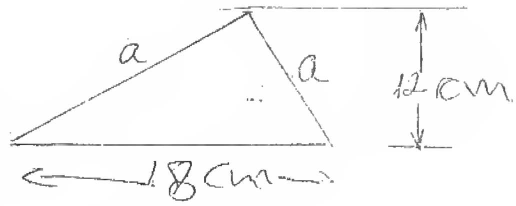


$\text{sen}\,\theta = \frac{6}{8}$

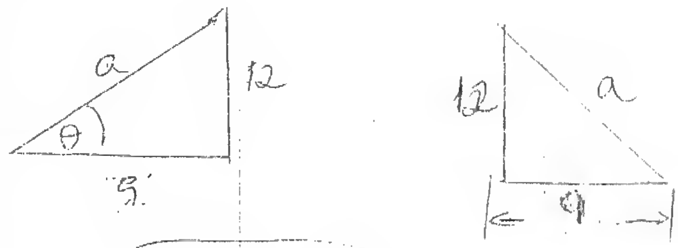
$\theta = \text{sen}^{-1}\left(\frac{6}{8}\right)$

$\theta = 48{,}6°$

7.-)



Por simetría



$$12^2 + 9^2 = a^2$$

$$a^2 = 225$$

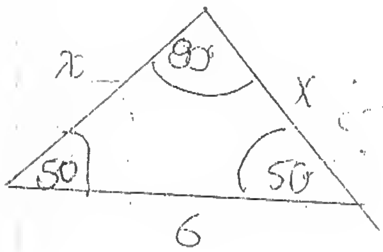
$$a = 15$$

$$\operatorname{sen}\theta = \frac{12}{15}$$

$$\theta = \operatorname{sen}^{-1}(4/5)$$

$$\theta = 53{,}13°$$

8.-



$$\alpha + 80 + \beta = 180$$

$$\beta = 180 - 80 - \alpha$$

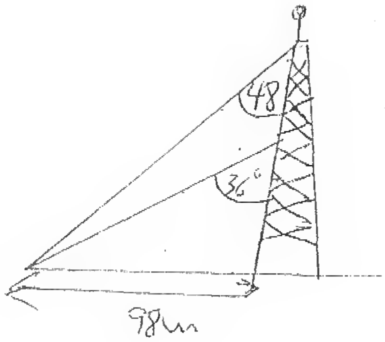
$$\beta = 100 - \alpha$$

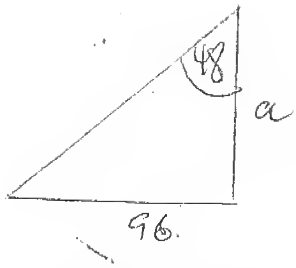
$$\frac{\operatorname{sen} 80}{6} = \frac{\operatorname{sen} 50}{x}$$

$$x = \frac{6 \operatorname{sen} 50}{\operatorname{sen} 80}$$

$$x = 4{,}66$$

9-



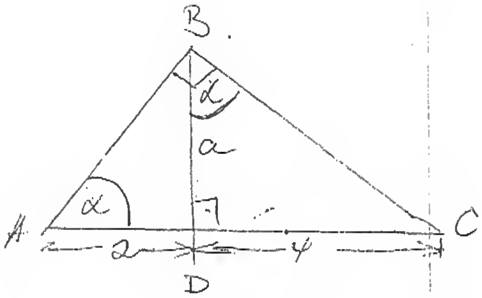


$$\tan 48 = \frac{96}{a}$$

$$a = \frac{96}{\tan 48}$$

$a = 86,44\,m$

10-



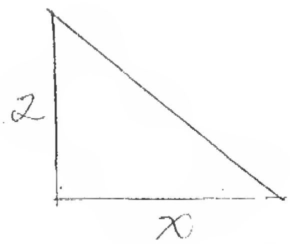
$$\tan\alpha = \frac{a}{2} \quad \Big\| \quad \tan\alpha = \frac{4}{a}$$

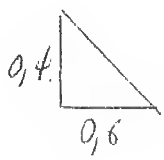
$$\frac{a}{2} = \frac{4}{a}$$

$$a^2 = 8$$

$a = \sqrt{8} \Rightarrow a = 2\sqrt{2}$ b) correcto

11-

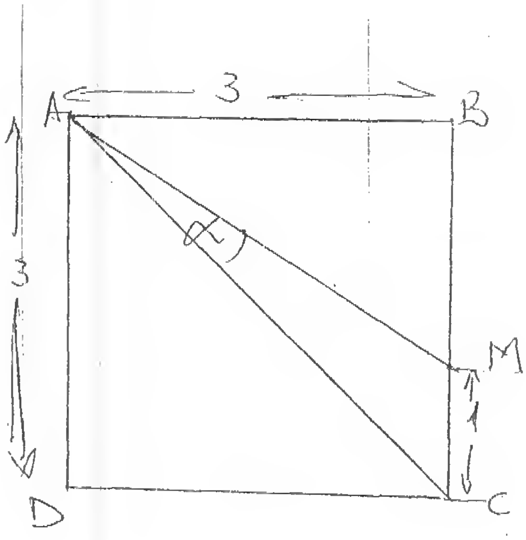


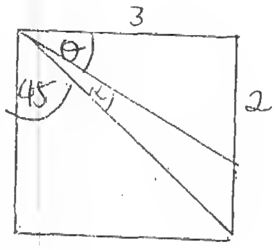


$$\frac{x}{2} = \frac{96}{0,4} \Rightarrow x = \frac{1,2}{0,4} \cdot \frac{10}{10} \Rightarrow \frac{\cancel{12}^{3}}{\cancel{4}_{1}}$$

$x = 3\,m$ e) correcto

12.—





$\theta + \alpha + 45 = 90$

$\alpha = 90 - 45 - \theta$

$\alpha = 45 - \theta$

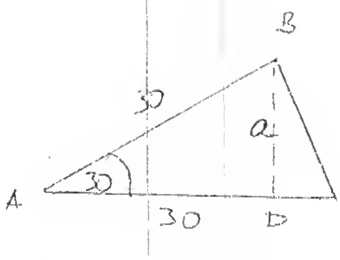
$\tan\alpha = \tan(45 - \theta)$

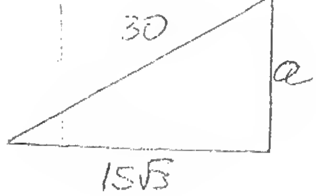
$\Rightarrow \dfrac{\tan 45 - \tan\theta}{1 + \tan\theta \tan 45}$

$\Rightarrow \dfrac{1 - 2/3}{1 + \frac{2}{3} \times 1} \Rightarrow \dfrac{\frac{1}{\cancel{3}}}{\frac{5}{\cancel{3}}}$

$\tan\alpha = 1/5$ d) CORRECTO

13.—





$AD = 30 \cos 30$

$AD = 30 \times \frac{\sqrt{3}}{2}$

$\boxed{AD = 15\sqrt{3}}$

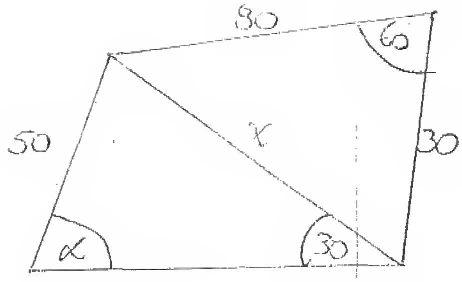
$a^2 = 30^2 - (15\sqrt{3})^2$

$a^2 = 900 - 675$

$a = \sqrt{225}$

$a = 15m$

14.-



$$x^2 = 80^2 + 30^2 - 2(80)(30)\cos 60$$

$$x^2 = 6400 + 900 - 2400$$

$$x = \sqrt{4900}$$

$$x = 70m$$

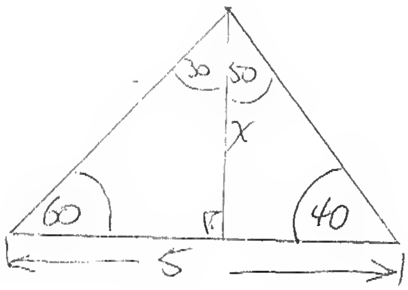
$$\frac{\operatorname{sen}\alpha}{x} = \frac{\operatorname{sen}30}{50}$$

$$\operatorname{sen}\alpha = \frac{x \cdot \operatorname{sen}30}{50} \Rightarrow \frac{7\cancel{0}}{5\cancel{0}} \cdot \frac{1}{2}$$

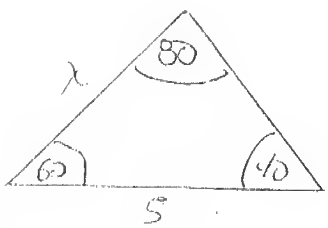
$$\alpha = \operatorname{sen}^{-1}(0{,}7)$$

$$\alpha = 44{,}42^\circ$$

15.-



a)



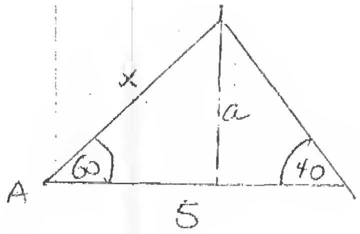
$\operatorname{sen}(2x) = 2\operatorname{sen}(x)\cos(x)$ [Angulo doble]

$$\frac{x}{\operatorname{sen}40} = \frac{5}{\operatorname{sen}80}$$

$$x = \frac{5\operatorname{sen}40}{\operatorname{sen}80} \Rightarrow \frac{5\cancel{\operatorname{sen}40}}{2\cancel{\operatorname{sen}40}\cos 40}$$

$$x = \frac{5}{2\cos 40}$$

6)



$$\operatorname{sen} 60 = \frac{a}{x}$$

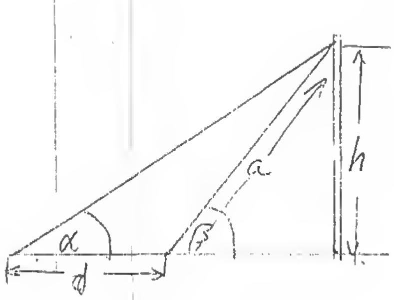
$$a = x \operatorname{sen} 60$$

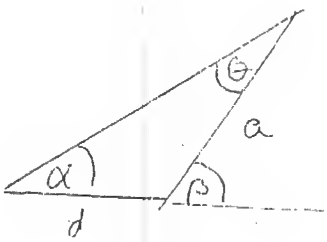
$$a = \frac{5}{2\cos 40} \cdot \operatorname{sen} 60$$

$$a = \frac{5}{2\cos 40} \cdot \frac{\sqrt{3}}{2}$$

$$a = \frac{5\sqrt{3}}{4\cos 40^\circ}$$

16.-

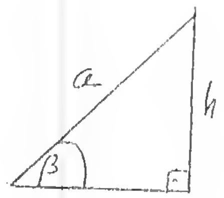




$$\beta = \alpha + \theta$$

$$\theta = \beta - \alpha$$

$$\frac{a}{\operatorname{sen}\alpha} = \frac{d}{\operatorname{sen}\theta} \Rightarrow a = \frac{d \operatorname{sen}\alpha}{\operatorname{sen}(\beta-\alpha)}$$

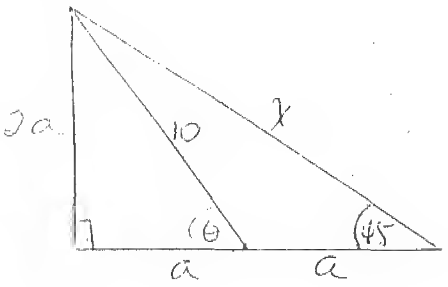


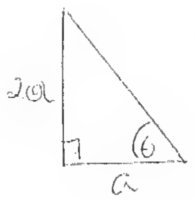
$$\operatorname{sen}\beta = \frac{h}{a}$$

$$h = a \operatorname{sen}\beta$$

$$h = \frac{d \operatorname{sen}\alpha}{\operatorname{sen}(\beta-\alpha)} \cdot \operatorname{sen}\beta$$

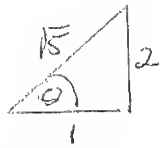
17.-



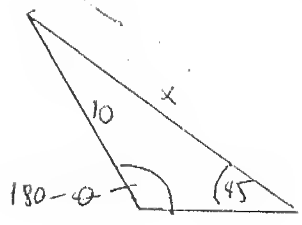


$\tan\theta = \frac{2a}{a}$

$\boxed{\tan\theta = 2}$



$\operatorname{sen}\theta = \frac{2}{\sqrt{5}}$



$$\frac{x}{\operatorname{sen}(180-\theta)} = \frac{10}{\operatorname{sen}45}$$

$$\frac{x}{\operatorname{sen}180\cos\theta - \operatorname{sen}\theta\cos180} = \frac{10}{\sqrt{2}/2}$$

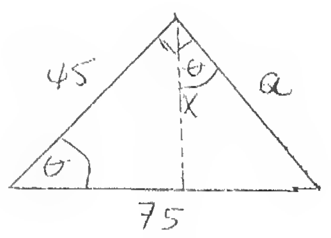
$$\frac{x}{\operatorname{sen}\theta} = \frac{20}{\sqrt{2}}$$

$$x = \frac{20}{\sqrt{2}}\operatorname{sen}\theta \Rightarrow \frac{20}{\sqrt{2}} \cdot \frac{2}{\sqrt{5}}$$

$$x = \frac{40}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} \Rightarrow \frac{40\sqrt{10}}{10}$$

$x = 4\sqrt{10}$

18.-



$a^2 = 75^2 - 45^2$

$a = \sqrt{3600}$

$\boxed{a = 60}$

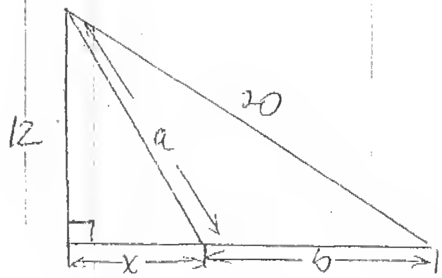
$\operatorname{sen}\theta = \frac{x}{45}$ $\quad$ $\operatorname{sen}\theta = \frac{a}{75}$

$$\frac{x}{45} = \frac{a}{75}$$

$$x = a\frac{\cancel{45}^{\cancel{9}\,3}}{\cancel{75}_{\cancel{15}\,5}} \Rightarrow \cancel{60}^{12} \cdot \frac{3}{\cancel{5}}$$

$x = 36$

19.-



$\boxed{a=b+8}$

$(x+b)^2 = 20^2 - 12^2$

$x+b = \sqrt{256}$

$\boxed{x+b=16}$

$\Downarrow$

$b = 16 - x$

$a = b + 8$

$a = 16 - x + 8$

$\boxed{a = 24 - x}$

$a^2 = 12^2 + x^2$

$a^2 = 144 + x^2$

$(24-x)^2 = 144 + x^2$

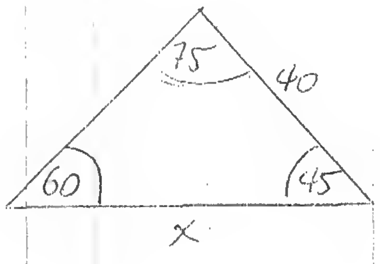
$576 - 48x + \cancel{x^2} = 144 + \cancel{x^2}$

$576 - 144 = 48x$

$432 = 48x$

$x = 9$

20.-



$\sin(75) = \sin(45+30)$

$\Rightarrow \sin 45 \cos 30 + \sin 30 \cos 45$

$\Rightarrow \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$\Rightarrow \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$\Rightarrow \frac{\sqrt{6}+\sqrt{2}}{4}$

$\frac{x}{\sin 75} = \frac{40}{\sin 60}$

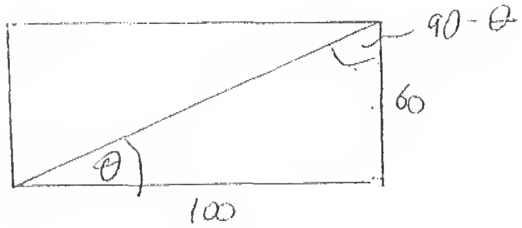
$x = 40 \frac{\sin 75}{\sin 60}$

$x = 40 \frac{(\sqrt{6}+\sqrt{2})}{\cancel{4}2} \cdot \frac{1}{\frac{\sqrt{3}}{\cancel{2}1}} \Rightarrow \frac{\overset{20}{\cancel{40}}(\sqrt{6}+\sqrt{2})}{\underset{1}{\cancel{2}}\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$x = \frac{20}{3}\sqrt{3}(\sqrt{6}+\sqrt{2})$

$\Rightarrow x = \frac{20\sqrt{6}}{3}(\sqrt{3}+1)$

21.-



$\tan\theta = \frac{\cancel{60}^{3}}{\cancel{100}_{5}}$

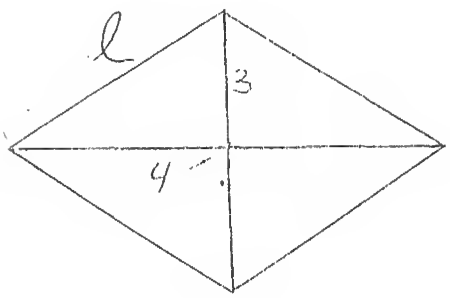
$\theta = \tan^{-1}\left(\frac{3}{5}\right)$

$\theta = 30{,}96°$

$\phi = 90-\theta$

$\phi = 59{,}04°$

22.-

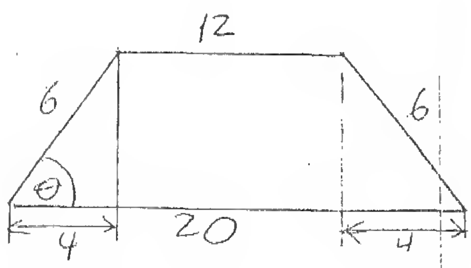


$l^2 = 3^2+4^2$

$l=\sqrt{9+16}$

$l=5cm$

23.-

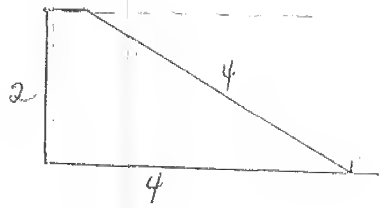


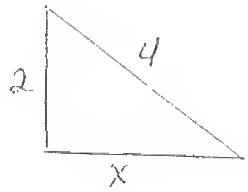
$\cos\theta = \frac{4}{6}$

$\theta = \cos^{-1}\left(\frac{2}{3}\right)$

$\theta = 48{,}19°$

24 -





$x^2 = 4^2 - 2^2$

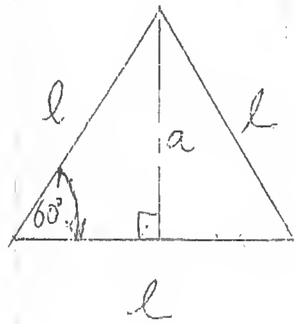
$x = \sqrt{16-4}$

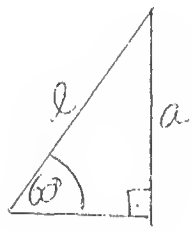
$x = \sqrt{12} \Rightarrow 2\sqrt{3}$

$b = 4 - 2\sqrt{3}$

$b = 2(2-\sqrt{3})$

25 -





$\operatorname{sen} 60 = \frac{a}{l}$

$a = l \operatorname{sen} 60$

$\boxed{a = l\frac{\sqrt{3}}{2}}$

$\text{Area} = \frac{b \times a}{2}$

$\Rightarrow \frac{1}{2} l \times \frac{l\sqrt{3}}{2}$

$\Rightarrow \frac{l^2\sqrt{3}}{4}$

$\Rightarrow \frac{5^2\sqrt{3}}{4}$

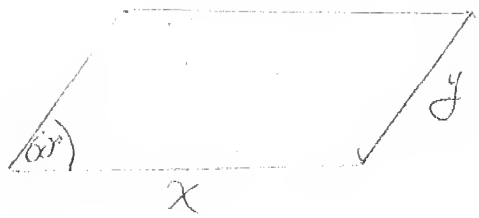
$\Rightarrow \frac{25\sqrt{3}}{4}$

26.-

$A = 90\sqrt{3}$

$x = ??$

$y = ??$



$x = 3y$

$$x \cdot y \operatorname{sen} 60 = 90\sqrt{3}$$

$$3y \cdot y \cdot \frac{\sqrt{3}}{2} = 90\sqrt{3}$$

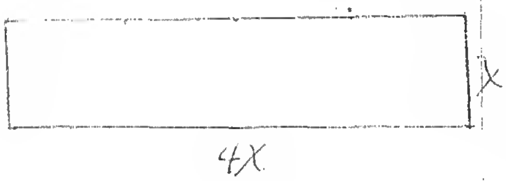
$$y^2 = \frac{2 \cdot \cancel{90}^{30}}{\cancel{3}_{1}}$$

$$\sqrt{y^2} = \sqrt{60}$$

$$y = 2\sqrt{15}$$

$$x = 6\sqrt{15}$$

27.-



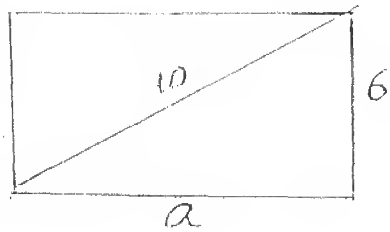
$$\text{Area} = 4x \cdot x = 100$$

$$x^2 = \frac{100}{4}$$

$$\sqrt{x^2} = \sqrt{25}$$

$$x = 5\,m$$

28.-



$$a^2 = 10^2 - 6^2$$

$$a^2 = 100 - 36$$

$$\sqrt{a^2} = \sqrt{64}$$

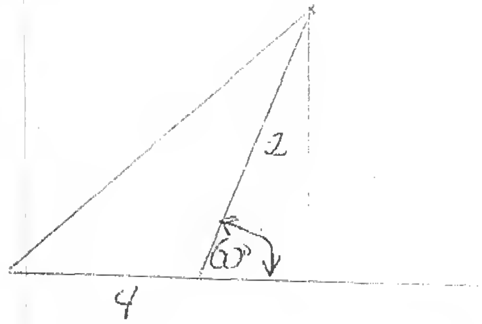
$$a = 8\,m$$

Área $= 8 \cdot 6$

Área $= 48\,m^2$

d) correcto

29.—

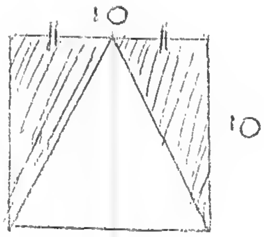


$$Área = \frac{b \times a}{2} \Rightarrow \frac{4 \times \cancel{2} \times sen60}{\cancel{2}}$$

$$\Rightarrow \cancel{4}^{2} \times \frac{\sqrt{3}}{\cancel{2}}$$

$$Área = 2\sqrt{3}\ u^2$$

30.—



$$A_R = \square - \triangle$$

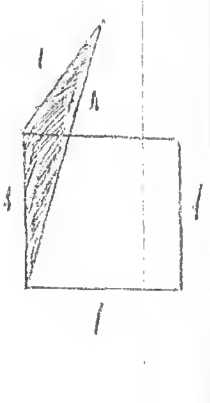
$$\Rightarrow 10^2 - \frac{b \times a}{2}$$

$$\Rightarrow 100 - \frac{10 \times 10}{2}$$

$$A_R = 50\ u^2$$

b) correcto

31.—



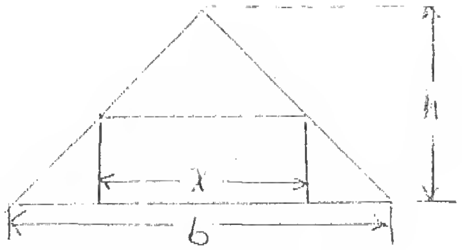
Si es $\triangle$ equilátero

$$Área = \frac{\sqrt{3}\, l^2}{4}$$

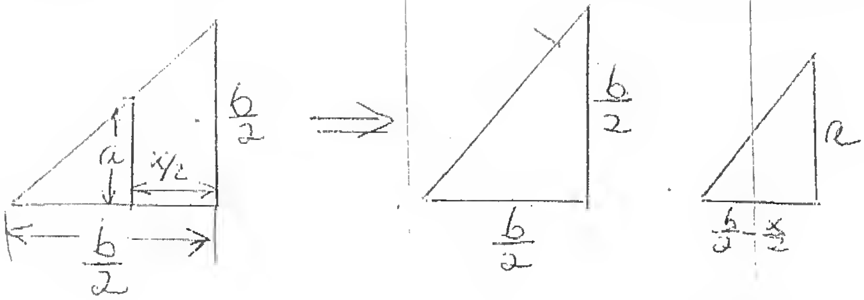
$$Área = \frac{\sqrt{3}\,(1)^2}{4}$$

$$Área = \frac{\sqrt{3}}{4}\ u^2$$

32.-



son △ isósceles

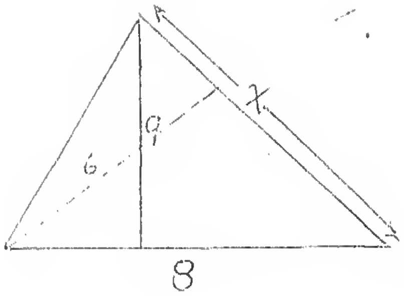


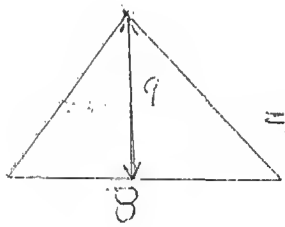
$$a = \frac{b}{2} - \frac{x}{2}$$

$$2a = b - x$$

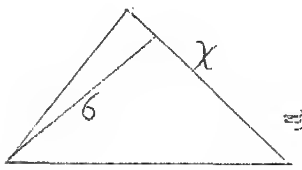
$$x = b - 2a$$

33.-





$\Rightarrow$ $\text{área}_1 = \dfrac{9 \times 8}{2}$



$\Rightarrow$ $\text{área}_2 = \dfrac{x \times 6}{2}$

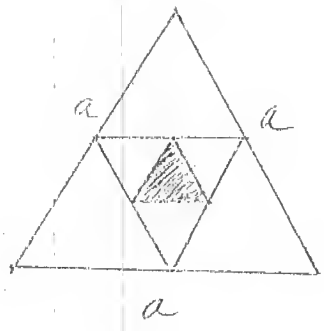
$$\text{área}_1 = \text{área}_2$$

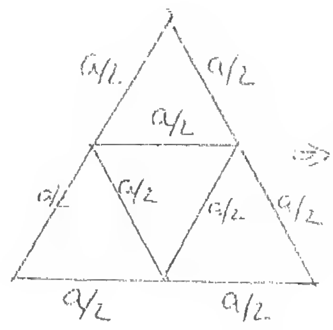
$$\frac{\cancel{9}^{3} \times 8}{\cancel{2}} = \frac{x \times \cancel{6}^{2}}{\cancel{2}}$$

$$24 = 2x$$

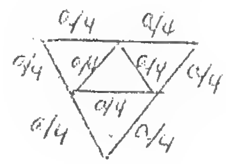
$$x = 12$$

34.-





$\Rightarrow$ son Δ equiláteros



$$\text{Area } \Delta_1 = \frac{\sqrt{3}(a)^2}{4}$$

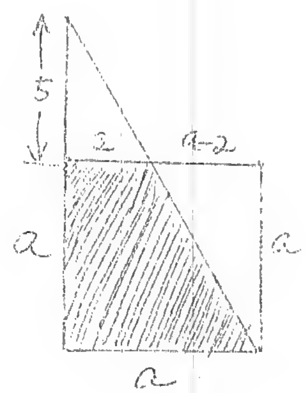
$$\Rightarrow \frac{\sqrt{3}a^2}{4}$$

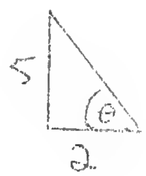
$$\text{Area}\Delta_2 = \frac{\sqrt{3}\,(a/4)^2}{4}$$

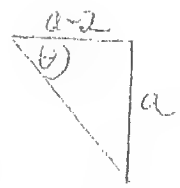
$$\Rightarrow \frac{1}{16}\left(\frac{\sqrt{3}a^2}{4}\right) \text{ area 1}$$

$$\text{Area2} = \frac{1}{16}\text{ area 1}$$

35.-







$$\tan\theta = \frac{a}{a-2}$$

$$\tan\theta = \frac{5}{2}$$

$$\frac{5}{2} = \frac{a}{a-2}$$

$$5(a-2) = 2a$$

$$5a - 10 = 2a$$

$$3a = 6$$

$$a = 10/3$$

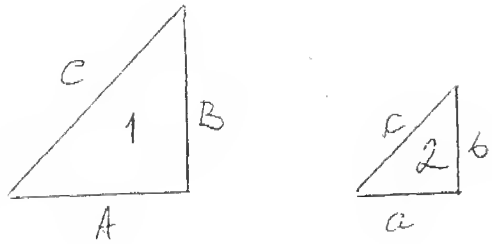
$$A_R = \square - \Delta$$

$$A_R = \left(\frac{10}{3}\right)^2 - \left(\frac{10}{3}\right)\left(\underbrace{\frac{10}{3}-2}_{4/3}\right)\cdot\frac{1}{2}$$

$$= \frac{100}{9} - \frac{\cancel{40}\ 20}{\cancel{18}\ 9}$$

$$A_R = 80/9$$

36.-



$$\left.\begin{array}{l} A = 2a \\ B = 2b \\ C = 2c \end{array}\right\}$$

$$A = 2a$$

$$AB = 2a \cdot 2b$$

$$\frac{AB}{2} = \frac{4ab}{2}$$

$$A_{\triangle 1} = 4 A_{\triangle 2}$$

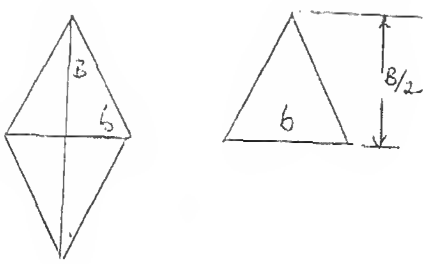
$$A_{\triangle 2} = 4u^2$$

$$A_{\triangle 1} = 4(4)$$

$$A_{\triangle 1} = 16u^2$$

b) FALSO

37.-



$$A_\triangle = \frac{1}{2} b \left(\frac{B}{2}\right)$$

$$A_\triangle = \frac{bB}{4}$$

$$A_\Diamond = 2A_\triangle$$

$$A_\Diamond = \cancel{2}^{1}\left(\frac{bB}{\cancel{4}_{2}}\right)$$

$$A_\Diamond = \frac{bB}{2}$$

$$A_\Diamond = \frac{\cancel{6}^{3} \times 8}{\cancel{2}_{1}}$$

$$A_\Diamond = 24u^2$$

b) FALSO

38.- $A_\square = \ell^2$ $\qquad A_\triangle = \frac{\sqrt{3}}{4}\ell^2$

$$A_\square = 2A_\triangle$$

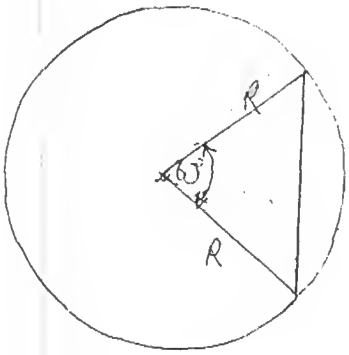
$$\ell^2 = \cancel{2}\left(\frac{\sqrt{3}}{\cancel{4}_2}\ell_1^2\right)$$

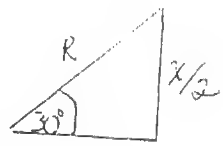
$$\ell^2 = \frac{\sqrt{3}}{2}\ell_1^2$$

$$\sqrt{\ell^{\cancel{2}}} = \sqrt{\frac{\sqrt{3}}{2}\ell_1^{\cancel{2}}}$$

$$\ell = \frac{\sqrt[4]{3}}{\sqrt{2}}\ell_1$$ b) FALSO

39.-

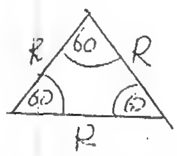




$$\sin 30 = \frac{x/2}{R}$$

$$R = \frac{x/2}{\sin 30}$$

$$R = \frac{x/\cancel{2}}{1/\cancel{2}}$$

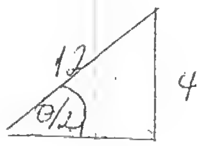


△ equilátero

$$x = R$$

$$x = 40\,cm$$

40.-



$$\sin\left(\frac{\theta}{2}\right) = \frac{\cancel{4}}{\cancel{12}_3}$$

$$\frac{\theta}{2} = \sin^{-1}\left(\frac{1}{3}\right)$$

$$\theta = 2\sin^{-1}\left(\frac{1}{3}\right)$$

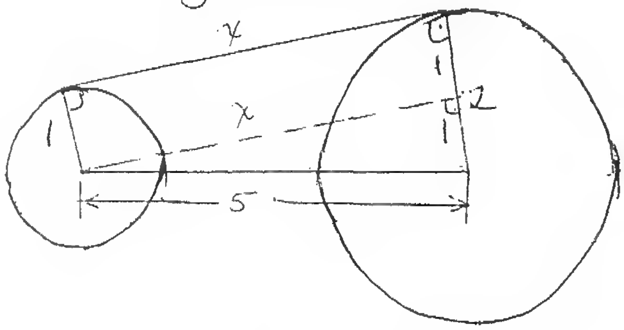
41.- Los datos corresponden al ejercicio anterior,

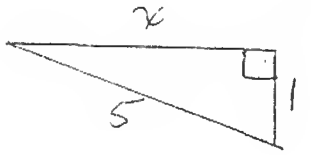
$$\theta = 2\,\text{sen}^{-1}\left(\frac{1}{5}\right)$$

$$\theta \neq \pi/3$$

b) FALSO

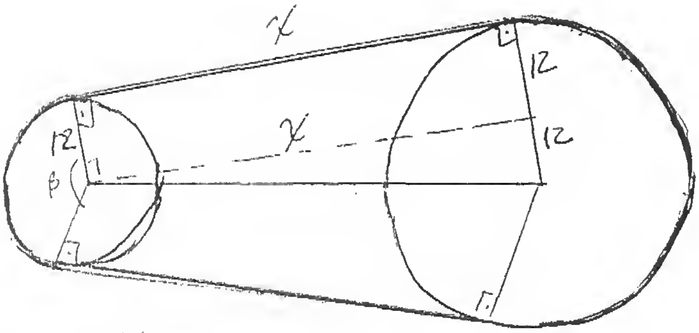
42.-

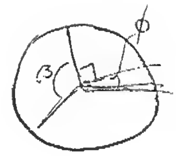


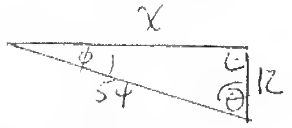


$$x = \sqrt{5^2 - 1^2}$$
$$x = \sqrt{24}$$
$$x = 2\sqrt{6}$$

43.-

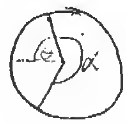






$$x = \sqrt{54^2 - 12^2}$$
$$x = \sqrt{2772}$$
$$x = 6\sqrt{77}$$

$$\theta = \cos^{-1}\left(\frac{\cancel{12}^{\,2}}{\cancel{54}_{\,9}}\right)$$
$$\theta = 1{,}35 \text{ rad}$$



$$\alpha = 2\pi - 2\theta$$
$$\alpha = 3{,}589 \text{ rad}$$

$$S_1 = \theta R$$
$$S_1 = 3{,}599(24)$$
$$S_1 = 86{,}15 \text{ cm.}$$

$$\phi = \pi - \frac{\pi}{2} - \theta$$
$$\phi = 0{,}22 \text{ rad}$$

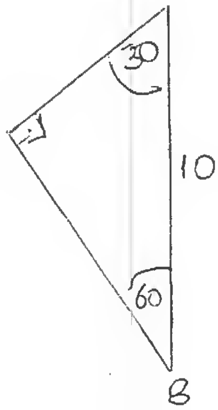
$$\beta + \frac{\pi}{2} + \phi + \frac{\pi}{2} + \phi = 2\pi$$
$$\beta = 2\pi - \pi - 2\phi$$
$$\beta = 2{,}7 \text{ rad}$$
$$S_2 = \theta R \Rightarrow 2{,}7(12)$$
$$S_2 = 32{,}4 \text{ cm}$$

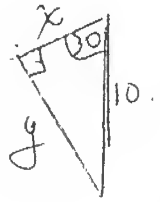
$$L = 2x + S_1 + S_2$$

$$L = 2(6\sqrt{77}) + 86{,}15 + 32{,}4$$

$$\boxed{L = 223{,}85\,cm}$$

44.-





$$\cos 30 = \frac{x}{10}$$

$$x = 10\cos 30$$

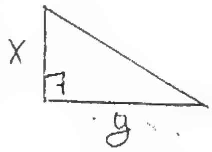
$$x = 10 \cdot \frac{\sqrt{3}}{2}$$

$$x = 5\sqrt{3}$$

$$\sin 30 = \frac{y}{10}$$

$$y = 10\sin 30$$

$$y = 5$$

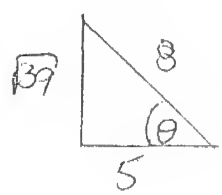


$$Area = \frac{5 \cdot 5\sqrt{3}}{2}$$

$$\boxed{\Rightarrow \frac{25\sqrt{3}}{2}}$$

45.-

$S_1 =$



$\theta = 0{,}895610$

$$(5\sqrt{3})^2 = 10^2 + 10^2 - 2(10)(10)\cos\theta$$

$$75 = 200 - 200\cos\theta$$

$$200\cos\theta = 200 - 75$$

$$\cos\theta = \frac{125}{200}$$

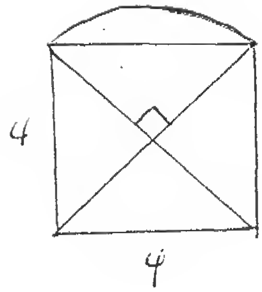
$$\cos\theta = \frac{5}{8}$$

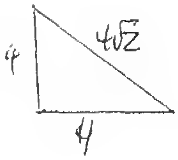
$$\frac{\theta R^2}{2} - \frac{1}{2}\,l^2 \sin\theta$$

$$\Rightarrow \frac{(0{,}8956)10^2}{2} - \frac{1}{2}(10^2)\frac{\sqrt{39}}{8}$$

$$\boxed{\Rightarrow 5{,}75\,m^2}$$

46.-



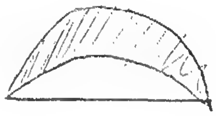


$= \frac{\theta R^2}{2} - \frac{ba}{2}$

$= \cancel{\frac{1}{2}}\frac{\pi}{2}(2\sqrt{2})^2 - \frac{1}{\cancel{2}}(4)(2)$

$= \frac{8\pi}{4} - 4$

$\boxed{= 2\pi - 4}$



$=$

$\Rightarrow \frac{\pi R^2}{2} -$

$\Rightarrow \frac{\pi(2)^2}{2} - (2\pi - 4)$

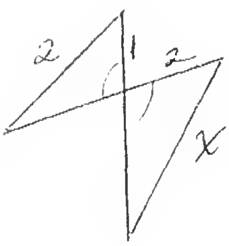
$\Rightarrow \cancel{2}\pi - \cancel{2}\pi + 4$

$\Rightarrow 4$

$A_R = 4 * 4$

$\Rightarrow 16u^2$

47.-



$\frac{x}{2} = \frac{2}{1}$

$x = 4u^2$

48.- b) FALSO

$\Delta_{ACE} \approx \Delta_{DBE}$

49.- $R = 3$ cm

$\widehat{AB} = 2\pi$ cm

$\theta = \frac{2\pi}{3}$ rad

$S = \frac{\theta R}{2}$

$\frac{2S}{R} = \theta$

$\theta = \frac{2(2\pi)}{3}$

$\theta = \frac{4\pi}{3}$ rad

b) FALSO

50.-

$A_R = (\bigcirc - \square)/4$

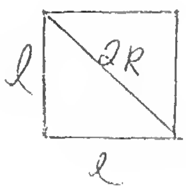
$A_R = \frac{\pi R^2 - l^2}{4}$

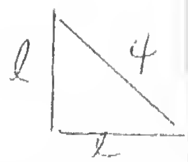
$A_R = \frac{\pi (2)^2 - (2\sqrt{2})^2}{4}$

$A_R = \frac{\overset{1}{\cancel{4}}\pi - \overset{2}{8}}{\underset{1}{\cancel{4}}}$

$A_R = \pi - 2$

c) CORRECTO





$4 = \sqrt{2}\, l$

$l = \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$l = 2\sqrt{2}$

51.-

$R = 2\,m$

$r = 1\,m$ $\qquad A_R =$ 

$\theta = \frac{\pi}{3}$ $\qquad A_R = \frac{\theta R^2}{2} - \frac{\theta r^2}{2}$

$$A_R = \frac{\theta}{2}(R^2 - r^2)$$

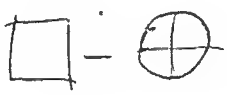
$$A_R = \frac{1}{2}\frac{\pi}{3}(2^2 - 1^2)$$

$$A_R = \frac{\pi}{\cancel{6}_2}\cancel{3}^1$$

$$A_R = \frac{\pi}{2}\,m^2$$ a) CORRECTO

52.-

$l = 12\,cm.$

$R = 6\,cm$ $\qquad A_R =$ 

$$A_R = l^2 - \pi R^2$$

$$A_R = 12^2 - \pi(6)^2$$

$$A_R = 144 - 36\pi$$

$$A_R = 36(4 - \pi)$$ c) CORRECTO

53.-

$\theta = \frac{\pi}{6}\,rad$ $\qquad A_S = \frac{\theta R^2}{2}$ $\qquad \boxed{R = 4\,M}$

$A_S = \frac{4\pi}{3}M^2$ $\qquad \frac{2A_S}{\theta} = R^2$

$R = ¿?$ $\qquad R = \sqrt{\frac{2A_S}{\theta}}$

$$R = \sqrt{\frac{2\cdot 4\cancel{\pi}\cdot\cancel{6}}{\cancel{3}\cdot\cancel{\pi}}}$$

$Arco = \theta R$

$Arco = \frac{\pi}{\cancel{6}_3}(4)^2$

$Arco = \frac{2\pi}{3}$

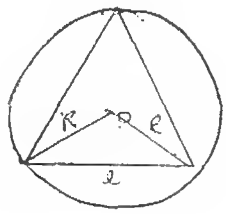
$Perimetro = 2R + arco$

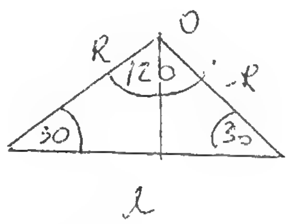
$\Rightarrow 2(4) + \frac{2\pi}{3}$

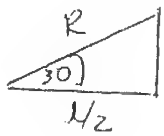
$\Rightarrow \frac{24 + 2\pi}{3}$

54.- $R = 8\,cm$

$A\Delta = ?$
equilátero







$\cos 30 = \frac{l/2}{R}$

$R\cos 30 = \frac{l}{2}$

$\frac{l}{\cancel{2}} = 8\left(\frac{\sqrt{3}}{\cancel{2}}\right)$

$\boxed{l = 8\sqrt{3}}$

$A\Delta = \frac{\sqrt{3}\,l^2}{4} \Rightarrow \frac{\sqrt{3}(8\sqrt{3})^2}{4}$

$A\Delta = \frac{\cancel{64}^{16} \cdot 3\sqrt{3}}{\cancel{4}_1} \Rightarrow 48\sqrt{3}$

55.—

$A_{hex} = 2\, A_\Delta$

$A_{hex} = 2(48\sqrt{3})$

$\cancel{6}\Delta = 2(\cancel{48})\sqrt{3}$ (8)

$A_\Delta = 16\sqrt{3}$

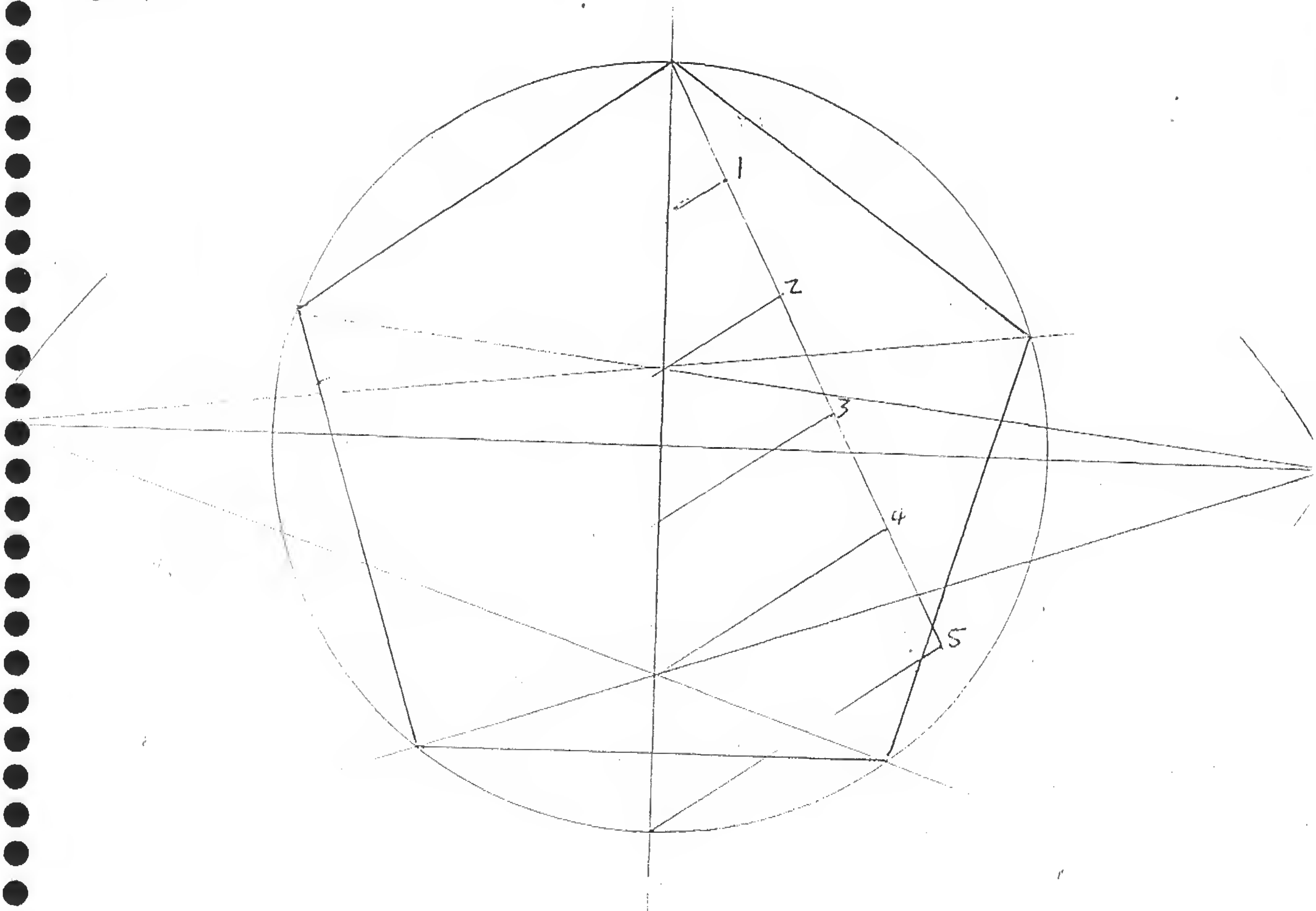
$\frac{\cancel{\sqrt{3}}L^2}{4} = 16\cancel{\sqrt{3}}$

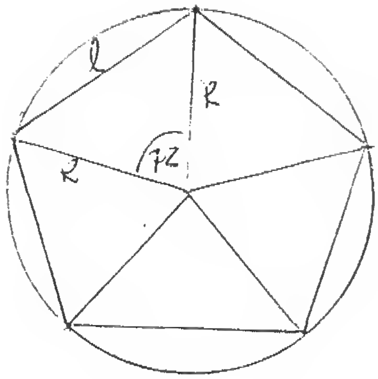
$\sqrt{L^2} = \sqrt{16(4)}$

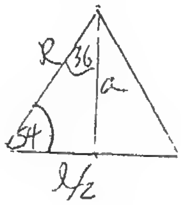
$L = 4(2)$

$L = 8\,cm$

56.—







$\cos 54 = \frac{l}{2R}$

$R = \frac{l}{2\cos 54}$

$R = 6\,cm$

$P = 5l$

$P = 35\,cm$

$a = \sqrt{R^2 - 3{,}5^2}$

$a = \sqrt{36 - 3{,}5^2}$

$a = 4{,}87\,cm$

$Area = 5\,A_i$

$\Rightarrow 5\,\frac{1}{2}\,l^2 \operatorname{sen}\theta$

$\Rightarrow 5\left(\frac{1}{2}\right)(6)^2 \operatorname{sen} 72$

$\Rightarrow 85{,}6\,cm^2$

Pentágono circunscrito

radio = apotema

$r = 4{,}87\,cm$

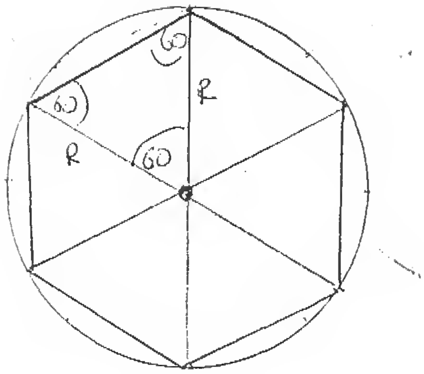
las demás mediciones son iguales

$P = 35\,cm$

$a = 4{,}87\,cm$

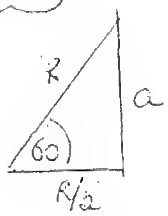
$Area = 85{,}6\,cm^2$

b)



$R = l$

$R = 5cm$



$a = \sqrt{R^2 - \left(\frac{R}{2}\right)^2}$

$a = \sqrt{5^2 - 2{,}5^2}$

$a = \sqrt{18{,}75}$

$a = 4{,}33cm$

Perímetro $= 6R$

$\Rightarrow 6(5)$

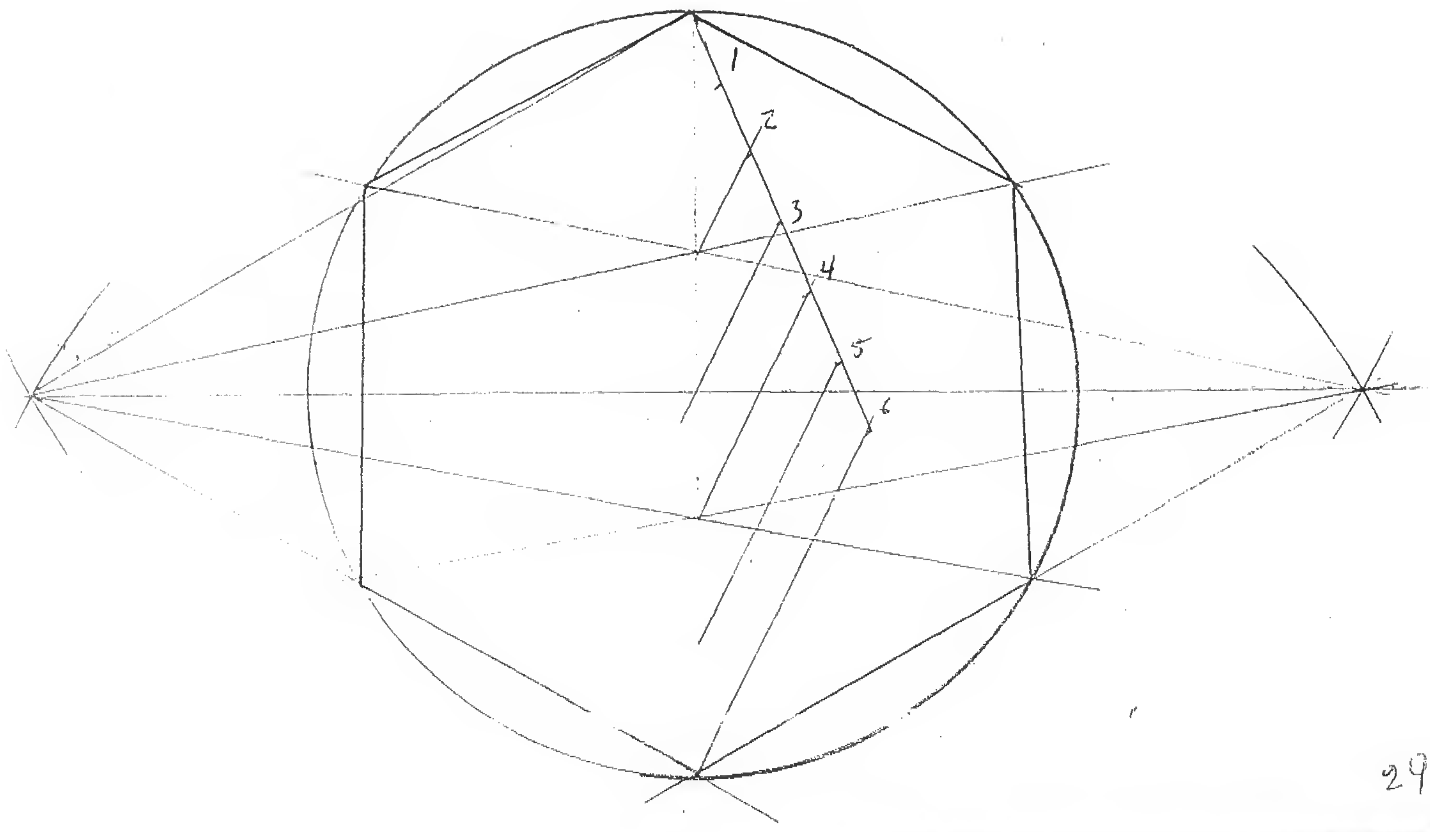
$\Rightarrow 30cm$

Área $= 6A\triangle$

$\Rightarrow 6 \frac{\sqrt{3}}{4} l^2$

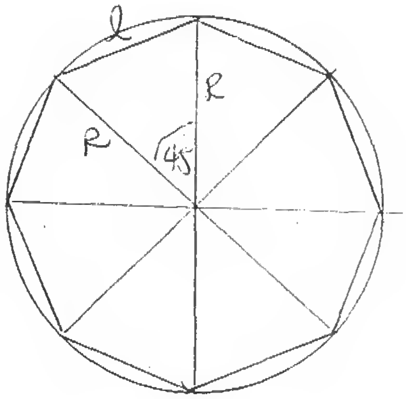
$\Rightarrow \cancel{6}^{3} \frac{\sqrt{3}}{\cancel{4}_{2}} (5)^2$

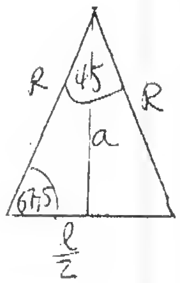
$\Rightarrow \frac{75\sqrt{3}}{2}$



c)

$l = 6$ cm





$\cos 67{,}5 = \frac{l}{2R}$

$R = \frac{l}{2\cos 67{,}5}$

$R = 7{,}84$ cm

$\sin 67{,}5 = \frac{a}{R}$

$a = R \sin 67{,}5$

$a = 7{,}24$ cm

Perímetro $= 8l$

$\Rightarrow 8(6)$

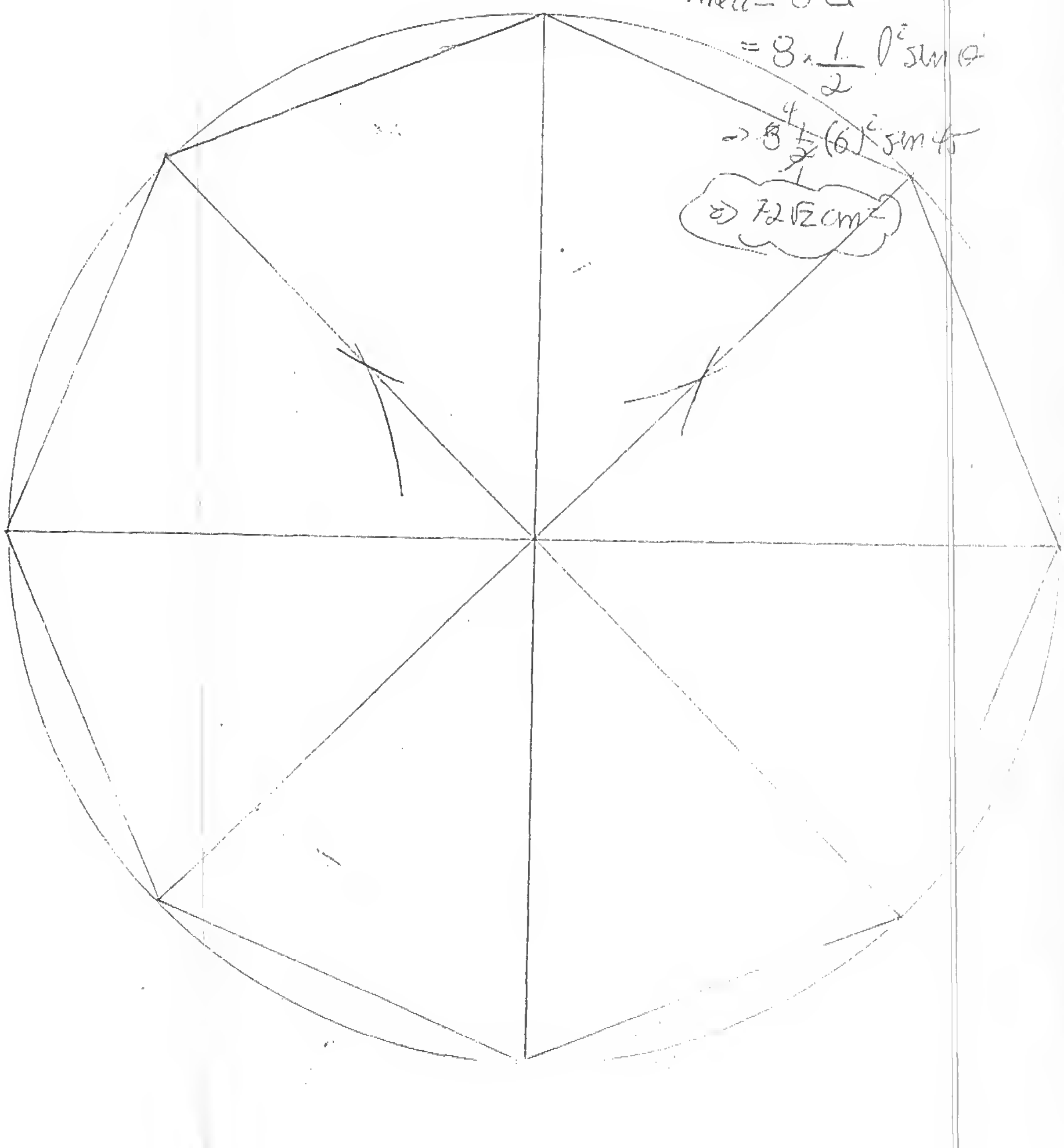
$\Rightarrow 48\ u^2$

Area $= 8\,\Delta$

$= 8 \cdot \frac{1}{2} l^2 \sin\theta$

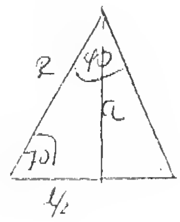
$\Rightarrow \overset{4}{\cancel{8}}\, \frac{1}{\cancel{2}} (6)^2 \sin 45$

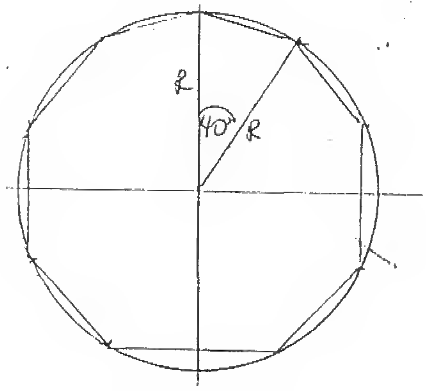
$\Rightarrow 72\sqrt{2}\ \text{cm}^2$



d)

$l = 4\,cm$





$\cos 70 = \dfrac{l}{2R}$

$R = \dfrac{l}{2\cos 70}$

$R = \dfrac{4}{2\cos 70}$

$R = 5{,}87\,cm$

$\operatorname{sen} 70 = \dfrac{a}{R}$

$a = R \operatorname{sen} 70$

$a = 5{,}5\,cm$

Perímetro $= 9 . l$

$\Rightarrow 9(4)$

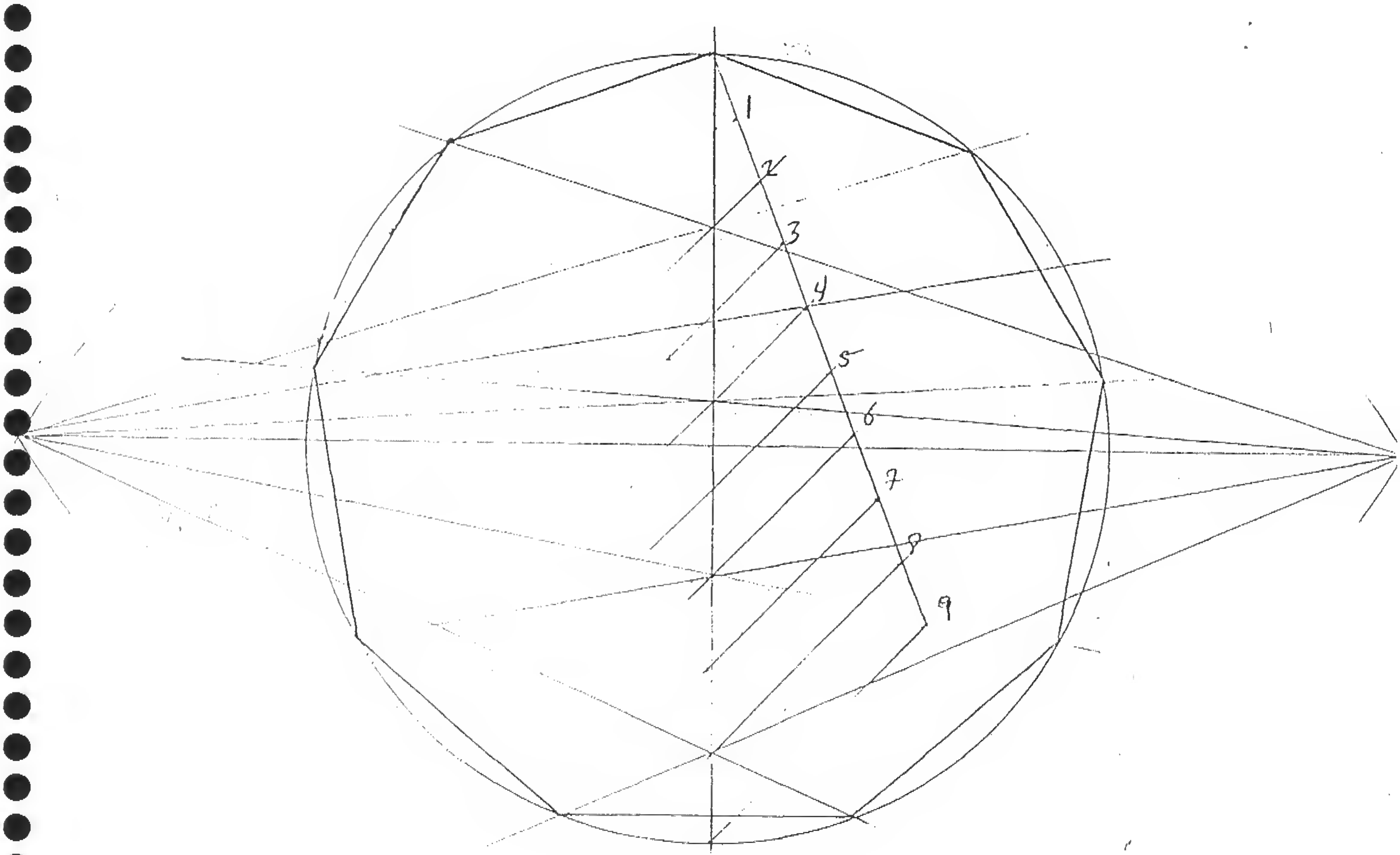
$\Rightarrow 36\,cm^2$

Área $= 9\,A_\Delta$

$\Rightarrow 9 \times \dfrac{1}{2} l^2 \operatorname{sin} \theta$

$\Rightarrow \dfrac{9}{2} (5{,}87)^2 \operatorname{sen} 40$

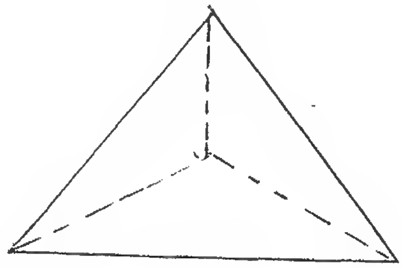
Área $= 99{,}67\,u^2$



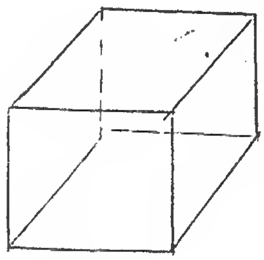
0.60

SOLUCION
EJERCICIOS PROPUESTOS
CAPITULO OCHO
MATEMATICAS
GEOMETRIA DEL ESPACIO.

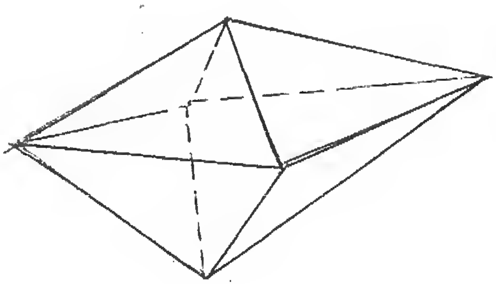
1.- TETRAEDRO REGULAR



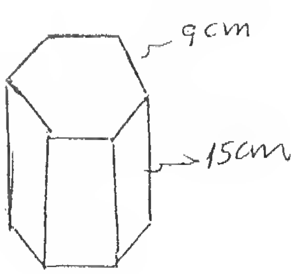
2.- HEXAEDRO REGULAR



3.- OCTAEDRO

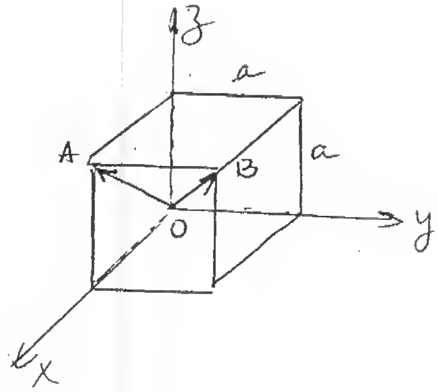


4.-



las aristas de las bases ≠ aristas laterales

5.-



$O(0,0,0)$

$A=(a,0,a)$

$B=(a,a,a)$

$\vec{A}=a\hat{\imath}+a\hat{k}$

$\vec{B}=a\hat{\imath}+a\hat{j}+a\hat{k}$

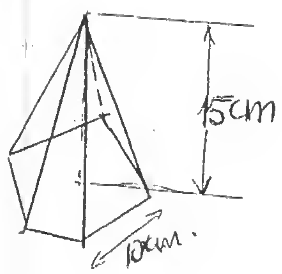
$$\theta=\cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right)$$

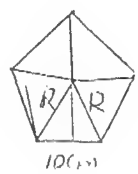
$$\theta=\cos^{-1}\frac{a^2+a^2}{\sqrt{a^2+a^2}\cdot\sqrt{a^2+a^2+a^2}}$$

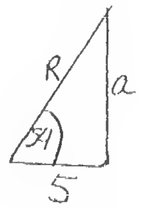
$$\theta=\cos^{-1}\frac{2a^2}{\sqrt{2a^2\times 3a^2}}\Rightarrow\cos^{-1}\frac{2\cancel{a^2}}{\sqrt{6}\,\cancel{a^2}}$$

$$\theta=\cos^{-1}\left(\frac{2}{\sqrt{6}}\right)$$

6.-



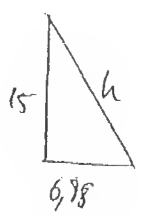




$\tan 54=\frac{a}{5}$

$a=5\cdot\tan 54$

$\{a=6{,}88\}$



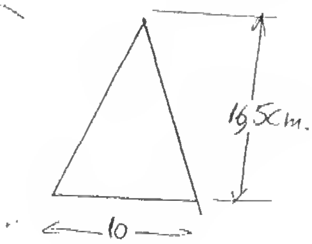
$h=\sqrt{15^2+6{,}98^2}$

$h=16{,}5\,cm$

$\cos 54=\frac{5}{R}$

$\left\{R=\frac{5}{\cos 45}\right\}$

caras laterales -



c) Octaedro ⇒ 8△ equiláteros

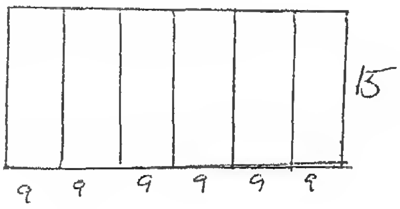
$$A_L = A_T = 8\frac{\sqrt{3}l^2}{4}$$

$$\Rightarrow 2\sqrt{3}\,l^2$$

$$\Rightarrow 2\sqrt{3}(8)^2$$

$$\Rightarrow 128\sqrt{3}\ cm^2$$

d) A_L



$$A_L = 15 \times 9(6)$$

$$A_L = 810 cm^2$$

$$\therefore A_T = A_L + 2⬡$$

$$A_T = 810 + 2(6)\triangle$$

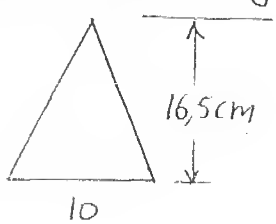
$$A_T = 810 + \cancel{12}^{3}\frac{\sqrt{3}l^2}{\cancel{4}_1}$$

$$A_T = 810 + 3\sqrt{3}(9)^2$$

$$A_T = 243\sqrt{3} + 810$$

$$A_T = 81(3\sqrt{3}+10)$$

e) Tomando datos del ejercicio #6



$$A_L = 5\triangle$$

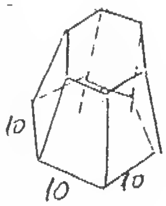
$$A_L = 5 \times \frac{10 \times 16{,}5}{2}$$

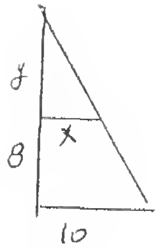
$$A_L = 412{,}5 cm^2$$

$$A_T = A_L + 5\triangle$$

$$A_T = 412{,}5 + 5 \cdot \frac{1}{2} l^2 \operatorname{sen}\theta$$

7.-





$$\frac{y+8}{10} = \frac{y}{x}$$

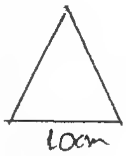
$$x(y+8) = 10y$$

$$xy + 8x = 10y$$

$$x = \frac{10y}{y+8}$$

Las aristas de las bases dependen de la inclinación. de las caras laterales.

8.-a) Alateral =>



$$\rightarrow A = \frac{\sqrt{3}\,l^2}{4}$$

$$A = \frac{\sqrt{3}\,(10)^2}{4}$$

$$A = 25\sqrt{3}$$

$$A_L = 25\sqrt{3}\,(3)$$

$$A_L = 75\sqrt{3}$$

$A_T = 4A$

$\Rightarrow 4(25\sqrt{3})$

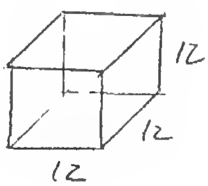
$\Rightarrow 100\sqrt{3}\,cm^2$

b) $A_L \Rightarrow$ cuadrado

$A = l^2$

$A = (12)^2$

$A = 144 cm^2$

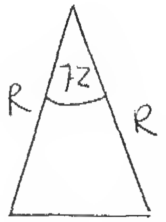


$A_L = 4(144)$

$A_L = 576 cm^2$

$A_T = 6(144)$

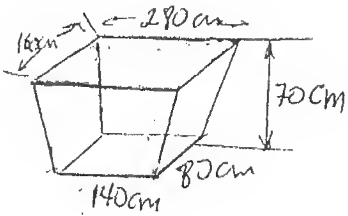
$A_T = 864 cm^2$



$$A_T = 42{,}54 + \frac{5}{2}(R^2)\,\text{sen}\,72$$

$$A_T = 161{,}38$$

9.-



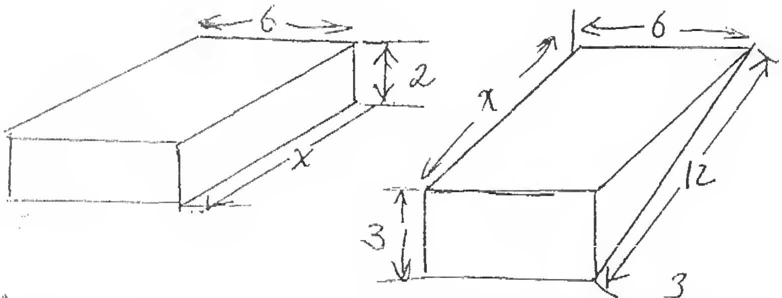
$$V_T = \frac{h}{3(A-a)}\left(A^2B - a^2b\right)$$

$$V_T = \frac{70}{3(280-140)}\left(280^2(160) - 140^2(80)\right)$$

$$V_T = \frac{\cancel{70}^{\,1}}{3(\cancel{140}_{\,2})}\left(\cancel{10976000}^{\,548800}\right)$$

$$V_T = \frac{548800}{3}$$

10.-



$$x = \sqrt{12^2 - 3^2}$$

$$x = \sqrt{144 - 9}$$

$$x = \sqrt{135}$$

$$x = 3\sqrt{15}$$

$$V_1 = 6(2)(3\sqrt{15})$$

$$V_1 = 36\sqrt{15}$$

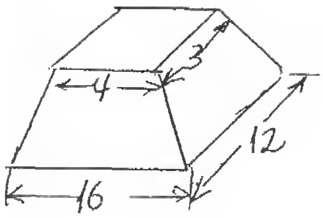
$$V_2 = \frac{1}{\cancel{2}_{\,1}}(3)\,3\sqrt{15}\,(\cancel{6}^{\,3})$$

$$V_2 = 27\sqrt{15}$$

$$V_T = V_1 + V_2$$

$$V_T = 63\sqrt{15}$$

11.-



$$V_T = \frac{h}{3(A-a)}(A^2B - a^2b)$$

$$V_T = \frac{\cancel{18}^{6}}{\cancel{3}_{1}(16-4)}(16^2(12) - 4^2(3))$$

$$V_T = \frac{\cancel{6}^{1}}{\cancel{12}_{2}}(256(12) - 16(3))$$

$$V_T = \frac{1}{2}(3024)$$

$$V_T = 1512\,u^3$$

12.- $V_C = 512\,cm^3$

$$\sqrt[3]{l^3} = \sqrt[3]{V_C}$$

$$l = \sqrt[3]{512}$$

$$l = 8\,cm$$

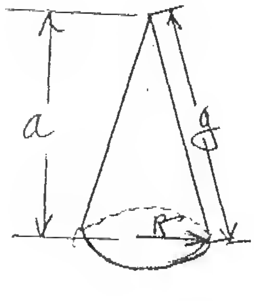
13.- $V = V_1 - V_2$

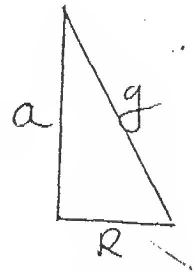
$$V = a^3 - \left(\frac{a}{2}\right)^3$$

$$V = a^3 - \frac{a^3}{8}$$

$$V = \frac{7a^3}{8}$$

14.-





$$g = \sqrt{a^2 + R^2}$$

$$g = \sqrt{8^2 + 6^2}$$

$$g = 10\ cm$$

15.- $R_1 = 4cm$

$R_2 = 7cm$

$$V_1 = \frac{4}{3}\pi(4)^3 \qquad V_2 = \frac{4}{3}\pi(7)^3$$

$$V_1 = \frac{256}{3}\pi\ cm^3 \qquad V_2 = \frac{1372}{3}\pi\ cm^3$$

$$V = V_1 + V_2$$

$$\Rightarrow \frac{256}{3}\pi + \frac{1372}{3}\pi$$

$$V = \frac{1628}{\cancel{3}}\cancel{\pi} = \frac{4}{\cancel{3}}\cancel{\pi}R^3$$

$$4R^3 = \frac{1628}{1} \Rightarrow R^3 = \frac{\cancel{1628}}{\cancel{4}}$$

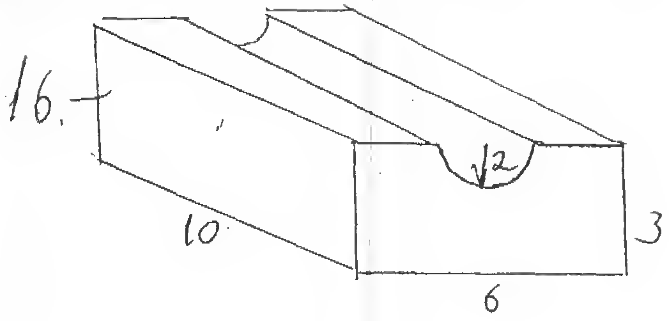
$$\cancel{814}\ 407$$

$$R^3 = 407$$

$$\sqrt[3]{R^3} = \sqrt[3]{407}$$

$$R = \sqrt[3]{407}$$

16.-



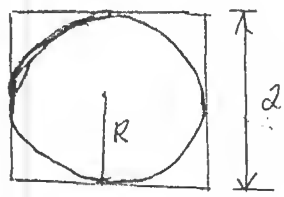
$V =$  $-$

$V = 6 \times 3 \times 10 - \frac{\pi(2)^2}{2} \times 10$

$V = 180 - 20\pi$

$V = 20(9 - \pi)$ a) CORRECTO

17.-



$2R = a$

$V_C = l^3$

$V_C = a^3$

$V_E = \frac{4}{3}\pi R^3$

$V_E = \frac{4}{3}\pi\left(\frac{a}{2}\right)^3$

$V_E = \frac{4\pi a^3}{3(8)}$

$V_E = \frac{\pi a^3}{6}$

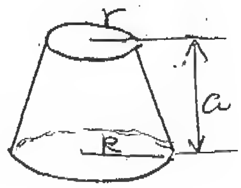
$2R = a$

$R = a/2$

18.- $\Delta V = a^3 - \frac{\pi a^3}{6}$

$\Rightarrow \frac{6a^3 - \pi a^3}{6} \Rightarrow \Delta V = \frac{a^3}{6}(6a^3 - \pi)$

19.— $\forall = \frac{a\pi}{3(R-r)}(R^3 - r^3)$



$\forall = \frac{\cancel{6}^{2}\pi}{\cancel{3}(\cancel{4-2})}(4^3 - 2^3)$

$\forall = \pi(64-8)$

$\forall = 56\pi$ b) CORRECTO

20.— $\forall = \forall_{C1} - \forall_{C2}$

$\forall = \pi R^2 \cdot l - \pi r^2 l$

$\forall = \pi l (R^2 - r^2)$

$\forall = \pi(10)(8^2 - 6^2)$

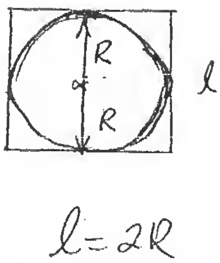
$\forall = 280\pi \, cm^3$

21.— $\forall_C = 64 cm^3$

$l^3 = 64$

$\sqrt[3]{l^3} = \sqrt[3]{64}$

$\boxed{l = 4cm}$



$l = 2R$

$R = \frac{l}{2}$

$\boxed{R = 2cm}$

$\forall_e = \frac{4}{3}\pi R^3$

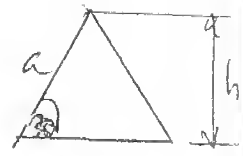
$\forall_e = \frac{4}{3}\pi(2)^3$

$\forall_e = \frac{32}{3}\pi \, cm^3$

c) CORRECTO

22.-

$$\forall_T = \forall_{\ominus} + 2\forall_{\Delta} + \forall_{\square}$$



$$\forall_T = \frac{4}{3}\pi R^3 + 2\left(\frac{1}{3}\pi R^2 h\right) + \pi R^2 a$$

$\operatorname{sen}30 = \frac{h}{a}$

$h = a\operatorname{sen}30$

$\boxed{h = \frac{a}{2}}$

$$\forall_T = \frac{4}{3}\pi a^3 + \frac{\cancel{2}}{3}\pi (a^2)\times\frac{a}{\cancel{2}} + \pi a^2 a$$

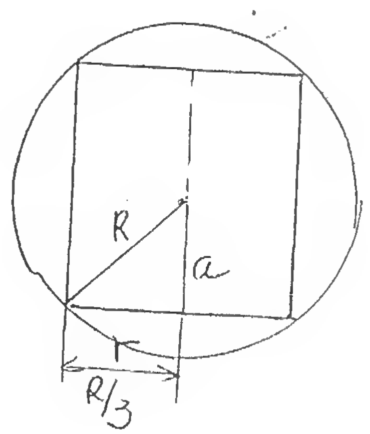
$$\forall_T = \frac{4}{3}\pi a^3 + \frac{a^3\pi}{3} + \pi a^3$$

$$\forall_T = \pi a^3\left(\frac{4}{3} + \frac{1}{3} + 1\right)$$

$$\forall_T = \pi a^3\left(\frac{5}{3} + 1\right)$$

$$\forall_T = \frac{8}{3}\pi a^3$$

23.-



$$a = \sqrt{R^2 - \left(\frac{R}{3}\right)^2}$$

$$a = \sqrt{R^2 - \frac{R^2}{9}}$$

$$a = \sqrt{\frac{9R^2 - R^2}{9}}$$

$$a = \sqrt{\frac{8R^2}{9}}$$

$$\boxed{a = \frac{2R\sqrt{2}}{3}}$$

$$\forall_c = \pi r^2 (2a)$$

altura

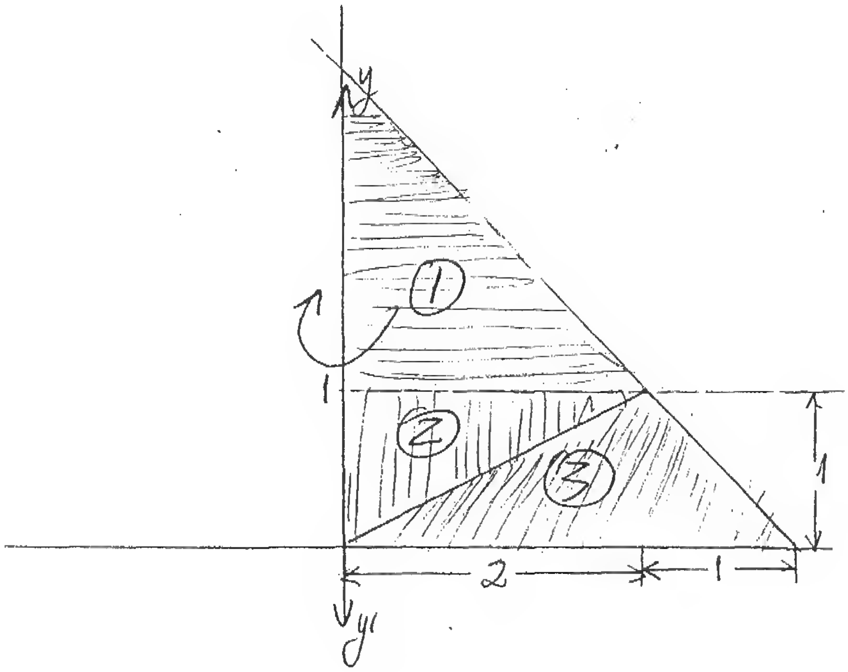
$$\forall_c = \pi\left(\frac{R}{3}\right)^2\left(2\left(\frac{2R\sqrt{2}}{3}\right)\right)$$

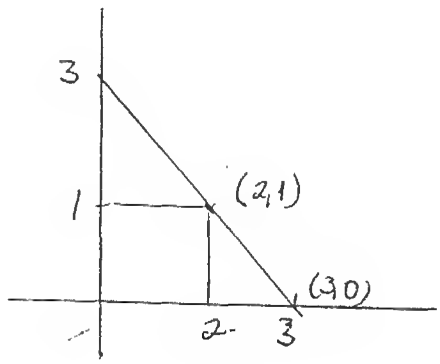
d) CORREC

$$\forall_c = \frac{R^2}{9}\pi \cdot \frac{4R\sqrt{2}}{3}$$

$$\forall_c = \frac{4\sqrt{2}\pi R^3}{27}$$

-24.-





$m = \frac{0-1}{3-2} = -1$

$y = -x + b$

$0 = -3 + b$

$\boxed{b = 3}$

$V = V_T - V_1 - V_2$

$V = \frac{1}{3}\pi R_1^2 a_1 - \frac{1}{3}\pi R_2^2 a_2 - \frac{1}{3}\pi R_3^2 a_3$

$V = \frac{\pi}{3}\left(3^2(3) - 2^2(2) - (2)^2(1)\right)$

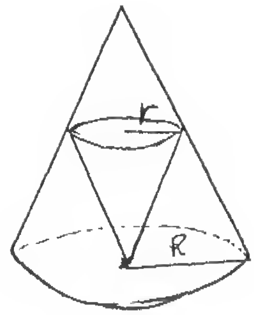
$V = \frac{\pi}{3}(27 - 8 - 4)$

$V = \frac{\cancel{15}^{5}}{\cancel{3}_{1}}\pi$

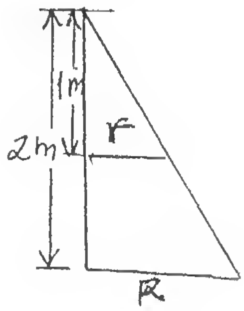
$V = 5\pi$

a) CORRECTO.

25.- $\begin{cases} \Phi = 2m \Rightarrow R = 1m \\ a = 2m \end{cases}$



$a_1 = 1m.$



$V_c = \frac{1}{3}\pi r^2 a_1$

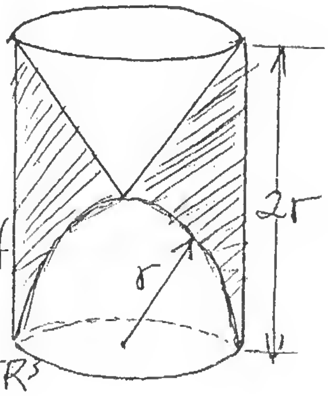
$\frac{r}{1} = \frac{R}{2}$

$V_c = \frac{1}{3}\pi\left(\frac{1}{2}\right)^2(1)$

$\boxed{r = \frac{1}{2}m}$

$V_c = \frac{\pi}{12} m^3$ b) CORRECTO

26.-



$V = V_{cil} - V_{cono} - V_{semiesf}$

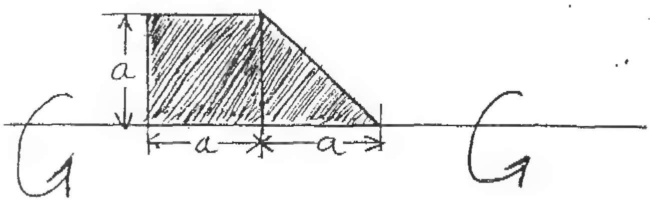
$V = \pi R^2 a - \frac{1}{3}\pi R^2 a - \frac{2}{3}\pi R^3$

$V = \pi(r)^2(2r) - \frac{1}{3}\pi r^2 \cdot r - \frac{2}{3}\pi r^3$

$V = \pi\left(2r^3 - \frac{r^3}{3} - \frac{2}{3}r^3\right)$

$V = \pi r^3$ d) CORRECTO

27.-



$V_{cilin} + V_{cono}$

$\pi R^2 a + \frac{1}{3}\pi R^2 a$

$\pi(a^2)a + \frac{1}{3}\pi a^2 a$

$a^3\pi + \frac{1}{3}a^3\pi$

$\frac{4}{3}a^3\pi$

28.- $R = 7$ pulg

$r = 5$ pulg.

$a = 8$ pulg

$V_{hueco} = V_{C1} - V_{C2}$

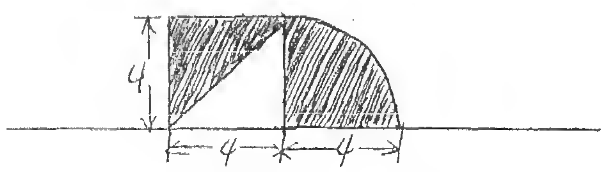
$\Rightarrow \pi R^2 a - \pi r^2 a$

$\Rightarrow \pi a(R^2 - r^2)$

$\Rightarrow \pi(8)(7^2 - 5^2)$

$\Rightarrow 192\pi$ d) CORRECTO.

29.-



$V = V_{cil} - V_{cono} + \frac{1}{2} V_{esfera}$

$V = \pi R^2 a - \frac{1}{3}\pi R^2 a + \frac{1}{\cancel{2}} \cdot \frac{\cancel{4}^{2}}{3}\pi R^3$

$V = \pi(4)^2(4) - \frac{\pi}{3}(4)^2(4) + \frac{2\pi}{3}4^3$

$V = 64\pi\left(1 - \frac{1}{3} + \frac{2}{3}\right)$

$V = \frac{256\pi}{3}$ b) CORRECTO

30.- $R = 2u.$

$V = 16\pi.$

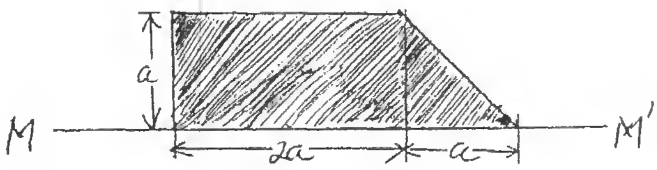
$V_{esfera} = \frac{4}{3}\pi R^3$

$\Rightarrow \frac{4}{3}\pi(2)^3$

$\Rightarrow \frac{32}{3}\pi$

b) CORRECTO

31.-



$V = V_{cil} + V_{cono}$

$\Rightarrow \pi R^2 a + \frac{1}{3}\pi R^2 a$

$\Rightarrow \pi(a)^2(2a) + \frac{1}{3}\pi a^2 a$

$\Rightarrow 2\pi a^3 + \frac{\pi}{3}a^3$

$\Rightarrow \frac{7\pi a^3}{3}$ a) CORRECTO

32.- $V = 125cm^3$

$a = 10cm$

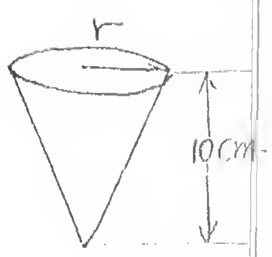
$R = ¿?$

$V = \frac{1}{3}\pi R^2 a$

$\frac{3V}{\pi a} = R^2$

$R = \sqrt{\frac{3(125)}{\pi(10)}}$

$R = 3.45cm$



33.- $V_{cil} = 400 cm^3$

$R = 7,5 cm$

$a = ?$

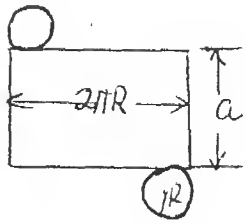
$A_L = ?$

$V_{cil} = \pi R^2 a$

$a = \frac{V_{cil}}{\pi R^2}$

$a = \frac{400}{\pi (7,5)^2}$

$a = 2,26 cm$



$A_L = 2\pi R a + 2\pi R^2$

$A_L = 2\pi (Ra + R^2)$

$A_L = 2\pi (7,5(2,26) + 7,5^2)$

$A_L = 460 cm^2$

SOLUCION EJERCICIOS

PROPUESTOS

CAPITULO NUEVE

VECTORES EN EL PLANO Y ESPACIO

1.-

a) $A(2,1)$; $B(3,4)$

$\overrightarrow{AB}=(1,3)$

$A(x_1,y_1)$

$B(x_2,y_2)$

$\overrightarrow{AB}=(x_2-x_1,\ y_2-y_1)$

b) $A:(-3,0)$; $B(10,-5)$

$\overrightarrow{AB}=(13,-5)$

c) $A:(-1,-2)$; $B(2,1)$

$\overrightarrow{AB}=(3,3)$

d) $A(0,1)$; $B(0,4)$

d) $\overrightarrow{AB}=(0,3)$

e) $A:(2,0)$; $B:(-7,0)$

$\overrightarrow{AB}=(-9,0)$

f) $A:(1,1,0)$; $B:(3,4,2)$

$\vec{AB}=(2,3,2)$

g) $A:(0,-3,4)$; $B:(1,-1,1)$

$\vec{AB}=(1,2,-3)$

h) $A:(3,2,3)$; $B:(3,1,3)$

$\vec{AB}=(0,-1,0)$

i) $A:(0,1,0)$; $B:(0,1,0)$

$\vec{AB}=(0,0,0)$ (vector nulo)

j) $A:(8,0,0)$; $B:(0,-1,2)$

$\vec{AB}=(-8,-1,2)$

2.- a) $\vec{V_1}=(2a+b-1, 3a-b)$ $\quad \vec{V_2}=(2b+3, 2)$

$2a+b-1=2b+3$ $\quad$ (2) $3a-b=2$

$2a+b-2b=3+1$ $\quad$ (1)(-1) + (2)

(1) $2a-b=4$ $\quad$ $-2a+b=-4$

$3a-b=2$

$a=-2$

$3a-b=2$

$3a-2=b$

$b=3(-2)-2$

$b=8$

b) $\vec{V}_1=(c-2b+1,\ 4a,\ c)$ $\quad V_2=(a-c,-a,1-b)$

$c-2b+1=a-c$

$1=a-c-c+2b$

(1) $a+2b-2c=1$

$4a=-a$

$4a+a=0$

$5a=0$

$a=0$ (3)

$c=1-b$

$b+c=1$ (2)

(3) en (1)

(4) $2b-2c=1$

(2)$*(+2)+$(4)

$$\begin{array}{r} +2b+2c=2 \\ 2b-2c=1 \\ \hline 4b=3 \end{array}$$

$b=3/4$

c) $\vec{V}_1=(-a-b,\ -a-b+1)$ $\quad \vec{V}_2=(2,3)$

$\boxed{-a-b=2}$

$-a-b+1=3$

$\boxed{-a-b=2}$

Son la misma ecuación

$-a=2+b$

$a=-(2+b)$

d) $\vec{V_1}=(4a+3b,\ a+2b)$; $\vec{V_2}=(3a, -b-1)$

$4a+3b=3a$ $\quad$ $a+2b=-b-1$

$4a-3a+3b=0$ $\quad$ $a+2b+b=-1$

$\boxed{a+3b=0} \longleftrightarrow \boxed{a+3b=-1}$

Es inconsistente el ejercicio

3.-

a) $\vec{V_1}=(a+b-3+c, -2a-b+2c, 3c)$

$\vec{V_2}=(b+c, 2c-3, \phi)$

No se puede igualar un vector en el espacio ($\vec{V_1}$) con un vector en el plano ($\vec{V_2}$)

b) $\vec{V_1}=(c-2b+1, 4a, c)$ $\quad$ $\vec{V_2}=(a-c, -a, 1-b)$

$1+\cancel{c}-2b=a-\cancel{c}$

$a+2b-c-c=0+1$

$\boxed{a+2b-2c=1}$

$\Downarrow$

$0+2b-2c=1$

$2(b-c)=1$

$\boxed{b-c=1/2}$ ①

$4a=-a$

$4a+a=0$

$5a=0$

$\boxed{a=0}$

$c=1-b$

② $\boxed{c+b=1}$

① + ②

$b-c=0,5$

$b+c=1$

$2b=1,5$

$b=1,5/2$

$\boxed{b=0,75}$

$c+b=1$

$c=1-b$

$c=1-0,75$

$\boxed{c=0,25}$

c) $\vec{V}_1 = (10a+2b-c,\ -a-b,\ 1)$ $\quad\quad$ $\vec{V}_2 = (b-2c-1,\ 3,\ b)$

$10a+2b-c = b-2c-1$ $\quad\quad$ $-a-b=3$ $\quad\quad$ $1=b$

$10a+2b-b-c+2c=-1$ $\quad\quad\quad\quad$ $\Leftarrow$ $b=1$

$10a+b+c=-1$

$-a=3+b$

$a=-(3+b)$

$a=-4$

$c=-1-10a-b$

$c=-1-10(-4)-1$

$c=-2+40$

$c=38$

d) $\vec{V} = (ab,\ ac,\ bc)$ $\quad\quad$ $\vec{V}_2 = (2,\ -b-c)$

$ab=2$ $\quad\quad$ $ac=-b$ $\quad\quad$ $bc=-c$

$a=\frac{2}{b}$ $\quad\quad$ $ac=-1$ $\quad\quad$ $bc+c=0$

$a=-2$ $\quad\quad$ $c=-\frac{1}{a}$ $\quad\quad$ $c(b+1)=0$

$c\neq 0 \therefore b+1=0$

$b=-1$

$c=-\frac{1}{(-2)}$

$c=1/2$

4.- $\vec{A}=(2,1,1)$

$\vec{B}=(-3,5,1)$

$\vec{C}=(2,-1,0)$

$\vec{D}\ (-5,6,4)$

a) $2\vec{A}=(4,2,2)$

$-3\vec{B}=(9,-15,-3)$

$2(\vec{C}+\vec{D})=(-6,10,8)$

$\vec{R}=(7,-3,7)$

b) $\vec{A} = (2, 1, 1)$

$2\vec{C} = (4, -2, 0)$

$-2(\vec{C}+\vec{A}) = (-8, 0, -2)$

$-\vec{B} = (3, -5, -1)$

$\vec{R} = (1, -6, -2)$

c) $A-D = (7, -5, -3)$

$-2(A-D) = (-14, 10, 6)$

$B-2(A-D) = (-17, 15, 7)$

$3(B-2(A-D)) = (-51, 45, 21)$

$3C = (6, -3, 0)$

$3(B-2(A-D)) + 3C \Rightarrow (-51, 45, 21) + (6, -3, 0)$

$\Rightarrow (-45, 42, 21)$

d) $3A = (6, 3, 3)$

$3A - C = (4, 4, 0)$

$\vec{X}+\vec{D} = (x_1 - 5;\ x_2 + 6;\ x_3 + 4)$

$4(X+B) = 4(x_1 - 3, x_2 + 5, x_3 + 1)$

$\Rightarrow (4x_1 - 12;\ 4x_2 + 20;\ 4x_3 + 4)$

$(1, -6, 0)$

$3A - C = (4, 4, 0)$

$+\ 4(X+B) = (4x_1 - 12;\ 4x_2 + 20;\ 4x_3 + 4)$

$3A - C + 4(X+B) = (4x_1 - 8;\ 4x_2 + 24;\ 4x_3 + 4) = (x_1 - 5;\ x_2 + 6;\ x_3 + 4)$

$4x_1 - 8 = x_1 - 5$ $\qquad$ $4x_2 + 24 = x_2 + 6$ $\qquad$ $4x_3 + \cancel{4} = x_3 + \cancel{4}$

$4x_1 - x_1 = -5 + 8$ $\qquad$ $4x_2 - x_2 = 6 - 24$ $\qquad$ $4x_3 - x_3 = 0$

$3x_1 = 3$ $\qquad$ $3x_2 = -18$ $\qquad$ $3x_3 = 0$

$x_1 = 1$ $\qquad$ $x_2 =$ [illegible] $\qquad$ $x_3 =$ [illegible]

e) $y+A=(y_1+2;\ y_2+1;\ y_3+1)$

$2y-C=(2y_1-2;\ 2y_2+1;\ 2y_3)$

$2(B+y) \Rightarrow 2(y_1-3;\ y_2+5;\ y_3+1)$

$2y-C+D=(2y_1-7;\ 2y_2+7;\ 2y_3+4)$

$\Rightarrow (2y_1-6;\ 2y_2+10;\ 2y_3+2)$

$3(2y-C+D)=(6y_1-21;\ 6y_2+21;\ 6y_3+12)$

$$\begin{array}{rl} & y+A = (y_1+2;\ y_2+1;\ y_3+1) \\ - & 2(B+y) = (2y_1-6;\ 2y_2+10;\ 2y_3+2) \\ \hline & y+A-2(B+y) = (-y_1+8;\ -y_2-9;\ -y_3-1) \end{array}$$

$$(-y_1+8;\ -y_2-9;\ -y_3-1) = (6y_1-21;\ 6y_2+21;\ 6y_3+12)$$

$-y_1+8=6y_1-21$

$-y_1-6y_1=-21-8$

$-7y_1=-29$

$y_1=\frac{29}{7}$

$-y_2-9=6y_2+21$

$-y_2-6y_2=21+9$

$-7y_2=30$

$y_2=-\frac{30}{7}$

$-y_3-1=6y_3+12$

$-y_3-6y_3=12+1$

$-7y_3=13$

$y_3=-\frac{13}{7}$

$$y=\left(\frac{29}{7},\ -\frac{30}{7},\ -\frac{13}{7}\right)$$

5.- $\underbrace{a(2,1,1)\ ;\ b(-3,5,1)\ ;\ c(2,-1,0)\ ;\ d(-5,6,4)}_{(2,-10,7)}$

$$\begin{aligned} 2a-3b+2c-5d&=2 \\ a+5b-c+6d&=-10 \\ a+b+4d&=7 \end{aligned} \Rightarrow \begin{array}{c} \\ (-2) \\ (-2) \end{array} \left(\begin{array}{cccc|c} 2 & -3 & 2 & -5 & 2 \\ 1 & 5 & -1 & 6 & -10 \\ 1 & 1 & 0 & 4 & 7 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 2 & -3 & 2 & -5 & 2 \\ 0 & -13 & 4 & -17 & 22 \\ 0 & -5 & 2 & -13 & -12 \end{array}\right)$$

Si falta una ecuación el sistema tendrá infinitas soluciones, si es combinación lineal.

6.- $\vec{V}=(2x, 3, y+1)$ $\quad \vec{W}=(x^2+1, 3, 2y)$

$$\vec{V}=\vec{W}$$

$$(2x, 3, y+1)=(x^2+1, 3, 2y)$$

$x^2+1=2x$ $\quad 3=3$ $\quad y+1=2y$

$x^2-2x+1=0$ $\quad 1=2y-y$

$(x-1)^2=0$ $\quad y=1$

$x=1$

$\vec{V}=(2, 3, 2)$ $\quad \vec{W}=(2, 3, 2)$

$$\vec{V}+\vec{W}=(4, 6, 4)$$

$$\Rightarrow 2(2, 3, 2)$$

a) CORRECTO

7.- $\vec{V}_1=(-3, 2, 4)$

$\vec{V}_2=(3/2, -1, -2)$

Si $\vec{V}_1 \parallel \vec{V}_2$

$\cancel{\theta}=0°$

$$\theta=\cos^{-1}\frac{\vec{V}_1 \cdot \vec{V}_2}{V_1 \times V_2}$$

$$\theta=\cos^{-1}\frac{-\frac{9}{2}-2-8}{\sqrt{9+4+16}\times\sqrt{\frac{9}{4}+1+4}}$$

$$\theta=\cos^{-1}\frac{-\frac{29}{2}}{\sqrt{29}\sqrt{\frac{29}{4}}} \Rightarrow \theta=\cos^{-1}\frac{-\frac{\cancel{29}}{\cancel{2}}}{\frac{\cancel{29}}{\cancel{2}}}$$

$$\theta=\cos^{-1}(-1) \Rightarrow \theta=180°$$

b) CORRECTO.

8-a) $\vec{P} \parallel Q$

$\vec{P}\cdot\vec{Q} = PQ$

$\vec{P} = (K, 2, K)$ $\qquad$ $\vec{P}\cdot\vec{Q} = 2K - 2 + 2K \Rightarrow 4K - 2$

$\vec{Q} = (2, -1, 2)$ $\qquad$ $P = \sqrt{K^2+4+K^2} \Rightarrow \sqrt{2K^2+4}$

$Q = \sqrt{2^2+1^2+2^2} = 3$

$\vec{P}\cdot\vec{Q} = PQ$

$4K - 2 = \sqrt{2K^2+4} \times 3$

$(4K-2)^2 = (3\sqrt{2K^2+4})^2$

$16K^2 - 16K + 4 = 9(2K^2+4)$

$16K^2 - 16K + 4 = 18K^2 + 36$

$18K^2 - 16K^2 + 16K + 36 - 4 = 0$

$2K^2 + 16K - 32 = 0$

$2(K^2 + 8K - 16) = 0$

$K^2 + 8K - 16 = 0$

$K = \dfrac{-8 \pm \sqrt{8^2 - 4(1)(-16)}}{2(1)} \Rightarrow \dfrac{-8 \pm \sqrt{64+64}}{2}$

$K = \dfrac{-\cancel{8}^{4} \pm \cancel{8}^{4}\sqrt{2}}{\cancel{2}} \Rightarrow -4 \pm 4\sqrt{2}$

b) $\vec{A} \parallel \vec{B}$

$\vec{A}\cdot\vec{B} = AB$

$\vec{A} = (K, 1, -1)$ $\qquad$ $\vec{A}\cdot\vec{B} = 3K + 1 - 4$ $\qquad$ $|\vec{A}| = \sqrt{K^2+1+1} = \sqrt{K^2+2}$

$\vec{B} = (3, 1, 4)$ $\qquad$ $\Rightarrow 3K - 3$ $\qquad$ $B = \sqrt{3^2+1+16} = \sqrt{26}$

$\vec{A}\cdot\vec{B} = AB$

$(3K-3)^2 = (\sqrt{26}\sqrt{K^2+2})^2$

$9K^2 - 18K + 9 = 26(K^2+2)$

$9K^2 - 18K + 9 = 26K^2 + 52$

$$26K^2 - 9K^2 + 19K + 52 - 9 = 0$$

$$17K^2 + 19K + 43 = 0$$

$$K = \frac{-19 \pm \sqrt{19^2 - 4(17)(43)}}{2(17)}$$

NO HAY SOLUCION.

9.- $\vec{A} = (1, 1, 5)$; $\vec{B} = (3, 2, -3)$

a)

$$\theta = \cos^{-1} \frac{\vec{A} \cdot \vec{B}}{AB}$$

$$\theta = \cos^{-1} \frac{3 + 2 - 15}{\sqrt{1 + 1 + 25}\ \sqrt{9 + 4 + 9}} \Rightarrow \cos^{-1} \frac{-10}{\sqrt{27}\sqrt{22}}$$

$$\theta = \cos^{-1} \frac{-10}{\sqrt{27 \times 22}} \Rightarrow \cos^{-1} \frac{-10}{3\sqrt{66}}$$

$$\theta = 114{,}22^\circ$$

b) $\vec{C} = (2, 10, -2)$; $\vec{D} = (2, 2, 1)$

$$\theta = \cos^{-1} \frac{4 + 20 - 2}{\sqrt{4 + 100 + 4}\ \sqrt{4 + 4 + 1}} \Rightarrow \cos^{-1} \frac{22}{\sqrt{108}\ \sqrt{9}}$$

$$\theta = \cos^{-1} \frac{22}{3\sqrt{108}}$$

$$\theta = 45{,}11^\circ$$

c) $\vec{E}=(-2,-10,2)$ $\vec{F}=(2,2,1)$

$$\theta=\cos^{-1}\frac{-4-20+2}{\sqrt{4+100+4}\sqrt{4+4+1}}$$

$$\theta=\cos^{-1}\frac{-22}{\sqrt{108}\sqrt{9}}$$

$$\theta=\cos^{-1}\frac{-22}{3\sqrt{108}}$$

$$\theta=134{,}88^{\circ}$$

d) $\vec{P}=(0,0,1)$

$\vec{Q}=(1,0,1)$

$$\theta=\cos^{-1}\frac{0+0+1}{\sqrt{1}\sqrt{1+1}}$$

$$\theta=\cos^{-1}\frac{1}{\sqrt{2}}$$

$$\theta=45^{\circ}$$

e) $\vec{R}=(1,0,0)$

$\vec{S}=(0,1,0)$

$$\theta=\cos^{-1}\frac{0+0+0}{\sqrt{1}\sqrt{1}}$$

$$\theta=\cos^{-1}0$$

$$\theta=90^{\circ}$$

10.- $\vec{V}_1 = (10, 2, -1)$

$\vec{V}_2 = (2, -8, 4)$

$\vec{V}_1 \perp \vec{V}_2 \Rightarrow \vec{V}_1 \cdot \vec{V}_2 = 0$

$\Rightarrow 20 - 16 - 4$

$\Rightarrow 0$

a) CORRECTO

11.- a) $\vec{A} = (4, 2k, 5)$

$\vec{B} = (3k, -10, 1)$ $\quad \vec{A} \cdot \vec{B} = 0$

$12k - 20k + 5 = 0$

$-8k = -5$

$k = 5/8$

b.) $\vec{P} = (5k+1, k-1, 0)$

$\vec{Q} = (1, k, 4)$

$\vec{P} \cdot \vec{Q} = 0$

$5k+1 + k(k-1) + 0 = 0$

$5k + 1 + k^2 - k = 0$

$k^2 + 4k + 1 = 0$

$k = \dfrac{-4 \pm \sqrt{4^2 - 4(1)(1)}}{2(1)} \Rightarrow \dfrac{-4 \pm \sqrt{12}}{2}$

$k = \dfrac{-\cancel{4}^{2} \pm \cancel{2}^{1}\sqrt{3}}{\cancel{2}} \Rightarrow k = -2 \pm \sqrt{3}$

12 – $\vec{V}=(4,2,t)$

$\vec{w}=(-1,1,2)$ $\quad \vec{V}\perp\vec{w}$

$\vec{V}\cdot\vec{w}=0$

$-4+2+2t=0$

$2t=2$

$t=1$ $\quad$ b) CORRECTO

13 – $\vec{V_1}=(1,2,x)$

$\vec{V_2}=(3,-1,-2)$

$\vec{V_1}\perp\vec{V_2}$

$\vec{V_1}\cdot\vec{V_2}=0$

$3-2-2x=0$

$-2x=-1$

$x=1/2$ $\quad$ a) CORRECTO

14 – $\vec{V_1}=(V_{1x},V_{1y},V_{1z})$

$\vec{V_2}=(V_{2x},V_{2y},V_{2z})$ $\quad \vec{V_1}\perp\vec{V_2}$

$\vec{V_1}\cdot\vec{V_2}=0$

$V_{1x}V_{2x}+V_{1y}+V_{2y}+V_{1z}V_{2z}=0$

$\vec{U}=\vec{V_1}+\lambda\vec{V_2}$ $\qquad \vec{W}=\vec{V_1}-\vec{V_2}$

$\vec{U}=(V_{1x}+V_{1y},V_{1z})+(\lambda V_{2x},\lambda V_{2y},\lambda V_{2z})$

$\vec{U}=(V_{1x}+\lambda V_{2x},\ V_{1y}+\lambda V_{2y},\ V_{1z}+\lambda V_{2z})$

$\vec{W}=(V_{1x}-V_{2x},\ V_{1y}-V_{2y},\ V_{1z}-V_{2z})$

$$U \perp W$$

$$\vec{U} \cdot \vec{W} = 0$$

$$(V_{1x} + \lambda V_{2x})(V_{1x} - V_{2x}) + (V_{1y} + \lambda V_{2y})(V_{1y} - V_{2y}) + (V_{1z} + \lambda V_{2z})(V_{1z} - V_{2z}) = 0$$

$$V_{1x}^2 - V_{1x}V_{2x} + \lambda V_{1x}V_{2x} - \lambda V_{2x}^2 + V_{1y}^2 - V_{1y}V_{2y} + \lambda V_{1y}V_{2y} - \lambda V_{2y}^2 + V_{1z}^2 - V_{1z}V_{2z} + \lambda V_{1z}V_{2z} - \lambda V_{2z}^2 = 0$$

$$V_{1x}^2 + V_{1y}^2 + V_{1z}^2 - \lambda V_{2x}^2 + \lambda V_{2y}^2 - \lambda V_{2z}^2 - \underbrace{(V_{1x}V_{2x} + V_{1y}V_{2y} + V_{1z}V_{2z})}_{0} + \lambda \underbrace{(V_{1x}V_{2x} + V_{1y}V_{2y} + V_{1z}V_{2z})}_{0} = 0$$

$$V_{1x}^2 + V_{1y}^2 + V_{1z}^2 - \lambda(V_{2x}^2 + V_{2y}^2 + V_{2z}^2) = 0$$

$$-\lambda\underbrace{(V_{2x}^2 + V_{2y}^2 + V_{2z}^2)}_{V_2^2} = -\underbrace{(V_{1x}^2 + V_{1y}^2 + V_{1z}^2)}_{V_1^2}$$

$$\lambda = \frac{V_1^2}{V_2^2} \Rightarrow \lambda = \left(\frac{V_1}{V_2}\right)^2$$

15.— $\vec{X} = -\vec{y}$

$\vec{X} + \vec{y} = 0 \qquad X + y \Rightarrow 2X$

$|\vec{x} + \vec{y}| \neq |\vec{x}| + |\vec{y}|$

b) CORRECTO

16.— $K\vec{X} \Rightarrow \vec{X} = (X_1K, X_2K, X_3K)$

$|\vec{X}| = \sqrt{X_1^2K^2 + X_2^2K^2 + X_3^2K^2}$

$|\vec{X}| = \sqrt{K^2(X_1^2 + X_2^2 + X_3^2)}$

$\Rightarrow K\sqrt{X_1^2 + X_2^2 + X_3^2}$

$\Rightarrow K|\vec{X}|$

a) CORRECTO

17.– $\vec{X} = (X_1, X_2, X_3)$

$X = \sqrt{X_1^2 + X_2^2 + X_3^2}$

$X \neq \vec{X_1} \cdot \vec{X_1}$

$\vec{X} \cdot \vec{X} = X_1^2 + X_2^2 + X_3^2$

b) CORRECTO

18.– $\vec{X} + \vec{y} = (X_1 + y_1, X_2 + y_2, X_3 + y_3)$

$\vec{X} - \vec{y} = (X_1 - y_1, X_2 - y_2, X_3 - y_3)$

$|\vec{X} + \vec{y}|^2 + |\vec{X} - \vec{y}|^2 = (X_1 + y_1)^2 + (X_2 + y_2)^2 + (X_3 + y_3)^2 + (X_1 - y_1)^2 + (X_2 - y_2)^2 + (X_3 - y_3)^2$

$4 \vec{X} \cdot \vec{y} = 4(X_1 y_1 + X_2 y_2 + X_3 y_3)$

$|\vec{X} + \vec{y}|^2 + |\vec{X} - \vec{y}|^2 \neq 4 \vec{X} \cdot \vec{y}$

b) CORRECTO

19.– a) $\vec{V} = (1, 1)$

$V = \sqrt{1+1} \Rightarrow \sqrt{2}$

$\vec{\mu}_V = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \Rightarrow \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

b) $\vec{V} = (-1, 3)$

$V = \sqrt{1 + 3^2} = \sqrt{10}$

$\vec{\mu}_V = \left(-\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right) \Rightarrow \left(-\frac{\sqrt{10}}{10}, \frac{3\sqrt{10}}{10}\right)$

c) $\vec{v}=(10,0)$

$v=\sqrt{10^2}=10$ $\quad \vec{u}_v=(1,0)$

d) $\vec{v}=(0,-2)$

$v=\sqrt{0+2^2}=2$

$\vec{u}_v=(0,-1)$

e) $\vec{v}=(2,0,1)$

$v=\sqrt{2^2+1^2}=\sqrt{5}$

$$\vec{u}_v=\left(\frac{2}{\sqrt{5}},0,\frac{1}{\sqrt{5}}\right)\Rightarrow\left(\frac{2\sqrt{5}}{5},0,\frac{\sqrt{5}}{5}\right)$$

f) $\vec{v}=(0,-10,1)$

$v=\sqrt{10^2+1^2}\Rightarrow\sqrt{101}$

$$\vec{u}_v=\left(0,\frac{-10}{\sqrt{101}},\frac{1}{\sqrt{101}}\right)\Rightarrow\left(0,\frac{-10\sqrt{101}}{101},\frac{\sqrt{101}}{101}\right)$$

g) $\vec{v}=(0,0,0)\Rightarrow$ NO HAY VECTOR

h) $\vec{v}=(0,-2,-2)$

$|\vec{v}|=\sqrt{2^2+2^2}=2\sqrt{2}$

$$\vec{u}_v=\left(0,\frac{-\cancel{2}^{1}}{\cancel{2}\sqrt{2}},\frac{-\cancel{2}^{1}}{\cancel{2}\sqrt{2}}\right)\Rightarrow\left(0,-\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$$

20.– a) $\vec{U}=(2,3)$ $\qquad$ $\vec{V}=(5,-1)$

$V=\sqrt{5^2+1}$

$$P_{U/V}=\frac{\vec{U}\cdot\vec{V}}{V} \Rightarrow \frac{10-3}{\sqrt{25+1}} \Rightarrow \frac{7}{\sqrt{26}} \Rightarrow \frac{7\sqrt{26}}{26}$$

$$\vec{P}_{U/V}=P_{U/V} * \frac{\vec{V}}{V} \Rightarrow \frac{7\cancel{\sqrt{26}}}{26} \cdot \frac{(5,-1)}{\cancel{\sqrt{26}}}$$

$$\Rightarrow \left(\frac{35}{26}, -\frac{7}{26}\right)$$

b) $\vec{U}=(-3,10)$ $\qquad$ $\vec{V}(2,0)$

$V=\sqrt{2^2+0}$

$\boxed{V=2}$

$$P_{U/V}=\frac{\vec{U}\cdot\vec{V}}{V} \Rightarrow \frac{-6+0}{2} \Rightarrow 3$$

$$\vec{P}_{U/V}=P_{U/V} \cdot \frac{\vec{V}}{V} \Rightarrow 3\,\frac{(2,0)}{2} \Rightarrow (3,0)$$

c) $\vec{U}=(1,8)$ $\qquad$ $\vec{V}=(0,3)$

$V=\sqrt{0+3^2}$

$\boxed{V=3}$

$$P_{U/V}=\frac{\vec{U}\cdot\vec{V}}{V} \Rightarrow \frac{0+24}{3} \Rightarrow 8$$

$$\vec{P}_{U/V}=P_{U/V} \cdot \frac{\vec{V}}{V} \Rightarrow 8\,\frac{(0,3)}{3}$$

$$\Rightarrow (0,8)$$

d) $\vec{U} = (3,1,3)$; $\vec{V} = (0,5,-1)$ $V = \sqrt{5^2+1}$

$V = \sqrt{26}$

$$P_{U/V} = \frac{\vec{U} \cdot \vec{V}}{V} \Rightarrow \frac{0+5-3}{\sqrt{26}}$$

$$\Rightarrow \frac{2}{\sqrt{26}}$$

$$\vec{P}_{U/V} = P_{U/V} \times \frac{\vec{V}}{V} \Rightarrow \frac{2}{\sqrt{26}} \times \frac{(0,5,-1)}{\sqrt{26}}$$

$$\Rightarrow \frac{2}{26}(0,5,-1)$$

$$\Rightarrow \left(0, \frac{5}{13}, -\frac{1}{13}\right)$$

e) $\vec{U} = (1,-5,2)$; $\vec{V} = (2,0,4)$; $V = \sqrt{2^2+4^2}$

$\boxed{V = \sqrt{20}}$

$$P_{U/V} = \frac{\vec{U} \cdot \vec{V}}{V} \Rightarrow \frac{2+0+8}{\sqrt{20}}$$

$$\Rightarrow \frac{10}{\sqrt{20}} \Rightarrow \frac{10\sqrt{20}}{20}$$

$$\vec{P}_{U/V} = P_{U/V} \times \frac{\vec{V}}{V} \Rightarrow \frac{10}{\sqrt{20}} \frac{(2,0,4)}{\sqrt{20}} \Rightarrow \frac{10(2,0,4)}{20}$$

$$\Rightarrow (1,0,2)$$

f) $\vec{U} = (7,1,3)$; $\vec{V} = (4,-3,2)$ $V = \sqrt{4^2+3^2+2^2}$

$V = \sqrt{29}$

$$P_{U/V} = \frac{\vec{U} \cdot \vec{V}}{V} \Rightarrow \frac{28-3+6}{\sqrt{29}} \Rightarrow \frac{31}{\sqrt{29}}$$

$$\vec{P}_{U/V} = P_{U/V} \times \frac{\vec{V}}{V} \Rightarrow \frac{31}{\sqrt{29}} \times \frac{(4,-3,2)}{\sqrt{29}} \Rightarrow \frac{31}{29}(4,-3,2)$$

21.- $\vec{U} = (1,0)$
$\vec{V} = (-1,0)$

$\underline{\underline{\vec{U}+\vec{V}=0}}$

$|\vec{U}+\vec{V}| \neq 2$.

b) CORRECTO

22.- $\vec{U}=(1,0)$
$\vec{V}=(0,1)$

$\vec{U}\cdot\vec{V}=0$
entonces $\vec{U}\perp\vec{V}$

$\vec{U}+\vec{V}=(1,1)$

$|\vec{U}+\vec{V}|=\sqrt{1+1}$
$=\sqrt{2}$

a) CORRECTO

23.- $\vec{V}=(4,2,t)$
$\vec{W}=(-1,1,2) \Rightarrow W=\sqrt{1+1+4} \Rightarrow \sqrt{6}$

$$P_{V/W}=\frac{\vec{V}\cdot\vec{W}}{W} \Rightarrow \frac{-4+2+2t}{\sqrt{6}}$$

$$\Rightarrow \frac{-2+2t}{\sqrt{6}}=\sqrt{6}$$

$$-2+2t=\sqrt{6}\cdot\sqrt{6}$$

$$2t=6+2$$

$$t=\frac{8}{4}$$

$$t=4$$

a) VERDADERO

24.- $\vec{U}=(1,0)$ > son perpendiculares
$\vec{V}=(0,1)$

$\vec{U}-\vec{V}=(1,-1)$

$|\vec{U}-\vec{V}|=\sqrt{1+1}$

$\Rightarrow \sqrt{2}$

c) CORRECTO

25.- $\vec{V_1}=\hat{i}+4\hat{j}-2\hat{k}$

$\vec{V_2}=\hat{i}-\hat{j}+3\hat{k} \Rightarrow V_2=\sqrt{1+1+9} \Rightarrow \sqrt{11}$

$$P_{V_1/V_2}=\frac{\vec{V_1}\cdot\vec{V_2}}{V_2} \Rightarrow \frac{1-4-6}{\sqrt{11}} \Rightarrow \frac{9}{\sqrt{11}}\times\frac{\sqrt{11}}{\sqrt{11}}$$

$$\Rightarrow \frac{9\sqrt{11}}{11}$$

26.- a) $\vec{A}=(2,3,1)$
$\vec{B}=(-3,1,0)$

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 1\\ -3 & 1 & 0\end{vmatrix}=(0-1)\hat{i}-(0+3)\hat{j}+(2+9)$$

$$\vec{A}\times\vec{B}=-\hat{i}-3\hat{j}+11\hat{k} \Rightarrow (-1,-3,11)$$

b) $\vec{A}=(2,1,-1)$
$\vec{B}=(-2,2,1)$

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -1\\ -2 & 2 & 1\end{vmatrix}=(1+2)\hat{i}-(2-2)\hat{j}+(4+2)\hat{k}$$

$$\vec{A}\times\vec{B}=(3,0,6)$$

c) $\vec{E}=(2,10,-2)$; $\vec{F}=(1,5,-1)$

$$\vec{E}\times\vec{F}=\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 10 & -2 \\ 1 & 5 & -1 \end{vmatrix}=(-10+10)\hat{\imath}-(-2+2)\hat{\jmath}+(10-10)\hat{k}$$

$\vec{E}\times\vec{F}=(0,0,0) \Rightarrow$ son paralelos.

d) $\vec{P}=(0,0,1)$ $\vec{Q}(1,0,1)$.

$$\vec{P}\times\vec{Q}=\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{vmatrix}=(0-0)\hat{\imath}-(0-1)\hat{\jmath}+(0-0)\hat{k}$$

$\vec{P}\times\vec{Q}=(0,1,0)$

e) $\vec{R}=(1,0,0)$; $\vec{S}=(0,1,0)$

$$\vec{R}\times\vec{S}=\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix}=(0-0)\hat{\imath}-(0-0)\hat{\jmath}+(1-0)\hat{k}.$$

$\vec{R}\times\vec{S}=(0,0,1)$

27.- Por definición

$$|\vec{A}\times\vec{B}|=AB\sin\theta$$

a) CORRECTO

28.- $\vec{V_1}\cdot(\vec{V_2}\times\vec{V_3})=0$

Forman un plano y no un cuerpo.

$29.-\ \vec{A} = \hat{i} + 2\hat{j} + 3\hat{k} \Rightarrow A = \sqrt{1+4+9} \Rightarrow \sqrt{14}$

$\vec{B} = 3\hat{i} + 2\hat{j} + \hat{k} \Rightarrow B = \sqrt{9+4+1} \Rightarrow \sqrt{14}$

$\vec{C} = \hat{i} + 4\hat{j} + \hat{k} \Rightarrow C = \sqrt{1+16+1} \Rightarrow \sqrt{18}$

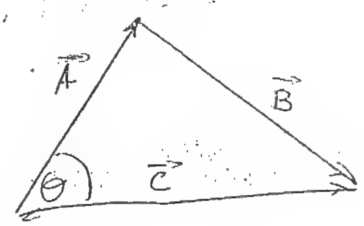
a) Si $\vec{A} \perp \vec{C}$ entonces $\vec{A} \cdot \vec{C} = 0$

$\vec{A} \cdot \vec{C} = 1 + 8 + 3 \neq 0$

$12 \neq 0$; FALSO

b)

$\vec{A} =$



$\theta = \cos^{-1} \dfrac{\vec{A} \cdot \vec{C}}{AC} \Rightarrow \cos^{-1} \dfrac{1+8+3}{\sqrt{14} \times \sqrt{18}} \Rightarrow \cos^{-1} \dfrac{12}{\sqrt{18 \times 14}}$

$\theta = 40{,}89^\circ$

el ángulo no es 60° o 120°; por lo tanto NO FORMAN UN Δ EQUILATERO

c)

$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = (2-6)\hat{i} - (1-9)\hat{j} + (2-6)\hat{k}$

$\vec{A} \times \vec{B} = -4\hat{i} + 8\hat{j} - 4\hat{k}$

$\vec{C} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 1 \\ 3 & 2 & 1 \end{vmatrix} = (4-2)\hat{i} - (1-3)\hat{j} + (2-12)\hat{k}$

$\vec{C} \times \vec{B} = 2\hat{i} + 2\hat{j} - 10\hat{k}$

$(\vec{A} \times \vec{B}) \cdot (\vec{C} \times \vec{B}) = -8 + 16 + 40$

$\neq 0$; FALSO

d) $(\hat{i}+2\hat{j}+3\hat{k})\cdot(-2\hat{i}-2\hat{j}+10\hat{k})$

$-2-4+30 \neq 1$; FALSO

e) $\theta = \cos^{-1} \dfrac{\vec{A}\cdot\vec{B}}{AB}$

$\theta = \cos^{-1} \dfrac{3+4+3}{\sqrt{14}\sqrt{14}} \Rightarrow \cos^{-1}\dfrac{10}{14}$

$\theta \neq 45°$; FALSO

30.- $\vec{A} = -\hat{i}+2\hat{j}-\hat{k}$

$\vec{B} = 2\hat{i}+2\hat{j}-3\hat{k}$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & -1 \\ 2 & 2 & -3 \end{vmatrix} = (-6+2)\hat{i}-(3+2)\hat{j}+(-2-4)\hat{k}$$

$\vec{A}\times\vec{B} = -4\hat{i}-5\hat{j}-6\hat{k}$

$|\vec{A}\times\vec{B}| = \sqrt{16+25+36} \Rightarrow \sqrt{77}$

$$\vec{U} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|} \Rightarrow \frac{-4\hat{i}-5\hat{j}-6\hat{k}}{\sqrt{77}} \Rightarrow -\frac{\sqrt{77}}{77}(4,5,6)$$

31.- FALTAN DATOS; NO SE PUEDE CALCULAR.

32.- a) $\vec{A} = (1,3,0)$

$\vec{B} = (-2,1,5)$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 0 \\ -2 & 1 & 5 \end{vmatrix} = (15-0)\hat{i}-(5-0)\hat{j}+(1+6)\hat{k}$$

$\Rightarrow 15\hat{i}-5\hat{j}+7\hat{k}$

área $= |\vec{A}\times\vec{B}| = \sqrt{15^2+5^2+7^2} = \sqrt{299}$

b) $\vec{C}=(1,-1,6)$

$\vec{D}=(-2,10,1)$

$$\vec{C}\times\vec{D}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 6\\ -2 & 10 & 1\end{vmatrix}=(-1-60)\hat{i}-(1+12)\hat{j}+(10+12)\hat{k}$$

$$\vec{C}\times\vec{D}=(-61,-13,12)$$

$$\text{área}=|\vec{C}\times\vec{D}|=\sqrt{61^2+13^2+12^2}$$

$$\Rightarrow\sqrt{4034}$$

c) $\vec{E}=(1,-4,-2)$

$F=(0,7,0)$

$$\vec{E}\times\vec{F}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -4 & -2\\ 0 & 7 & 0\end{vmatrix}=(0+14)\hat{i}-(0-0)\hat{j}+(7-0)\hat{k}$$

$$\vec{E}\times\vec{F}=(14,0,7)$$

$$\text{área}=|\vec{E}\times\vec{F}|=\sqrt{14^2+7^2}=\sqrt{245}$$

33.- a) $A=(1,1,1)$, $B(-1,2,3)$, $C(2,6,5)$

$\vec{P}=(-2,1,2)$

$\vec{Q}=(3,4,2)$

$$\vec{P}\times\vec{Q}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -2 & 1 & 2\\ 3 & 4 & 2\end{vmatrix}=(2-8)\hat{i}-(-4-6)\hat{j}+(-8-3)\hat{k}$$

$$\vec{P}\times\vec{Q}=-6\hat{i}+10\hat{j}-11\hat{k}$$

$$\text{area}\,\Delta=\frac{|\vec{P}\times\vec{Q}|}{2}=\frac{\sqrt{36+100+121}}{2}$$

$$\text{área}\,\Delta=\frac{\sqrt{257}}{2}$$

b) $A(1,4,3)$ $> \vec{P}=(-3,-2,-1)$
$B(-2,2,2)$ $> \vec{Q}=(1,4,3)$
$C(-1,6,5)$

$$\vec{P}\times\vec{Q}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -2 & -1 \\ 1 & 4 & 3 \end{vmatrix}=(-6+4)\hat{i}-(-9+1)\hat{j}+(-12+2)\hat{k}$$

$$\vec{P}\times\vec{Q}=(-2,8,-10)$$

$$\text{área}\,\Delta=\frac{|\vec{P}\times\vec{Q}|}{2}\Rightarrow\frac{\sqrt{4+64+100}}{2}\Rightarrow\frac{\sqrt{4\cdot 42}}{2}\Rightarrow\frac{2\sqrt{42}}{\cancel{2}}$$

$$\Rightarrow \sqrt{42}$$

c) $A(0,0,1)$ $> \vec{P}=(-1,0,1)$
$B(-1,0,0)$ $> \vec{Q}=(-1,1,0)$
$C(0,1,0)$

$$\vec{P}\times\vec{Q}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{vmatrix}=(0-1)\hat{i}-(0+1)\hat{j}+(-1+0)\hat{k}$$

$$\vec{P}\times\vec{Q}=(-1,-1,-1)$$

$$\text{área}\,\Delta=\frac{|\vec{P}\times\vec{Q}|}{2}\Rightarrow\frac{\sqrt{1+1+1}}{2}\Rightarrow\frac{\sqrt{3}}{2}$$

34.– a) $\vec{A}=(0,2,1)$
$\vec{B}=(3,-6,2)$
$\vec{C}=(1,0,3)$

$$\begin{vmatrix} 0 & 2 & 1 \\ 3 & -6 & 2 \\ 1 & 0 & 3 \\ 0 & 2 & 1 \\ 3 & -6 & 2 \end{vmatrix}=0+0+4-(-6+0+18)$$

$$\Rightarrow 4-12$$

$$\Rightarrow 8$$

b) $\vec{A}=(1,0,0)$
$\vec{B}=(0,-1,0)$
$\vec{C}=(1,7,2)$

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 7 & 2 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} = -2+0+0-(0+0+0$$

$\Rightarrow 2$

c) $\vec{A}=(2,1,4)$
$\vec{B}=(-1,1,0)$
$\vec{C}=(3,2,4)$

$$\begin{vmatrix} 2 & 1 & 4 \\ -1 & 1 & 0 \\ 3 & 2 & 4 \\ 2 & 1 & 4 \\ -1 & 1 & 0 \end{vmatrix} = 8-8+0-(12+0-4)$$

$\Rightarrow 0-(8)$

$\Rightarrow 8$

35.— $\vec{V}_1=(1,1,0)$
$\vec{V}_2=(0,1,1)$
$\vec{V}_3=(1,1,1)$

$$\begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = 1+0+1-(0+1+0)$$

$\Rightarrow 2-1$

$\Rightarrow 1$

b) CORRECTO

36.— $P(a, 2a, 3a)$

$Q(3a, a, -2a)$ $\qquad \vec{PQ} = (2a, -a, -5a)$

$R(-a, a, 2a)$ $\qquad \vec{PR} = (-2a, -a, -a)$

$S(2a, 5a, a)$ $\qquad \vec{PS} = (a, 3a, -2a)$

$$\text{Volumen} \Rightarrow \begin{vmatrix} 2a & -a & -5a \\ -2a & -a & -a \\ a & 3a & -2a \\ 2a & -a & -5a \\ -2a & -a & -a \end{vmatrix} \Rightarrow 4a^3+30a^3+a^3-(5a^3-6a^3-4a^3)$$

$$\Rightarrow 35a^3-(-5a^3)$$

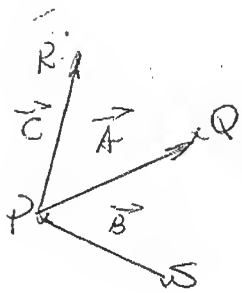
$$\Rightarrow 40a^3$$

37.— $P(2,1,3)$

$Q(4,-2,2)$

$R(1,1,3)$

$S(-4,0,2)$



$\vec{A} = (2,-3,-1)$

$\vec{B} = (-6,-1,-1)$

$\vec{C} = (-1,0,0)$

$$\text{Volumen} \begin{vmatrix} 2 & -3 & -1 \\ -6 & -1 & -1 \\ -1 & 0 & 0 \\ 2 & -3 & -1 \\ -6 & -1 & -1 \end{vmatrix} = 0+0-3-(-1+0+0)$$

$$\Rightarrow -3+1$$

$$\Rightarrow 2$$

$$\text{área base} = |\vec{A}\times\vec{B}| = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -3 & -1 \\ -6 & -1 & -1 \end{vmatrix} = (3-1)\hat{\imath}-(-2-6)\hat{\jmath}+(-2-18)\hat{k}$$

$$\vec{A}\times\vec{B} = 2\hat{\imath}+8\hat{\jmath}-20\hat{k}$$

$$\text{área base} = |\vec{A}\times\vec{B}| = \sqrt{4+64+400} \Rightarrow \sqrt{468}$$

$$\Rightarrow \sqrt{4\times117} = 2\sqrt{117}$$

$$\text{Volumen} = (\text{área base}) \times \text{altura}$$

$$\text{altura} = \frac{\text{Volumen}}{\text{área base}}$$

$$\text{altura} = \frac{\cancel{2}}{\cancel{2}\sqrt{117}} \times \frac{\sqrt{117}}{\sqrt{117}}$$

$$\text{altura} = \frac{\sqrt{117}}{117}$$

1 35

SOLUCION
EJERCICIOS PROPUESTOS
CAPITULO DIEZ
MATEMATICAS
GEOMETRIA ANALITICA

10.1 RECTAS EN EL PLANO

a) (1,2) , (-2,3)

$d = \sqrt{(-2-1)^2+(3-2)^2}$

$d = \sqrt{9+1}$

$d = \sqrt{10}$

$\bar{X} = \frac{-2+1}{2} \Rightarrow -1/2$

$\bar{Y} = \frac{3+2}{2} \Rightarrow 5/2$

$PM = (-1/2, 5/2)$

b) (0,3) ; (1,5)

$d = \sqrt{(1-0)^2+(5-3)^2}$

$d = \sqrt{1+4}$

$d = \sqrt{5}$

$\bar{X} = \frac{1+0}{2} = 1/2$

$\bar{Y} = \frac{5+3}{2} = 4$

$PM = (1/2, 4)$

c) $(-2,-1)$; $(-3,4)$

$d=\sqrt{(-3+2)^2+(4+1)^2}$

$d=\sqrt{1^2+5^2}$

$d=\sqrt{26}$

$\bar{X}=\frac{-3-2}{2}=-5/2$.

$\bar{Y}=\frac{4-1}{2}=\frac{3}{2}$

$PM=\left(-5/2,\ 3/2\right)$

d) $(2,4)$; $(3,4)$

$d=\sqrt{(3-2)^2+(4-4)^2}$

$d=\sqrt{1^2+0^2}$

$d=1$

$\bar{X}=\frac{3+2}{2}=5/2$

$\bar{Y}=\frac{4+4}{2}=4$

$PM=\left(5/2,\ 4\right)$

e) $(-4,6)$; $(-7,6)$

$d=\sqrt{(-7+4)^2+(6-6)^2}$

$d=\sqrt{3^2+0^2}$

$d=3$

$\bar{X}=\frac{-7-4}{2}=-11/2$

$\bar{Y}=\frac{6+6}{2}=6$

$PM=\left(-11/2;6\right)$

f.- $(a, 1)$; $(2a, 1)$

$d = \sqrt{(2a-a)^2 + (1-1)^2}$

$d = \sqrt{a^2 + 0^2}$

$d = a$

$\bar{X} = \frac{2a+a}{2} = \frac{3a}{2}$

$\bar{Y} = \frac{1+1}{2} = 1$

$PM = (3a/2 ; 1)$

g.- $(5a, 2a)$; $(a, 3a)$

$d = \sqrt{(a-5a)^2 + (3a-a)^2}$

$d = \sqrt{(4a)^2 + (2a)^2}$

$d = \sqrt{16a^2 + 4a^2}$

$d = \sqrt{20a^2}$

$d = \sqrt{4}\sqrt{a^2}\sqrt{5}$

$d = 2a\sqrt{5}$

$\bar{X} = \frac{a+5a}{2} = \frac{3a}{2}$

$\bar{Y} = \frac{3a+2a}{2} = 5/2\, a$

$PM = (3a/2 ; 5/2\, a)$

2.- a) $(-2,3)$; $(-3,1)$

$$m = \frac{1-3}{-3+2} \Rightarrow \frac{-2}{-1} \Rightarrow 2$$

Pto. Pendiente

$y - y_1 = 2(x - x_1)$

$y - 3 = 2(x+2) \Rightarrow$

$y - 3 = 2x + 4$

$y = 2x + 7$

Ec. Paramétrica

$$\frac{y-3}{2} = \frac{2x+4}{2}$$

$$\frac{y-3}{2} = \frac{x+2}{1} = t$$

Ec. General.

$y = 2x + 7$

$2x - y + 7 = 0$

b) $(2,0)$, $(4,5)$

$$m = \frac{5-0}{4-2} \Rightarrow \frac{5}{2}$$

$y - y_1 = \frac{5}{2}(x - x_1)$

$y - 0 = \frac{5}{2}(x-2) \Rightarrow \Rightarrow \Rightarrow$

$y = \frac{5x}{2} - 5$

Ec. Pto.-Pendiente

$2y = 5(x-2)$

$2y = 5x - 10$

$5x - 2y - 10 = 0$

Ec. General

$$\frac{2y}{10} = \frac{5x - 10}{10}$$

$$\frac{y}{5} = \frac{x-2}{2} = t$$

c) $(-1,1)$; $(1,1)$

$$m = \frac{1-1}{1+1} \Rightarrow 0$$

$y - y_1 = m(x - x_1)$

$y - y_1 = 0(x - x_1)$

$y - 1 = 0$ Ec. General

Ec. Paramétrica

$\Downarrow$

$y = 1$

Ec. Pto/pendiente

d) $(-2,4)$; $(1,4)$

$$m = \frac{4-4}{1+2} \Rightarrow 0$$

$y - y_1 = m(x - x_1)$

$y - 4 = 0(x+2)$

$y - 4 = 0$ Ec. General

Ec. Paramétrica

$\Downarrow$

$y = 4$

Ec. Pto/Pendiente

e) $(-4,5)$; $(-4,2)$

$m = \frac{2-5}{-4+4} \Rightarrow$ NO HAY PENDIENTE

$x - x_1 = 0$

$x + 4 = 0$ Ec. General

Ec. Paramétrica

$\Downarrow$

$x = -4$

3.- $P(1,3)$

$m = 9$

$y - y_1 = m(x - x_1)$

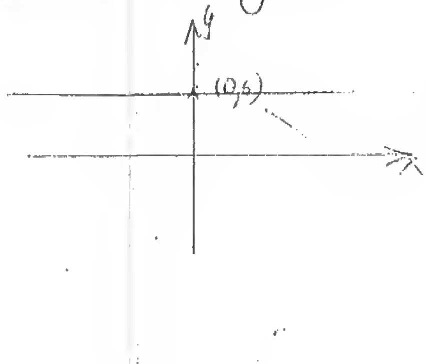
$y - 3 = 9(x - 1)$

$y - 3 = 9x - 9$

$y = 9x - 6$

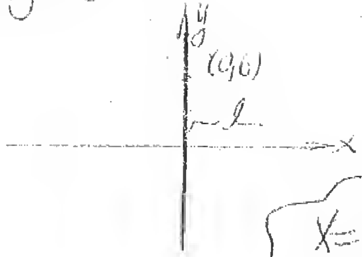
4.- $P(0,6)$

a) Paralela al eje x.



$y = 6$

b) Paralela al eje y



$$x=0$$

c) Paralela a la recta $3x-2y=6$

$$-2y=-3x+6$$

$$y=\tfrac{3}{2}x+3$$

$$m=\tfrac{3}{2}$$

$$y-y_1=m(x-x_1)$$

$$y-6=\frac{3}{2}(x-0)$$

$$2(y-6)=3(x-0)$$

$$2y-12=3x$$

$$3x-2y+12=0$$

d) Perpendicular

$$-2x+y-1=0$$

$$y=2x+1$$

$$m_1=2$$

$$m_2=-\frac{1}{2} \qquad y-y_1=-\frac{1}{2}(x-x_1)$$

$$y-6=-\frac{1}{2}(x-0)$$

$$2(y-6)=-x$$

$$2y-6=-x \Rightarrow x+2y-6=0$$

5.- a) $l: ax+by+c=0$ $\qquad \vec{V}=(a,b)$

$$ax=-by-c$$

$$\frac{ax}{ab}=\frac{-by-c}{ab}$$

$$\frac{x}{b}=-\frac{y+c/b}{a}=t$$

$$\frac{x}{b}=\frac{y+c/b}{-a}=t$$

un vector paralelo a l es: $(b,-a)$

(Falso)

b) $\vec{V}=(2a,2b)$

$$l: ax+by+c=0$$

$$ax=-by-c$$

$$\frac{ax}{ab}=-\frac{(by+c)}{ab}$$

$$\frac{x}{b}=-\frac{y+c/b}{a}=t$$

un vector paralelo a l: $\vec{V_1}=(b,-a)$

si $\vec{V}\perp\vec{V_1}$ entonces $\vec{V}\cdot\vec{V_1}=0$

$$(2a,2b)\cdot(b,-a)=0$$

$$2ab-2ab=0$$

$$0=0$$

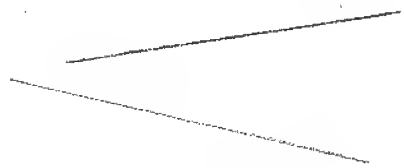
CORRECTO

c) $l_1 = A_1x + B_1y + C_1 = 0$

$l_2 = A_2x + B_2y + C_2 = 0$

$$l_1 \cap l_2 = \phi$$

no se intersectan.



no necesariamente son paralelas.

$$\underbrace{A_1 = A_2K \;;\; B_1 = B_2K \;;\; C_1 = C_2K}$$

Se cumple cuando son rectas paralelas.

(FALSO)

d) $y = kx + b \Rightarrow$ la pendiente es k

$y = mx + b$

VERDADERO

e) l_1: $x = x_0 + at$ l_2: $y = y_0 + bt$

$$l_1 \cap l_2 = (x_0, y_0)$$

Al existir intersección entre l_1 y l_2 se podrá trazar una recta paralela al vector (a,b), tomando en cuenta que por un punto pasan infinitas rectas

6.- recta vertical.- $\boxed{x-k=0}$

$x+1/2=0$

7.- $P(5,-1)$

$\parallel \ell: 2x+y-1=0$

$y=-2x+1$

$m_1=-2$

$y-y_1=m(x-x_1)$

$y+1=-2(x-5)$

$y+1=-2x+10$

$2x+y-9=0$

8.- $\vec{V_1}=(2,-3)$

$\vec{V_2}=(4,1)$

$\vec{V_1}-\vec{V_2}=(-2,-4)$ $P(1,2)$

$$\frac{x-1}{\cancel{-2}}=\frac{y-2}{-4}=t$$

$$x-1=\frac{y-2}{2}$$

$2(x-1)=y-2$

$2x-\cancel{2}=y-\cancel{2}$

$2x-y=0$

9.- $P(3,-5)$

$l \parallel \vec{V}=(-4,2)$

$$\frac{x-3}{\cancel{-4}_{-2}} = \frac{y+5}{\cancel{2}_{1}} = t$$

$$\frac{x-3}{-2} = y+5$$

$$x-3=-2(y+5)$$

$$x-3=-2y-10$$

$$x+2y+7=0$$

10.- $P_0(1,2)$

$l \perp \vec{V}=(-2,-4)$

$\vec{V_1} \perp \vec{V} \Rightarrow l \parallel \vec{V_1}$

$(a,b)\cdot(-2,-4)=0$

$-2a-4b=0$

$-2(a+2b)=0$

$a=-2b$

$\vec{V_1}=(-2b,b) \parallel l$

$$\frac{x-1}{-2b} = \frac{y-2}{b} = t$$

$$\frac{x-1}{-2\cancel{b}} = \frac{y-2}{\cancel{b}}$$

$$\frac{x-1}{-2} = y-2$$

$$x-1=-2(y-2)$$

$x-1=-2y+4 \Rightarrow x+2y-5=0$ b) CORRECTO

11.- $P_1(-1,1)$

$P_2(3,9)$

$m = \frac{9-1}{3+1} \Rightarrow \frac{8}{4} \Rightarrow 2$

$y - y_1 = 2(x - x_1)$

$y - 1 = 2(x+1)$

$y - 1 = 2x + 2$

$2x - y + 3 = 0$ $\quad P(x,0) \Rightarrow 2x - 0 + 3 = 0$

$2x = -3$

$x = -3/2$

a) CORRECTO

12.- $l_1: 2x - 5y + 6 = 0$ $\quad l_2: 4x - 2y - 5 = 0$

$5y = 2x + 6$ $\quad 2y = 4x - 5$

$P(x,0)$ $\quad y = 2x - 5/2$

$\boxed{m = 2}$

$0 = 2x + 6$

$2x = -6$ $\quad y - y_1 = m(x - x_1)$

$x = \frac{-6}{2}$ $\quad y - 0 = 2(x+3)$

$\boxed{x = -3}$ $\quad y = 2x + 6$

$P(-3,0)$ $\quad 2x - y + 6 = 0$

d) CORRECTO.

13.- $l: 3x+2y-5=0$

$2y=-3x+5$

$y=-\frac{3}{2}x+\frac{5}{2}$ $\begin{cases} m=-3/2 \\ \text{Intersección eje } y\ (0, 5/2) \\ \text{Intersección eje } x\ (5/3, 0) \end{cases}$

a) Falso

b) Falso

c) VERDADERO; $2x-3y-15=0$

$3y=2x-15$

$y=\frac{2}{3}x-5$

$\downarrow$

$m_1=\frac{2}{3}$ $\quad m_1 \cdot m_2=1$

$\frac{2}{3}\left(-\frac{3}{2}\right)=1$

14.- $f(x)=ax+b$

a) $a=0$

$\underbrace{f(x)=b}$ $\Rightarrow$ función constante.

$\Downarrow$

función creciente ; FALSO

b) $b=0$

$f(x)=ax$ $\Rightarrow$ función lineal

$\Downarrow$

NO ES FUNCION PAR ; FALSO

c) $a<0$

$f(x)=-ax+b$ $\Rightarrow$ función lineal

$\Downarrow$

es función decreciente ; VERDADERO

d) FALSO.

15.- $P_1(-3,0)$

$P_2(0,6)$

$$m = \frac{6-0}{0+3} \Rightarrow \frac{6}{3}$$

$\underline{m=2}$

$y-y_1=2(x-x_1)$

$y-0=2(x+3)$

$y=2x+6$ C) CORRECTO

16.-

$t=6$

$P_1(0,59000)$

$P_2(6,95000)$

$$m = \frac{95000-59000}{6-0}$$

$\underline{m=6000}$

$y-y_1=6000(x-x_1)$

$y-59000=6000(x-0)$

$y-59000=6000x$

$y=6000x+59000$

d) CORRECTO.

17.- $P_0 = 80.000 \Rightarrow P_1(0, 80.000)$

$t = 10$ años

$P = 2000 \Rightarrow P_2(10, 2000)$

$$m = \frac{2000 - 80000}{10 - 0} \Rightarrow \frac{-7800\cancel{0}}{1\cancel{0}}$$

$m = -7800$

$y - y_1 = m(x - x_1)$

$y - 2000 = -7800(x - 10)$

$y - 2000 = -7800x + 78000$

$y = -7800x + 80.000$

18.- $y = -7800x + 80.000$

$x = 5$

$y = -7800(5) + 80.000$

$y = -39000 + 80.000$

$y = 41000$ b) CORRECTO

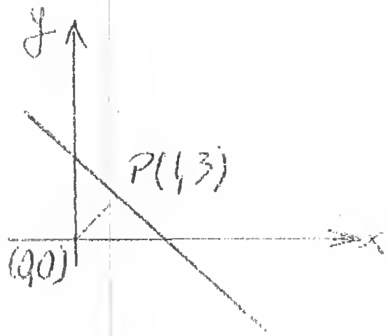
19.- $l: 3x - 4y + 12 = 0$

$P(4, -1)$

$$d = \frac{3(4) - 4(-1) + 12}{\sqrt{3^2 + 4^2}}$$

$$d = \frac{28}{5}$$

20.—

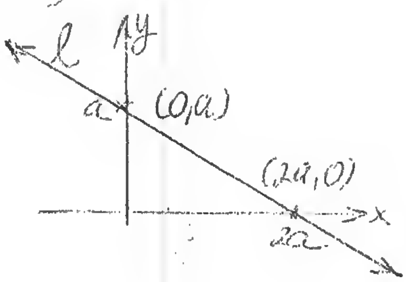


$$d = \sqrt{(1-0)^2 + (3-0)^2}$$

$$d = \sqrt{10}$$

a) FALSO; faltan datos
b) FALSO; faltan datos
c) FALSO; faltan datos
d) FALSO; faltan datos
e) CORRECTO

21.— $P\left(\frac{3}{2}, \frac{1}{2}\right) \in l.$



$$m = \frac{0-a}{2a-0} \Rightarrow -\frac{\cancel{a}}{2\cancel{a}}$$

$$m \Rightarrow -\frac{1}{2}$$

$$y - y_1 = m \cdot (x - x_1)$$

$$y - \frac{1}{2} = -\frac{1}{2}\left(x - \frac{3}{2}\right)$$

$$\frac{2y-1}{\cancel{2}} = -\frac{1}{\cancel{2}}\left(x - \frac{3}{2}\right)$$

$$2y - 1 = -\left(\frac{2x-3}{2}\right)$$

$$2(2y-1) = -(2x-3)$$

$$4y - 2 = -2x + 6$$

$$2x + 4y - 8 = 0 \Rightarrow x + 2y - 4 = 0$$

22.- $l_1 = 3x - 4y = 1 \rightarrow P(\frac{1}{3}, 0)$

$l_2 = 3x - 4y = 10$

distancia punto-recta P, l_2

$$d = \frac{\left|\left(\frac{1}{3}\right)(3) + 0(-4) - 10\right|}{\sqrt{3^2 + 4^2}}$$

$$d = \frac{9}{5}$$

23.- $m_1 = -2$

$l: ax + by + c = 0$

$d = 4$

$P(-2, 3)$

$$m = +\frac{a}{b} = +2$$

$$\boxed{a = 2b}$$

$$d = \frac{-2a + 3b + c}{\sqrt{a^2 + b^2}} = \pm 4$$

$$\Rightarrow \frac{-2(2b) + 3b + c}{\sqrt{(2b)^2 + b^2}} = \pm 4$$

$$\Rightarrow \frac{-b + c}{\sqrt{5}\, b} = \pm 4$$

$$-b + c = \pm 4\sqrt{5}\, b$$

$$\boxed{c = \pm 4\sqrt{5}\, b + b}$$

$ax + by + c = 0$

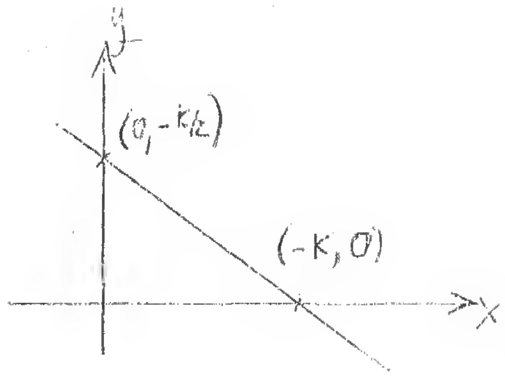
$\Rightarrow 2bx + by \pm 4\sqrt{5}\, b + b = 0$

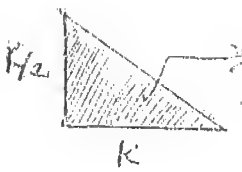
$b(2x + y \pm 4\sqrt{5} + 1) = 0$

$2x + y \pm 4\sqrt{5} + 1 = 0$

24- $l: x+2y+k=0$

área $= 16\,u^2$





$$\text{área} = \frac{1}{2}ab.$$

$$\text{área} = \frac{1}{2}k\left(\frac{k}{2}\right) = 16$$

$$\Rightarrow \frac{k^2}{4} = 16$$

$$\Rightarrow k^2 = 16 \times 4$$

$$\Rightarrow k = \sqrt{16 \times 4}$$

$$\Rightarrow k = 8$$

25- $l: 3x+ky-2=0$ $\quad P(1,1)$

a) $d=2$

$$d = \frac{3(1)+k(1)-2}{\sqrt{3^2+1^2}} = 2$$

$$\frac{1+k}{\sqrt{10}} = 2$$

$$k = 2\sqrt{10}-1$$

b) $d<5$

$$d = \frac{3(1)+k(1)-2}{\sqrt{3^2+(1)^2}} < 5$$

$$\frac{k+1}{\sqrt{10}} < 5$$

$$k+1 < 5\sqrt{10}$$

$$k < 5\sqrt{10}-1$$

c) $d > 1$

$$d = \frac{3(1) + K(1) - 2}{\sqrt{3^2+1^2}} > 1$$

$$\frac{1+K}{\sqrt{10}} > 1$$

$$K+1 > \sqrt{10}$$

$$K > \sqrt{10} - 1$$

26.- $x^2 + y^2 - 4x + 6y - 3 = 0$

$$(x^2 - 4x + \underline{4}) + (y^2 + 6y + \underline{9}) - 3 = 0 + 13$$

$$(x-2)^2 + (y+3)^2 = 13 + 3$$

$$(x-2)^2 + (y+3)^2 = 16$$

$$C(2, -3)$$

$$R = 4$$

d) CORRECTO

27.- $P_1(5, -2)$

$P_2(-3, -2)$

$$\bar{x} = \frac{5-3}{2} = 1$$

$$\bar{y} = \frac{-2-2}{2} = -2$$

$$C(1, -2)$$

$$R = d = \sqrt{(1-5)^2 + (-2+2)^2}$$

$$R = \sqrt{4^2 + 0}$$

$$R = 2$$

$$(x-1)^2 + (y+2)^2 = 4$$

$$x^2 - 2x + 1 + y^2 + 4y + 4 = 4$$

$$x^2 + y^2 - 2x + 4y + 1 = 0$$

28.- $C_1: x^2+y^2-2x+6y+9=0$

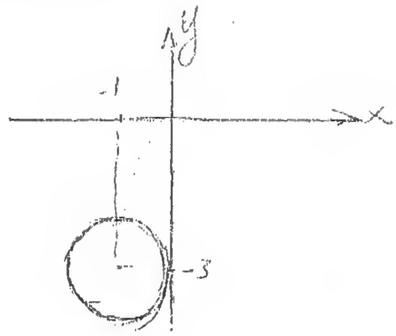
$C_2: x^2+y^2-4x-4y+4=0$

$\underline{C_1}$

$(x^2-2x+1)+(y^2+6y+9)\cancel{+9}=1+\cancel{9}$

$(x-1)^2+(y+3)^2=1$

$\begin{cases} C:(1,-3) \\ R=1 \end{cases}$

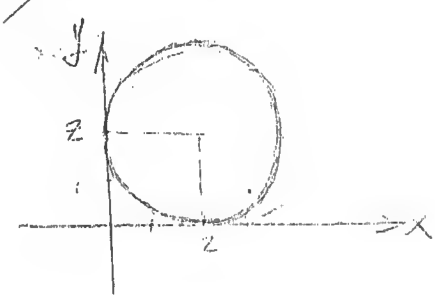


$\underline{C_2}$

$(x^2-4x+4)+(y^2-4y+4)\cancel{+4}=\cancel{4}+4$

$(x-2)^2+(y-2)^2=4$

$\begin{cases} C:(2,2) \\ R=2 \end{cases}$



a) FALSO

b) FALSO

c) CORRECTO

d) FALSO

e) FALSO

29.- $(x+1)^2+(y-9)^2=3$

$C(-1,9)$

$l: 3x-6y+1=0$

$6y=3x+1$

$y=\frac{3}{6}x+\frac{1}{6}$

$y=\frac{1}{2}x+\frac{1}{6}$

$m_1=\frac{1}{2}$

$\underline{l \perp l_1}$

$m_1 \times m=-1$

$\left(\frac{1}{2}\right)m=-1$

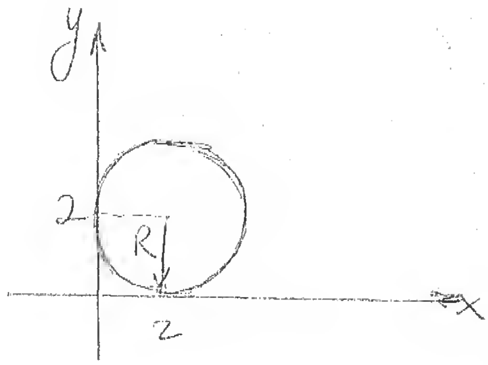
$\underline{m=-2}$

$(y-9)=-2(x+1)$

$y-9=-2x-2$

$2x+y-7=0$

30.-



$C(2,2)$

$R=2$

$(x-2)^2+(y-2)^2=2^2$

$x^2-4x+4+y^2-4y+4=4$

$\boxed{x^2+y^2-4x-4y+4=0}$

31.- $l: 2x-y+3=0$

$\boxed{y=2x+3}$

$(0,3)$ $(-1/5, 13/5)$

$(0,1)$ $(-1/5, 7/5)$

$(0,3)$ ✓

$y=0+3$

$\boxed{y=3}$

$(0,1)$ ✗

$(-1/5, \frac{13}{5})$

$y=2\left(-\frac{1}{5}\right)+3$

$y=-\frac{2}{5}+3$

$\boxed{y=\frac{13}{5}}$

$C: x^2+y^2-3x-4y+3=0$

$x^2+(2x+3)^2-3x-4(2x+3)+3=0$

$x^2+4x^2+12x+9-3x-8x-12+3=0$

$5x^2+x=0$

$x(5x+1)=0$

$\boxed{x=0}$ $5x+1=0$

$5x=-1$

$\boxed{x=-1/5}$

$x^2+y^2-3x-4y+3=0$

$0+y^2-4y+3=0$

$(y-3)(y-1)=0$

$\boxed{y=3}$ $\boxed{y=1}$

$\left(-\frac{1}{5}\right)^2+y^2-3\left(-\frac{1}{5}\right)-4y+3=0$

$\frac{1}{25}+y^2+\frac{3}{5}-4y+3=0$

$y^2-4y+\frac{91}{25}=0$

$y=\frac{4\pm\sqrt{16-4(1)(91/25)}}{2(1)}$

$y=\frac{4\pm\frac{6}{5}}{2}$

$y=\frac{\frac{20\pm 6}{5}}{\frac{2}{1}}$

$y=\frac{20\pm 6}{10}=\frac{10\pm 3}{5}$

2 puntos de la recta coinciden con la circunferencia

a) CORRECTO

32.- $C(4,1)$

tangente a $l: 2x-3y=15$.

$$d=\frac{|2(4)-3(1)-15|}{\sqrt{2^2+3^2}}$$

$$d=R=\frac{10}{\sqrt{13}}$$

$$(x-4)^2+(y-1)^2=\left(\frac{10}{\sqrt{13}}\right)^2$$

33.- $P(0,0)$

$$C_1 = x^2+y^2-6x+2y-5=0 \Rightarrow C$$

$$(x^2-6x+9)+(y^2+2y+1)=5+9+1$$

$$\underbrace{(x-3)^2+(y+1)^2=15}$$

$$C:(3,-1)$$

$$(x-3)^2+(y+1)^2=R^2$$

$$(0-3)^2+(0+1)^2=R^2$$

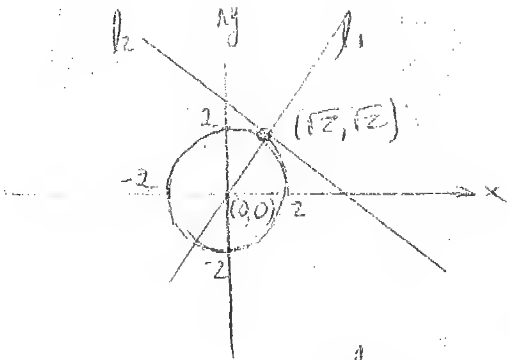
$$R^2=10$$

$$(x-3)^2+(y+1)^2=10$$

$$x^2-6x+\cancel{9}+y^2+2y+\cancel{1}=\cancel{10}$$

$$x^2+y^2-6x+2y=0$$

34.-



l_1

$m = \frac{\sqrt{2}-0}{\sqrt{2}-0} = 1$

$y - y_1 = m(x - x_1)$

$y - 0 = 1(x - 0)$

$y = x$

l_2

$m_2 = -\frac{1}{m_1}$

$m_2 = -1$

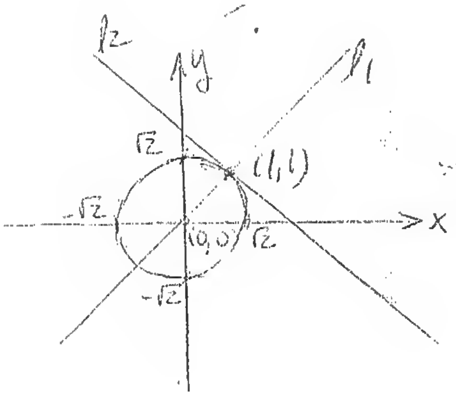
$y - y_1 = m_2(x - x_1)$

$y - \sqrt{2} = -(x - \sqrt{2})$

$y - \sqrt{2} = -x + \sqrt{2}$

$x + y - 2\sqrt{2} = 0$

35.-



l_1

$P_1(1,1)$

$P_0(0,0)$

$m = \frac{1-0}{1-0}$

$m = 1$

$l_1 \perp l_2$

$l_2 =$

$m_2 = -\frac{1}{m_1}$

$m_2 = -1$

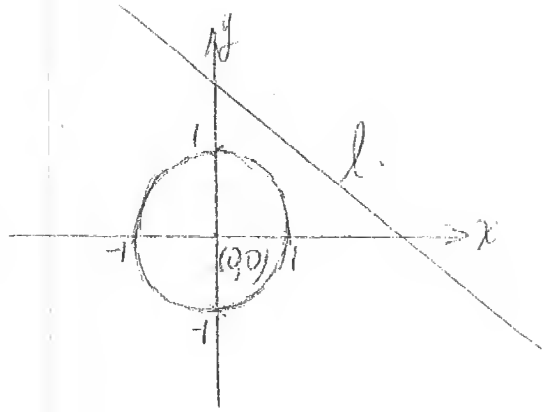
$y - y_1 = m(x - x_1)$

$y - 1 = -1(x - 1)$

$y - 1 = -x + 1$

$x + y - 2 = 0$

36.-



distancia pto-recta

$l: 4x+3y-15=0$

$P:(0,0)$

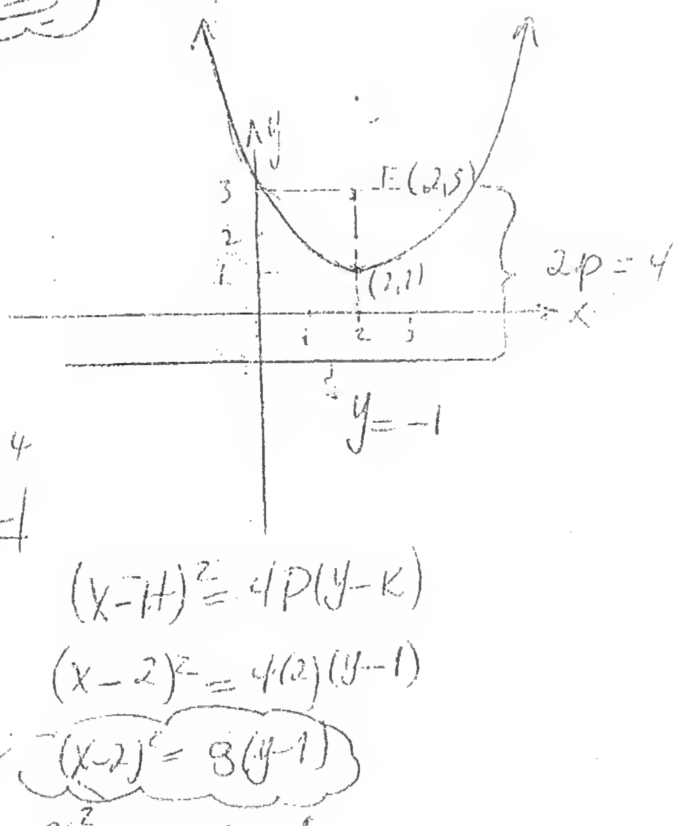
$$d=\frac{|4(0)+3(0)-15|}{\sqrt{4^2+3^2}}=\frac{15}{5}=3$$

distancia $=3-R$

$\Rightarrow 2$

37.- $F(2,3)$

$y=-1$



$2p=4$

$p=2$

$$(x-h)^2=4p(y-k)$$

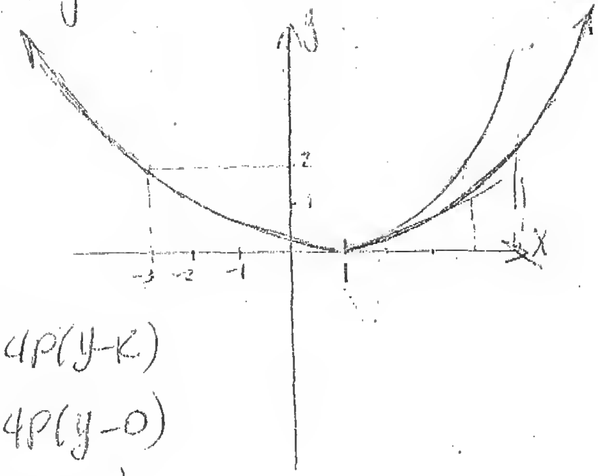
$$(x-2)^2=4(2)(y-1)$$

$$(x-2)^2=8(y-1)$$

$$x^2-4x+4=8y-8$$

$$x^2-4x-8y+12=0$$

38.- Parábola eje vertical



V(1,0)

P:(-3,2)

$(x-h)^2=4p(y-k)$

$(x-1)^2=4p(y-0)$

$(-3-1)^2=4p(2)$

$16=8p$

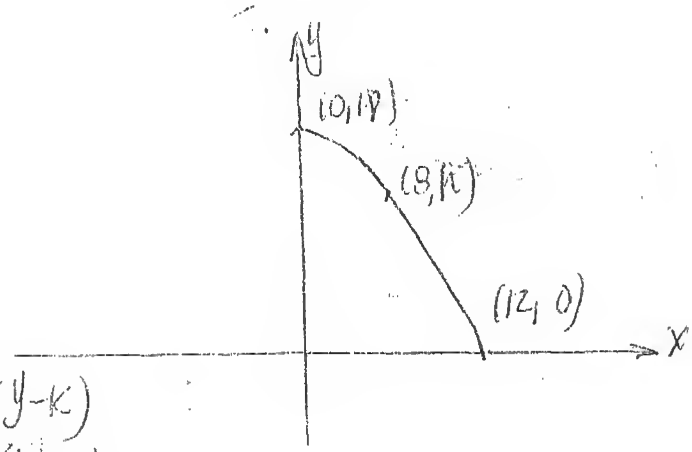
$\boxed{p=2}$

$(x-1)^2=4(2)y$

$x^2-2x+1=8y$

$x^2-2x-8y+1=0$

39.-



V:(0,18)

P:(12,0)

$(x-h)^2=4p(y-k)$

$(x-0)^2=4p(y-18)$

$(12)^2=4p(0-18)$

$\cancel{144}=\cancel{4}p(-\cancel{18})$

$\cancel{36}$

2

$2=-p$

$\boxed{p=-2}$

$x^2=4(-2)(y-18)$

$x^2=-8y+144$

$x^2+8y-144=0$

P(8,h)

$8^2+8h-144=0$

$64+8h-144=0$

$8h=80 \Rightarrow h=10$

40.- F=¿?

$$2x^2-4x+y+4=0 \Rightarrow (x-h)^2=4p(y-k)$$

$$2x^2-4x=-y-4$$

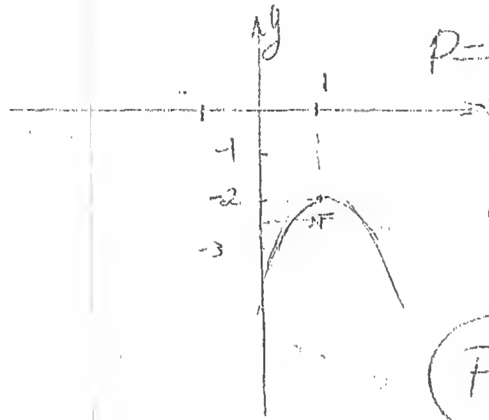
$$2(x^2-2x+1)=-y\underbrace{-4+2}_{-2}$$

$$2(x-1)^2=-(y+2)$$

$$(x-1)^2=-\frac{1}{2}(y+2)$$

$V(1,-2)$ $\quad 4p=-\frac{1}{2}$

$p=-\frac{1}{8}$



$F \Rightarrow -2-p$

$\Rightarrow -2-\frac{1}{8} \Rightarrow \frac{-16-1}{8}$

$F(1,-17/8)$

41.- $C: x^2+y^2+2x-8y+9=0$

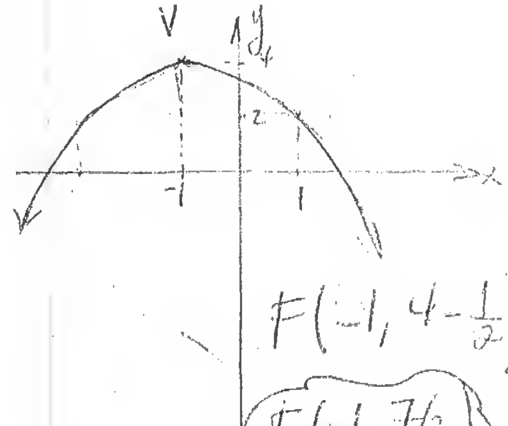
$$(x^2+2x+1)+(y^2-8y+16)=-9+1+16$$

$$(x+1)^2+(y-4)^2=8$$

$C(-1,4)$

$V(-1,4)$

$A(1,2)$



$$(x-h)^2=4p(y-k)$$

$$(x+1)^2=4p(y-4)$$

$$(1+1)^2=4p(2-4)$$

$$4=4p(-2)$$

$$\boxed{p=-\frac{1}{2}}$$

$F(-1,4-\frac{1}{2})$

$F(-1,7/2)$

c) CORRECTO

42 – $y=-x-1$

a) $(x+2)^2+(y+1)^2=4$

$(x+2)^2+(-x-\cancel{1}+\cancel{1})^2=4$

$x^2+4x+\cancel{4}+x^2=\cancel{4}$

$2x^2+4x=0$

$2x(x+2)=0$

$2x=0$ $\quad$ $x+2=0$

$\boxed{x=0}$ $\quad$ $\boxed{x=-2}$

$y=-x-1$

$P_1(0,-1)$; $P_2(-2,-3)$

$(x+2)^2+(y+1)^2=4$

$P_1(0,-1)$ ✓ Válido

$P_2(-2,-3)$ ✓ Válido

b) $(x-2)^2=y-1$

$(x-2)^2=-x-1-1$

$x^2-4x+4=-x-2$

$x^2-3x+6=0$

$$x=\frac{3\pm\sqrt{3^2-4(1)(6)}}{2(1)}$$

NO HAY INTERSECCIONES

c) $-(y+1)^2 = x$; $y = -x-1$

$-(-x-1+1)^2 = x$

$-(-x)^2 - x = 0$

$-x^2 - x = 0$

$-x(x+1) = 0$

$\boxed{x=0}$; $x+1=0$

$\boxed{x=-1}$

$y = -x-1$

$P_1(0,-1)$; $P_2(-1,0)$

$-(y+1)^2 = x$

$P_1(0,-1)$ válido

$P_2(-1,0)$ válido

43.- $C:(-3,1)$

$R: 3$

$(x+3)^2 + (y-1)^2 = 3^2$

$x^2 + 6x + \cancel{9} + y^2 - 2y + 1 = \cancel{9}$

$x^2 + y^2 + 6x - 2y + 1 = 0$

$V(-3,1)$

$P(1,3)$

$(y-1)^2 = 4p(x+3)$

$(3-1)^2 = 4p(1+3)$

$\cancel{4} = 4p(\cancel{4})$

$p = \frac{1}{4}$

$(y-1)^2 = \cancel{4}(\frac{1}{\cancel{4}})(x+3)$

$y^2 - 2y + 1 = x + 3$

$y^2 - 2y - x - 2 = 0$

44.– $V_1(0,2)$

$V_2(4,2)$ $\}$ $2a=4$

$\boxed{a=2}$

Centro $(2,2)$

$2b=2$

$\boxed{b=1}$

$a^2=b^2+c^2$

$c^2=a^2-b^2$

$c^2=4-1$

$\boxed{c=\sqrt{3}}$

$$\frac{(x-2)^2}{4}+\frac{(y-2)^2}{1}=1$$

$$\frac{(x-2)^2+4(y-2)^2}{4}=1$$

$$x^2-4x+\cancel{4}+4(y^2-4x+4)=\cancel{4}$$

$$x^2-4x+4y^2-16x+16=0$$

$$x^2+4y^2-4x-16x+16=0$$

45.– Centro $(2,3)$

$2b=4$

$\boxed{b=2}$

$a=\sqrt{13}$

$$\frac{(x-2)^2}{13}+\frac{(y-3)^2}{4}=1$$

a) $P(4,5)$ $\quad \frac{(4-2)^2}{13}+\frac{(5-3)^2}{4}\neq 1$

b) $P(3,0)$ $\quad \frac{(3-2)^2}{13}+\frac{(0-3)^2}{4}\neq 1$

c) $P(2,1)$ $\quad \frac{(2-2)^2}{13}+\frac{(1-3)^2}{4}=1$ CORRECTO

d) $(5,3)$ $\quad \frac{(5-2)^2}{13}+\frac{(3-3)^2}{4}\neq 1$

e) $(4,\sqrt{10})$ $\quad \frac{(4-2)^2}{13}+\frac{(\sqrt{10}-3)^2}{4}\neq 1.$

46.- $3x^2-4y^2+16y-18=0$

$$3x^2+(-4y^2+16y- \ )=18$$

$$3x^2-4(y^2-4y+4)=18-16$$

$$3x^2-4(y-2)^2=2$$

$$\frac{3x^2}{2}-\frac{\cancel{4}^{2}(y-2)^2}{\cancel{2}}=\frac{\cancel{2}^{1}}{\cancel{2}}$$

$$\frac{(x-0)^2}{2/3}-\frac{(y-2)^2}{1/2}=1$$

HIPERBOLA

$C(0,2)$

b) CORRECTO

47.- $6y^2-4x^2+12y+16x-34=0$

$$(6y^2+12y)-(4x^2-16x \ )=34$$

$$6(y^2+2y+1)-4(x^2-4x+4)=34+6-16$$

$$6(y+1)^2-4(x-2)^2=24$$

$$\frac{\cancel{6}(y+1)^2}{\cancel{24}_{3}}-\frac{\cancel{4}^{1}(x-2)^2}{\cancel{24}_{6}}=\frac{\cancel{24}^{1}}{\cancel{24}}$$

$$\frac{(y+1)^2}{3}-\frac{(x-2)^2}{6}=1$$

$a^2=3 \Rightarrow a=\sqrt{3}$

$b^2=6 \Rightarrow b=\sqrt{6}$

$c = \sqrt{a^2 + b^2}$

$c = \sqrt{3+6}$

$\boxed{c = 3}$

centro $(2, -1)$

$e = \frac{c}{a} \Rightarrow \frac{3}{\sqrt{3}}$

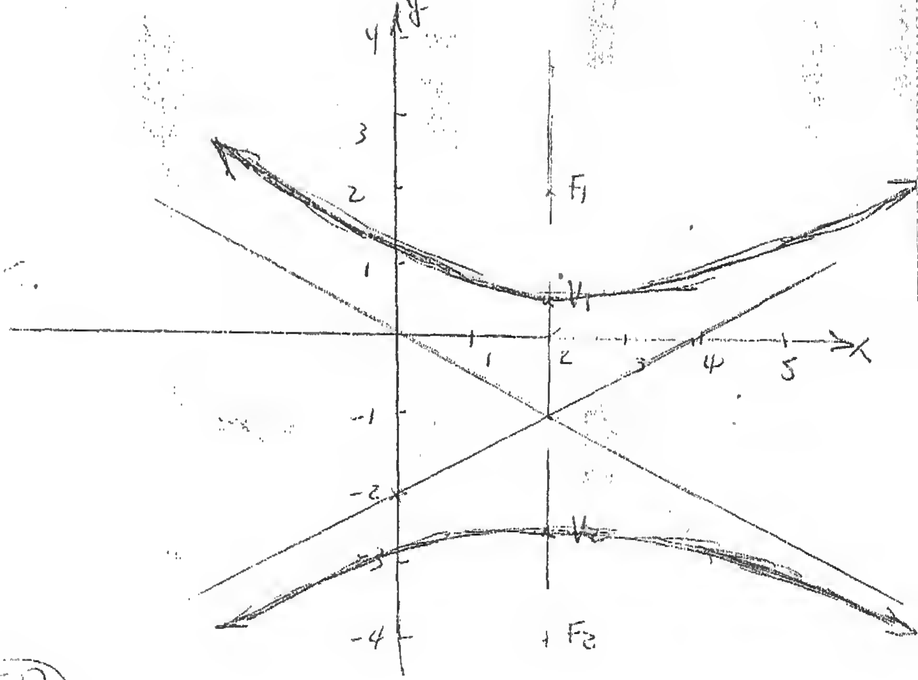
$e = \sqrt{3}$

$V_1 = (2, -1+\sqrt{3})$

$V_2 = (2, -1-\sqrt{3})$

$F_1 = (2, 2)$

$F_2 = (2, -4)$



asintota

$y - k = \pm \frac{a}{b}(x - h)$

$y + 1 = \pm \frac{\sqrt{3}}{\sqrt{6}}(x - 2)$

$y + 1 = + \frac{1}{2}(x - 2)$

$y = \frac{x}{2} - 1 - 1$

$\boxed{y = \frac{x}{2} - 2}$

$x = 0$, $y = -2$

$y = 0$, $x = 4$

$y = -\frac{1}{2}(x-2) - 1$

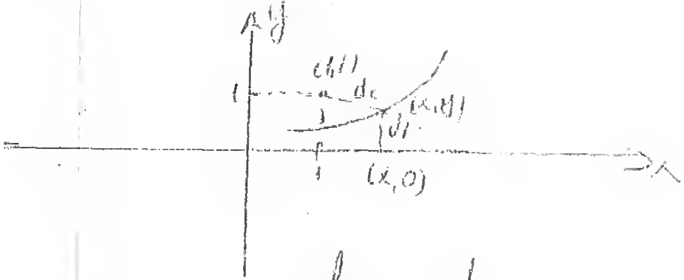
$y = -\frac{x}{2} + \cancel{1} \cancel{-1}$

$\boxed{y = -\frac{x}{2}}$

$x = 0$, $y = 0$

$x = 2$, $y = -1$

48.—



$$d_2 = 2\,d_1$$

$$\sqrt{(x-1)^2+(y-1)^2} = 2\sqrt{(x-x)^2+(y-0)^2}$$

$$\sqrt{(x-1)^2+(y-1)^2} = 2\sqrt{y^2}$$

$$\left(\sqrt{(x-1)^2+(y-1)^2}\right)^2 = (2y)^2$$

$$(x-1)^2+(y-1)^2 = 4y^2$$

$$x^2-2x+1+y^2-2y+1-4y^2=0$$

$$x^2-3y^2-2x-2y+2=0$$

$$3y^2+2y-x^2+2x=2$$

$$3\left(y^2+\frac{2}{3}y+\frac{1}{9}\right)-\left(x^2-2x+1\right)=2+\frac{1}{3}-1$$

$$3\left(y+\frac{1}{3}\right)^2-(x-1)^2=\frac{4}{3}$$

$$\frac{3\left(y+\frac{1}{3}\right)^2}{4/3}-\frac{(x-1)^2}{4/3}=\frac{\cancel{4/3}}{\cancel{4/3}}$$

$$\frac{\left(y+\frac{1}{3}\right)^2}{4/9}-\frac{(x-1)^2}{4/3}=1$$

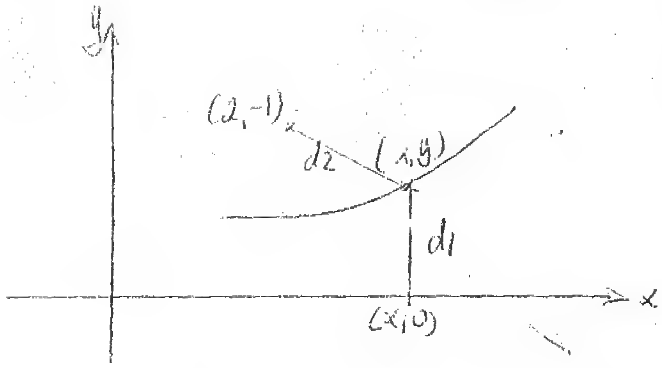
⇓

hipérbola

centro$(1, -1/3)$

b) CORRECTO.

49.-



$$d_1 = 3d_2$$

$$\sqrt{(x-x)^2+(y-0)^2} = 3\sqrt{(x-2)^2+(y+1)^2}$$

$$\sqrt{y^2} = 3\sqrt{(x-2)^2+(y+1)^2}$$

$$(y)^2 = \left(3\sqrt{(x-2)^2+(y+1)^2}\right)^2$$

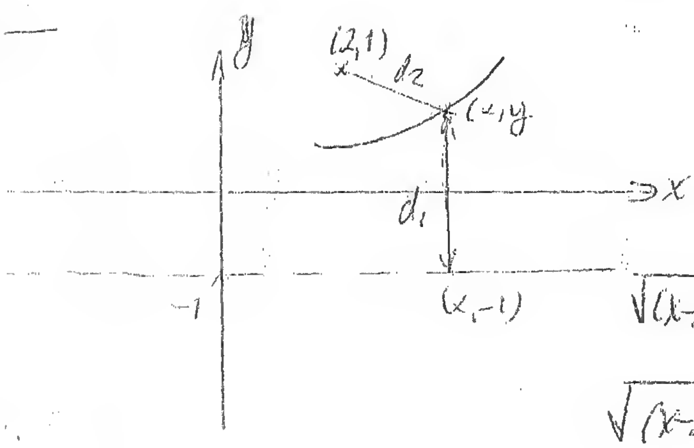
$$y^2 = 9\left[(x-2)^2+(y+1)^2\right]$$

$$y^2 = 9(x^2-4x+4+y^2+2y+1)$$

$$y^2 = 9x^2+9y^2-36x+18y+45$$

$$9x^2+8y^2-36x+18y+45=0$$

50.-



$$d_2 = 2d_1$$

$$\sqrt{(x-2)^2+(y-1)^2} = 2\sqrt{(y+1)^2+(x-x)^2}$$

$$\sqrt{(x-2)^2+(y-1)^2} = 2\sqrt{(y+1)^2}$$

$$\left(\sqrt{(x-2)^2+(y-1)^2}\right)^2 = (2(y+1))^2$$

$$(x-2)^2+(y-1)^2 = 4(y+1)^2$$

$$x^2-4x+4+y^2-2y+1=4(y^2+2y+1)$$

$$x^2-4x+y^2-2y+5=4y^2+8y+4$$

$$x^2-3y^2-4x-10y+1=0$$